1) Given ΔABC with $\angle C = 90^\circ$, $b = 5$, and $a = 5\sqrt{3}$, find the exact values of β and c.

A $\beta = 60^\circ,\ c = 5\sqrt{2}$

B $\beta = 30^\circ,\ c = 5\sqrt{2}$

C $\beta = 60^\circ,\ c = 10$

D $\beta = 30^\circ,\ c = 10$

E $\beta = 45^\circ,\ c = 10$

$\tan\beta = \frac{opp}{adj} = \frac{5}{5\sqrt{3}} = \frac{1}{\sqrt{3}}$

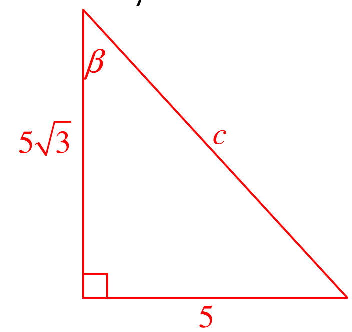
$\beta = 30^\circ$

$(5\sqrt{3})^2 + 5^2 = c^2$ or $\sin 30^\circ = \frac{5}{c}$

$25(3) + 25 = c^2$ $\quad c = \frac{5}{\sin 30^\circ}$

$100 = c^2$ $\quad c = 5 \div \frac{1}{2}$

$10 = c$



2) A taut wire is attached between a vertical post that is 6 feet off the level ground and a vertical pole. The angle of elevation between the top of the post and the wire is 67°. Approximate the height of the pole if the length of the wire is 39 feet. Round to the nearest tenth of a foot.

A 21.2 *ft.*

B 43.4 *ft.*

C 46.9 *ft.*

D 29.0 *ft.*

E None of the above.

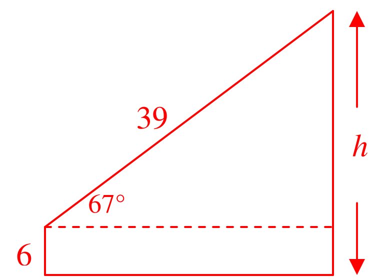
$\sin 67^\circ = \frac{h-6}{39}$

$39\sin 67^\circ = h - 6$

$39\sin 67^\circ + 6 = h$

$41.9 \approx h$

None of the Above.



3) A small ship leaves a port and travels in the direction **N 33° W** for 3 hours at 15 miles per hour. The captain hears a distress call from a sinking boat and changes direction to **S 57° W** in order to rescue the boat's passengers. After traveling 24 miles, the ship reaches the boat and rescues its passengers just before the boat sinks. What is the distance for the ship to return back to port with the passengers and how long is the trip to port, if the ship travels at 20 miles per hour? Round the distance to the nearest mile, if necessary. Convert any decimal part of an hour to minutes.

A 35.3 mi., 1 hr. 46 min.

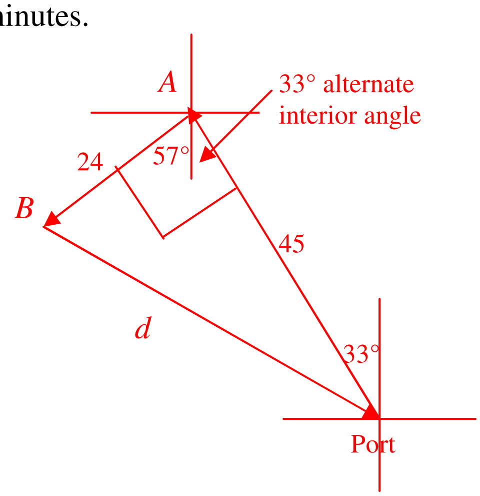
B 35.3 mi., 1 hr. 14 min.

C 51.0 mi., 2 hr. 33 min.

D 55.0 mi., 2 hr. 46 min.

E 51.0 mi., 2 hr. 55 min.

Since the angle at *A* is right, the Pythagorean Thm. can be used to find *d*. Then time can be determined by dividing *d* by *r*.



$$d^2 = 24^2 + 45^2 \qquad t = \frac{d}{r} = \frac{51}{20}$$

$$d^2 = 2604 \qquad t = 2.55$$

$$d = 51 \qquad t = 2\text{ hr.} + 0.55\text{ hr.}\left(\frac{60\text{ min.}}{1\text{ hr.}}\right)$$

$$t = 2\text{ hr. } 33\text{ min.}$$

4) A regular pentagon is inscribed inside a circle of radius 12 centimeters. Approximate the perimeter of the pentagon. Round to the nearest tenth of a centimeter. See the picture for help.

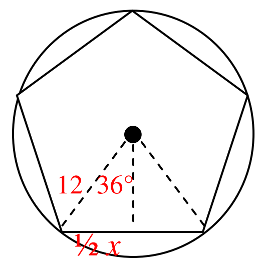
A $57.1\ cm$

B $70.5\ cm$

C $68.3\ cm$

D $35.3\ cm$

E $71.6\ cm$



Let x = length of each side

p = 5x

Each central angle is 72°. Drop a perpendicular bisector from a central angle to a side. A right triangle is formed with the bisector and ½ a side as legs and the radius of 12 as hypotenuse. The angle opposite ½ x is 36°.

$$\sin 36^\circ = \frac{\frac{1}{2}x}{12} = \frac{x}{24}$$

$$24\sin 36^\circ = x$$

$$p = 5(24\sin 36^\circ)$$

$$p = 120\sin 36^\circ$$

$$p \approx 70.5$$

5) Find all solutions of the equation below using n as an arbitrary integer.

$$\cot\left(\frac{1}{2}x\right) = -1$$

A $x = \frac{3\pi}{4} + \pi n$

B $x = \frac{3\pi}{2} + \pi n$

C $x = \frac{3\pi}{8} + \frac{\pi}{2} n$

D $x = \frac{\pi}{2} + 2\pi n$

E $x = \frac{3\pi}{2} + 2\pi n$

$$\cot\left(\frac{1}{2}x\right) = -1 \rightarrow \tan\left(\frac{1}{2}x\right) = -1$$

Tangent is negative in Q II and IV; the period between is π. The reference angle is $\frac{\pi}{4}$; in Q II $\frac{3\pi}{4}$. Add $\pi = \frac{7\pi}{4}$

$$\frac{1}{2}x = \frac{3\pi}{4} + \pi n$$

$$x = 2\left(\frac{3\pi}{4} + \pi n\right)$$

$$x = \frac{3\pi}{2} + 2\pi n$$

6) Find **all solutions in the interval [0, 2π)** for the equation below.

$$2\sin^2 t - \sin t - 1 = 0$$

A $t = \frac{\pi}{2}, \frac{7\pi}{6}, \frac{11\pi}{6}$

B $t = \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}$

C $t = 0, \pi, \frac{4\pi}{3}, \frac{5\pi}{3}$

D $t = \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{3\pi}{2}$

E $t = \frac{7\pi}{6}, \frac{11\pi}{6}$

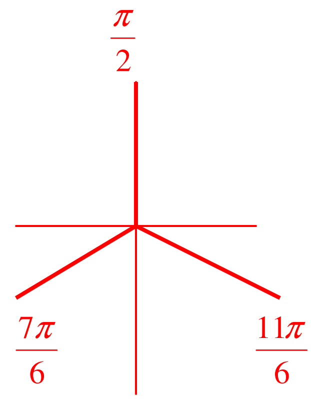
$(2\sin t + 1)(\sin t - 1) = 0$

$\sin t = -\frac{1}{2}$ $\quad \sin t = 1$

QIII and *QIV* and + *y*-axis

$t_R = \frac{\pi}{6}$

$t = \frac{\pi}{2}, \frac{7\pi}{6}, \frac{11\pi}{6}$



7) Which of the following is(are) equivalent to $\cos(u)$ if $u = \alpha + \beta$?

I	$\sin\left(\frac{\pi}{2} - u\right)$
II	$\cos^2\left(\frac{u}{2}\right) - \sin^2\left(\frac{u}{2}\right)$
III	$\cos\alpha\cos\beta - \sin\alpha\sin\beta$

A I and II only

B II and III only

C I and III only

D I, II, and III

E None are equivalent to $\cos(u)$.

I is true using a cofunction formula.

$\cos u = \sin\left(\frac{\pi}{2} - u\right)$

II is true using a double angle formula.

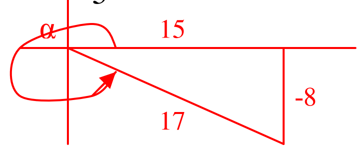
$\cos(2u) = \cos^2 u - \sin^2 u$ so

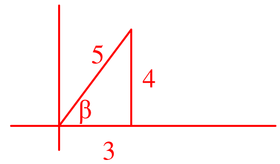
$\cos(u) = \cos^2\left(\frac{u}{2}\right) - \sin^2\left(\frac{u}{2}\right)$

III is true using the subtraction formula for the cosine function.

$\cos u = \cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

8) If $\tan\alpha = \frac{-8}{15}$ and $\cos\beta = \frac{3}{5}$ for a **QIV angle α and a QI angle β**, find the exact value of $\tan(\alpha+\beta)$.





A $\frac{36}{77}$

B $\frac{-12}{11}$

C $\frac{-84}{13}$

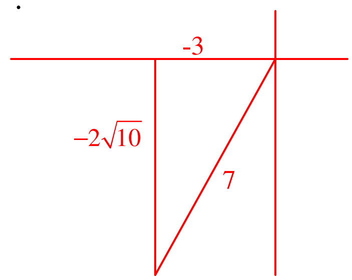
D $\frac{36}{13}$

E $\frac{12}{77}$

$$\tan(\alpha+\beta) = \frac{\tan\alpha + \tan\beta}{1-\tan\alpha\tan\beta} = \frac{\left(\frac{-8}{15}\right)+\left(\frac{4}{3}\right)}{1-\left(\frac{-8}{15}\right)\left(\frac{4}{3}\right)}$$

$$= \frac{\left(\frac{-8}{15}\right)+\left(\frac{20}{15}\right)}{1-\left(\frac{-32}{45}\right)} = \frac{\frac{4}{5}}{\frac{77}{45}} = \frac{4}{5}\cdot\frac{45}{77} = \frac{36}{77}$$

9) Find the exact value of $\sin(2\theta)$ if $\sec\theta = \frac{-7}{3}$ and $180^\circ < \theta < 270^\circ$.



A $\frac{12\sqrt{10}}{49}$

B $\frac{21}{49}$

C $\frac{-2\sqrt{10}}{49}$

D $\frac{\sqrt{10}}{49}$

E $\frac{-21}{29}$

$$\sec\theta = \frac{-7}{3} \rightarrow \cos\theta = \frac{-3}{7}$$
$$7^2 = (opp)^2 + 3^2$$
$$40 = (opp)^2$$
$$\sqrt{40} = 2\sqrt{10} = opp$$

$$\sin(2\theta) = 2\sin\theta\cos\theta$$
$$= 2\left(\frac{-2\sqrt{10}}{7}\right)\left(\frac{-3}{7}\right)$$
$$= \frac{12\sqrt{10}}{49}$$

10) Which statement is **false**?

A $\cos(45^\circ) = \cos(30^\circ)\cos(15^\circ) - \sin(30^\circ)\text{si}$

B $\tan\left(\frac{2\pi}{3}\right) = \frac{2\tan\left(\frac{\pi}{3}\right)}{1-\tan^2\left(\frac{\pi}{3}\right)}$

C $\cos\theta\left(\cos^{-1}\frac{\sqrt{3}}{2}\right) = \frac{-\sqrt{3}}{2}$

D $\cot(40^\circ 5' 12'') = \tan(49^\circ 54' 48'')$

E $\sin(\pi - \theta) = \sin\theta$

A is true (using the addition formula for the cosine function).

$\cos(45^\circ) = \cos(30^\circ + 15^\circ)$

$= \cos(30^\circ)\cos(15^\circ) - \sin(30^\circ)\sin(15^\circ)$

B is true (using the double angle formula for tangent function).

$\tan\left(\frac{2\pi}{3}\right) = \tan\left(2\left(\frac{\pi}{3}\right)\right) = \frac{2\tan\left(\frac{\pi}{3}\right)}{1-\tan^2\left(\frac{\pi}{3}\right)}$

C is false.

$\cos\left(\cos^{-1}\frac{\sqrt{3}}{2}\right) = \cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$ (not $-$)

D is true (using cofunction formula).

$\cot(40^\circ 5' 12'') = \tan(\text{complement})$

$= \tan(90^\circ - 40^\circ 5' 12'')$

$= \tan(89^\circ 59' 60'' - 40^\circ 5' 12'') = \tan(49^\circ 54' 48'')$

E is true (using subtraction formula for the sine function).

$\sin(\pi - \theta) = \sin\pi\cos\theta - \cos\pi\sin\theta$

$= 0(\cos\theta) - \cos\pi(-1)$

$= 0 + \sin\theta = \sin\theta$

Begin 'inside'. The cosine is negative in Q II, reference angle is 30°. The 'inside' is finding a value. Then, an angle is returned in **Q IV with a negative** measurement, since a negative sine value returns an angle in Q IV in the inverse sine function. The reference angle is 60°.

$\arcsin\left[\cos\left(\frac{7\pi}{6}\right)\right] = \arcsin\left[-\frac{\sqrt{3}}{2}\right] = -\frac{\pi}{3}$

11) Find the exact value of $\arcsin\left[\cos\left(\frac{7\pi}{6}\right)\right]$.

A $\frac{4\pi}{3}$

B $\frac{\pi}{3}$

C $\frac{5\pi}{3}$

D $\frac{-\pi}{3}$

E $\frac{-\pi}{6}$

12) Use an inverse trigonometric function and the quadratic formula to approximate the solutions of the following equation in the interval **[0, 2π)** to four decimal places. **Check mode on your calculator.**

$$3\sin^2\theta + 4\sin\theta - 1 = 0$$

A $\theta = 12.4302$

B $\theta = 0.2169,\ 2.9246$

C $\theta = 3.3585,\ 6.0663$

D No solution

E $\theta = 0.2169,\ 6.0663$

Use the quadratic function with $x = \sin\theta$.

$$\sin\theta = \frac{-4 \pm \sqrt{4^2 - 4(3)(-1)}}{2(3)} = \frac{-4 \pm \sqrt{16+12}}{6}$$

$$= \frac{-4 \pm \sqrt{28}}{6}$$

$$\sin\theta = \frac{-4+\sqrt{28}}{6} \approx 0.21525 \qquad \sin\theta = \frac{-4-\sqrt{28}}{6} \approx -1.54858$$

Put calculator in radian mode.

$\theta_R = 0.216948$ Sine value above is impossible.

Sine is positive in Q I and II.

$\theta = 0.2169,\ \pi - 0.2169$

$\theta = 0.2169,\ 2.9246$

13) The expression below is equivalent to which of the following?

$$\sin\left(\frac{\pi}{2} - x\right)\sec x$$

A $\cot x$

B $\tan x$

C 0

D 1

E $\sec^2 x$

$\sin\left(\frac{\pi}{2} - x\right)\sec x = \cos x \sec x$ (using a cofunction formula)

$= \cos x\left(\frac{1}{\cos x}\right)$ (using a reciprocal identity)

$= 1$

14) A builder wishes to construct a ramp 15 feet long that rises to a height of 3.35 feet above level ground. Approximate the angle to the nearest minute that the ramp should make with the level ground.

A $77^\circ\ 6'$

B $12^\circ\ 35'$

C $77^\circ\ 24'$

D $33^\circ\ 14'$

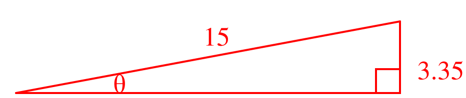
E $12^\circ\ 54'$

$\sin\theta = \frac{3.35}{15} = 0.2233333$

$\theta = 12.90489^\circ$

$\theta = 12^\circ + 0.90489^\circ\left(\frac{60'}{1^\circ}\right)$

$\theta = 12^\circ 54'$



15) If a projectile is fired from ground level with an initial velocity of v feet per second and at an angle of θ degrees with the horizontal, the range R of the projectile is given by $R = \frac{v^2}{16}\sin\theta\cos\theta$. If velocity if 90 feet per second, approximate **an angle** that results in a range of 200 feet. Round to the nearest tenth of a degree.

A 52.2°

B 63.9°

C 78.4°

D 23.3°

E 11.6°

$200 = \frac{90^2}{16}\sin\theta\cos\theta$

$200 = 506.25\sin\theta\cos\theta$

$0.395061728 = \sin\theta\cos\theta$

If the right side had a coefficient 2, it would equal $\sin 2\theta$. Multiply both sides by 2.

$0.790123457 = 2\sin\theta\cos\theta$

$0.790123457 = \sin 2\theta$

$(2\theta)_R = 52.19705^\circ$ Sine is positive in Q I and II.

$2\theta = 52.19705^\circ$ $\quad 2\theta = 180^\circ - 52.19705^\circ = 127.8029^\circ$

$\theta = 26.1^\circ$ $\quad \theta = 63.9^\circ$