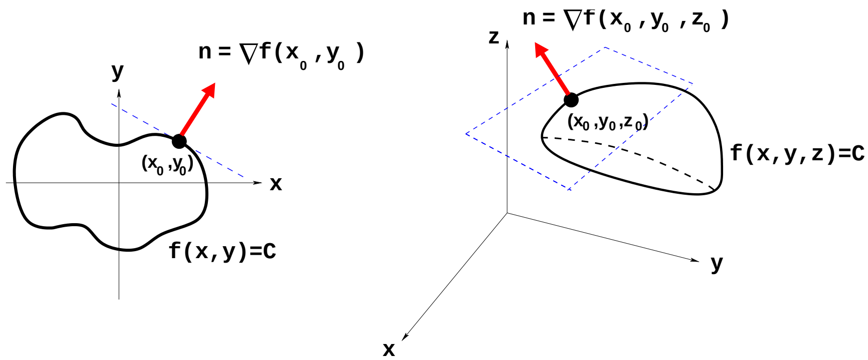
Study Guide # 2

1. Gradient vector for $f(x,y)$: $\nabla f(x,y) = \left\langle \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y} \right\rangle$, properties of gradients; gradient points in direction of maximum rate of increase of f; $\nabla f(x_0,y_0) \perp$ level curve $f(x,y) = C$ and, in the case of 3 variables, $\nabla f(x_0,y_0,z_0) \perp$ level surface $f(x,y,z) = C$:



2. Directional derivative of $f(x,y)$ at (x_0,y_0) in the direction $\vec{\mathbf{u}}$: $D_{\vec{\mathbf{u}}} f(x_0,y_0) = \nabla f(x_0,y_0) \cdot \vec{\mathbf{u}}$, where $\vec{\mathbf{u}}$ must be a *unit* vector; tangent planes to level surfaces $f(x,y,z) = C$ (a normal vector at (x_0,y_0,z_0) is $\vec{\mathbf{n}} = \nabla f(x_0,y_0,z_0)$).

3. Relative/local extrema; critical points ($\nabla f = \vec{\mathbf{0}}$ or ∇f does not exist); 2^{nd} Derivatives Test: A critical points is a local min if $D = f_{xx} f_{yy} - f_{xy}^2 > 0$ and $f_{xx} > 0$, local max if $D > 0$ and $f_{xx} < 0$, saddle if $D < 0$; absolute extrema; Max-Min Problems; **Lagrange Multipliers:** *Extremize* $f(\vec{\mathbf{x}})$ subject to a constraint $g(\vec{\mathbf{x}}) = C$, solve the system: $\nabla f = \lambda \nabla g$ and $g(\vec{\mathbf{x}}) = C$.

4. Double integrals; Midpoint Rule for rectangle : $\displaystyle\iint_R f(x,y)\,dA \approx \sum_{i=1}^{m} \sum_{j=1}^{n} f(\overline{x_i}, \overline{y_j})\ \Delta A$;

5. Type I region $D : \left\{ \begin{array}{c} g_1(x) \le y \le g_2(x) \\ a \le x \le b \end{array} \right.$; Type II region $D : \left\{ \begin{array}{c} h_1(y) \le x \le h_2(y) \\ c \le y \le d \end{array} \right.$;

iterated integrals over Type I and II regions: $\displaystyle\iint_D f(x,y)\,dA = \int_a^b \int_{g_1(x)}^{g_2(x)} f(x,y)\,dy\,dx$ and $\displaystyle\iint_D f(x,y)\,dA = \int_c^d \int_{h_1(y)}^{h_2(y)} f(x,y)\,dx\,dy$, respectively; Reversing Order of Integration (regions that are both Type I and Type II); properties of double integrals.

6. Integral inequalities: $mA \le \displaystyle\iint_D f(x,y)\,dA \le MA$, where $A =$ area of D and $m \le f(x,y) \le M$ on D.

7. Change of Variables Formula in Polar Coordinates: if $D : \begin{cases} h_1(\theta) \le r \le h_2(\theta) \\ \alpha \le \theta \le \beta \end{cases}$, then

$$\iint_D f(x,y)\,dA = \int_\alpha^\beta \int_{h_1(\theta)}^{h_2(\theta)} f(r\cos\theta,\ r\sin\theta)\,\underset{\uparrow}{r}\,dr\,d\theta.$$

8. Applications of double integrals:

(a) Area of region D is $A(D) = \iint_D dA$

(b) Volume of solid under graph of $z = f(x,y)$, where $f(x,y) \ge 0$, is $V = \iint_D f(x,y)\,dA$

(c) Mass of D is $m = \iint_D \rho(x,y)\,dA$, where $\rho(x,y) =$ density (per unit area); sometimes write $m = \iint_D dm$, where $dm = \rho(x,y)\,dA$.

(d) Moment about the x-axis $M_x = \iint_D y\,\rho(x,y)\,dA$; moment about the y-axis $M_y = \iint_D x\,\rho(x,y)\,dA$.

(e) Center of mass $(\overline{x}, \overline{y})$, where $\overline{x} = \dfrac{M_y}{m} = \dfrac{\iint_D x\,\rho(x,y)\,dA}{\iint_D \rho(x,y)\,dA}$, $\overline{y} = \dfrac{M_x}{m} = \dfrac{\iint_D y\,\rho(x,y)\,dA}{\iint_D \rho(x,y)\,dA}$

<u>Remark</u>: centroid = center of mass when density is constant (this is useful).

9. Elementary solids $E \subset \mathbb{R}^3$ of Type 1, Type 2, Type 3; triple integrals over solids E: $\iiint_E f(x,y,z)\ dV$; volume of solid E is $V(E) = \iiint_E dV$; applications of triple integrals, mass of a solid, moments about the coordinate planes M_{xy}, M_{xz}, M_{yz}, center of mass of a solid $(\overline{x}, \overline{y}, \overline{z})$.
