Some homework solutions

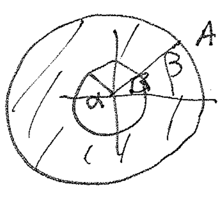
Page 129 #1. Remember – the important harmonic functions are:

$$\mathrm{Re}(z^n) \quad \mathrm{Im}(z^n)$$

$$e^x \cos y, \quad e^x \sin y,$$

and especially $\mathrm{Log}|z|$, $\mathrm{Arg}\, z$ (so long as the second is a function!)

In #1 we have a washer. So if in the w-plane the washer looks like



(so $u = A$ on $|w| = \beta$, $u = B$ on $|w| = \alpha$),

then

$$u = c \log|w| + d \quad (= c \log|w - 0| + d).$$

But here the washer is centered at $1+i$. So we think of $w = z-(1+i)$ and our function then is

$$u = c \log|z-(1+i)| + d,$$

and to find c and d, we have the boundary conditions

$$0 = c \log|1| + d,$$
$$10 = c \log|3| + d$$

(look at Fig 3.11 to see how to get this)

Then $d = c$, $c = \dfrac{10}{\log 3}$, and

$$w = \frac{10}{\log 3} |z-(1+i)|$$

P129 #2: Basic function:

$A \,\mathrm{Arg}\, w + B$

Now Fig 3.12 is 3/4 of the plane with 'center' at $1+i$. So we want to get to our basic situation. So the solution is

$$u = A \,\mathrm{Arg}\left(z-(1+i)\right)^{2/3} + B,$$

where

$$A \cdot \pi + B = 0$$
$$A \cdot 0 + B = 10$$