## TRAPEZOIDAL RULE

$$
\int_{a}^{b} f(x) d x \equiv \frac{\Delta x}{2}\left[f\left(x_{1}\right)+2 f\left(x_{2}\right)+2 f\left(x_{3}\right)+\cdots+2 f\left(x_{n}\right)+f\left(x_{n+1}\right)\right]
$$

where $a=x_{1}, x_{2}, x_{3}, \ldots, x_{n+1}=b$ subdivides $[a, b]$ into $n$ equal subintervals of length $\Delta x=\frac{b-a}{n}$.

## THE SECOND DERIVATIVE TEST

Suppose $f$ is a function of two variables $x$ and $y$, and that all the second-order partial derivatives are continuous. Let

$$
D=f_{x x} f_{y y}-\left(f_{x y}\right)^{2}
$$

and suppose $(a, b)$ is a critical point of $f$.

1. If $D(a, b)<0$, then $f$ has a saddle point at $(a, b)$,
2. If $D(a, b)>0$ and $f_{x x}(a, b)<0$, then $f$ has a relative maximum at $(a, b)$.
3. If $D(a, b)>0$ and $f_{x x}(a, b)>0$, then $f$ has a relative minimum at $(a, b)$.
4. If $D(a, b)=0$, the test is inconclusive.
