MA 22400 FORMULAS

CONSUMERS' AND PRODUCERS' SURPLUS

$$\text{CS} = \int_0^{q_0} D(q)dq - p_o q_0$$

$$\text{PS} = p_o q_0 - \int_0^{q_0} S(q)dq$$

TRAPEZOIDAL RULE

$$\int_a^b f(x)dx \equiv \frac{\Delta x}{2}\Big[f(x_1) + 2f(x_2) + 2f(x_3) + \cdots + 2f(x_n) + f(x_{n+1})\Big],$$

where $a = x_1, x_2, x_3, \ldots, x_{n+1} = b$ subdivides $[a, b]$ into n equal subintervals of length $\Delta x = \dfrac{b-a}{n}$.

THE SECOND DERIVATIVE TEST

Suppose f is a function of two variables x and y, and that all the second-order partial derivatives are continuous. Let

$$D = f_{xx} f_{yy} - (f_{xy})^2$$

and suppose (a, b) is a critical point of f.

1. If $D(a, b) < 0$, then f has a saddle point at (a, b),
2. If $D(a, b) > 0$ and $f_{xx}(a, b) < 0$, then f has a relative maximum at (a, b).
3. If $D(a, b) > 0$ and $f_{xx}(a, b) > 0$, then f has a relative minimum at (a, b).
4. If $D(a, b) = 0$, the test is inconclusive.

LAGRANGE EQUATIONS

For the function $f(x, y)$ subject to the constraint $g(x, y) = k$, the Lagrange equations are

$$f_x = \lambda g_x \qquad f_y = \lambda g_y \qquad g(x, y) = k$$

LEAST-SQUARES LINE

The equation of the least-squares line for the n points (x_1,y_1), (x_2,y_2), ... , (x_n,y_n), is $y = mx + b$, where

$$m = \frac{n\sum xy - \sum x \sum y}{n\sum x^2 - \left(\sum x\right)^2} \qquad b = \frac{\sum x^2 \sum y - \sum x \sum xy}{n\sum x^2 - \left(\sum x\right)^2}$$

GEOMETRIC SERIES

If $0 < |r| < 1$, then

$$\sum_{n=0}^{\infty} ar^n = \frac{a}{1-r}$$

TAYLOR SERIES

The Taylor series of $f(x)$ about $x = a$ is the power series

$$\sum_{n=0}^{\infty} a_n (x-a)^n \quad \text{where} \quad a_n = \frac{f^{(n)}(a)}{n!}$$

Examples:

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}, \text{ for } -\infty < x < \infty; \qquad \ln x = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n}(x-1)^n, \text{ for } 0 < x \le 2$$