Summer Lesson 28 MA 15200, Appendix I Sections 5.5 and 5.6

You are familiar with the simple interest formula, $I = prt$. However, in many accounts the interest is left in the account and earns interest also. We say the account earns **compound interest.**

For example: Suppose Bob invests $100 at 10% simple interest. At the end of 1 year, Bob has earned $I = 100(.10)(1) = \$10$. He now has $110. ($A = P + Prt$ or $A = P(1+rt)$)
At the end of the 2nd year, Bob has earned $I = 110(.10)(1) = \$11$. He now has $121.
At the end of the 3rd year, Bob has earned $I = 121(.10)(1) = \$12.10$. He has a total of $143.10. I'm sure you get the idea of what is happening.

Formula for Compound Interest with **<u>Annual</u>** compound interest:
$S = P(1+r)^t$, where P is the initial investment (principal),
t is the number of years, r is the annual interest rate, and S is the future value or final value.

<u>Ex 1:</u> Assume that $1500 is deposited in an account in which interest is compounded annually at a rate of 6%. Find the accumulated amount after 5 years.

<u>Ex 2:</u> Assume that $1500 is deposited in an account in which interest is compounded annually for 5 years. Find the accumulated amount, if the interest rate is 8 ½ %.

Many banks or financial institutions figure interest more often than once a year; quarterly monthly, semiannually, daily, etc. For example, if the annual rate or **nominal rate** is 12% and interest is compounded quarterly, that is equivalent to 3% every 3 months. 3% is called the **periodic rate**.

Formula for Periodic Rate: Periodic Rate = $\dfrac{\text{annual rate}}{\text{number of periods per year}}$

$i = \dfrac{r}{k}$, where r is annual interest rate, k is the number of times interest is paid each year, and i is the periodic rate.

<u>Ex 3:</u> Find the periodic rate in each example.

a) annual rate: 10%, compounded quarterly

b) annual rate: 3.6%, compounded monthly

Compound Interest Formula (Future Value of an Investment):
Let P be principal earning interest compounded k times per year for n years at an annual rate of r. Then, the final or future value will be

* $S = P(1+i)^{kt}$, where $i = \dfrac{r}{k}$

*Earlier in the semester, when we had this formula, it was written $A = P\left(1+\dfrac{r}{n}\right)^{nt}$, where A is the final amount, P is principal or beginning amount, r is annual interest rate, n is number of compounding periods a year, and t is time in years. This lesson the formula is simply written differently.

<u>Ex 4:</u> Assume that $1500 is deposited in an account in which interest is compounded monthly at an annual rate of 6%.

a) Find the accumulated amount after 8 years.

b) How much interest was earned during the 8 years?

Financial institutions are required to provide customers with the **effective rate of interest**, that rate at which, if compounded annually, would provide the same yield as the plan where interest is compounded more frequently.
In other words: For what interest rate is $P(1+r)^n = P(1+i)^{kt}$? If this equation is solved for *r*, we get the following formula.

Effective Rate of Interest: The effective rate of interest *R* for an account paying a nominal or annual interest rate *r*, compounded *k* times per year is….

$$E = (1+i)^k - 1, \text{ where } i \text{ (the periodic rate)} = \frac{r}{k}.$$

Ex 5: Find the effective rate of interest given the annual rate and the compounding frequency.

a) $r = 9\%$, $k = 2$

b) $r = 11\ ½\ \%$, $k = 4$

We studied the continuously compounded formula for an investment earlier in lesson 27. It was given as $A = Pe^{rt}$. For this lesson, it will be written $S = Pe^{rt}$, where *S* is the final amount of the investment.

Ex 6: Jake has the option of investing \$1200 at an annual rate of 4.8% compounded quarterly or at an annual rate of 4.6% compounded continuously. Which would result in the best investment in a year's time?

Often people need to know what amount must be invested (principal) in order to end up with a certain future or final value.

$S = P(1+i)^{kt}$

Solve the formula above for *P*.

$S = P(1+i)^{kt}$

Divide both sides by $(1+i)^{kt}$

$\frac{S}{(1+i)^{kt}} = P$

Since an exponent is the opposite when moved from denominator to numerator...

$S(1+i)^{-kt} = P$

This is the formula for present value, when you need to find what principal or investment now would result in a given final value.

Present Value Formula: The present value *P* that must be deposited now in order to result in a future value *S*, in *t* years is given by…

$P = S(1+i)^{-kt}$, where interest is compounded k times per year at an annual rate r, and $i = \frac{r}{k}$

Ex 7: Find the present value of $15,000 due in 8 years, at the annual rate of 11% and compounding daily. **Note: Compounded daily is counted as 365 times a year.**

Applied Problems

Ex 8: After the birth of their first granddaughter, the Fields deposited $8000 in a savings account paying 6% interest, compounded quarterly. How must will be available for this granddaughter for college, when she turns 18? How much interest was earned during that time?

Ex 9: A financial institution offers two different accounts. The NOW account has a 7.2% annual interest rate, compounded quarterly and the Money Market account is 6.9% annual rate, compounded monthly. Compare the effective interest rates for the two accounts.

Ex 10: A businessman estimates the computer he needs for his business that he plans to buy in 18 months will cost $5500. To meet this cost, how much should he deposit now in an account paying 5.75% compounded monthly?

Any of you who are familiar with financial plans or retirement investments know about annuities.
An annuity is a plan involving payments made at regular intervals. An **ordinary annuity** is one in which the payments are made at the end of each time interval. We will be discussing ordinary annuities.

The **future value** of an annuity is the **sum of all the payments and the interest those payments earn.** Suppose a person makes a payment of $500 every 3 months for 20 years. The amount of money in that account at the end of the 20 years is the future value of the annuity.

Formula for the Future Value of an Annuity: The future value *S* of an ordinary annuity with deposits or payments of *R* made regularly *k* times per year for *t* years, with interest compounded *k* times per year at an annual rate *r*, is given by…

$$S = R\left[\frac{(1+i)^{kt}-1}{i}\right], \text{ where } i = \frac{r}{k}$$

Note: The frequency of compounding per year always equals the types of payments. For example, if a person makes monthly payments, then the interest is compounded monthly. If a person makes payments every 6 monthly, then the interest is compounded semiannually.

Ex 11: Assume that $1200 is deposited at the end of each year into an account in which interest is compounded annually at a rate of 5%. Find the accumulated amount (future value) after 6 years.

Ex 12: Assume that $1200 is deposited every 3 months into an account in which interest is compounded quarterly 3 ½ % annual interest. Find the accumulated amount (future value) after 8 years.

Ex 13: Assume that \$1200 is deposited monthly in an account in which interest is compounded monthly at an annual rate of 8%. Find the accumulated amount (future value) after 12 years.