Trigonometry Information

Angles

1. An angle has an initial side and a terminal side. A positive angle rotates counterclockwise and a negative angle rotates clockwise. Angles that *look* the same but only differ by number of rotations either direction are called **coterminal angles**. These angles may be found by adding or subtracting multiples of 360° or 2π.

2. A **radian** is the measurement of a central angle (angle in standard position in a circle) whose measure of the radius of the circle (sides of the angle) equals the length of the arc of the circle that the angle subtends. One radian is approximately 57.3°. If no label is present, the measurement is assumed to be radians.

$$\boxed{\begin{array}{l} 360^\circ = 2\pi \\ 180^\circ = \pi \\ 1^\circ = \frac{\pi}{180} \\ \left(\frac{180}{\pi}\right)^\circ = 1 \end{array}}$$

3. To convert degrees to radians: Multiply the degree measurement by $\frac{\pi}{180^\circ}$.

   To convert radians to degrees: Multiply the radian measurement by $\frac{180^\circ}{\pi}$.

4. Commonly used angles:

   $0^\circ = 0 \qquad 30^\circ = \frac{\pi}{6} \qquad 45^\circ = \frac{\pi}{4} \qquad 60^\circ = \frac{\pi}{3}$

   $90^\circ = \frac{\pi}{2} \qquad 180^\circ = \pi \qquad 270^\circ = \frac{3\pi}{2} \qquad 360^\circ = 2\pi$

5. The relationship between a central angle of θ **in radians** (angle in standard position), the length of the radius *r* of the circle (side), and the length of the subtended arc *s* is $s = r\theta$. The relationship between a central angle of θ **in radians**, the length of the radius *r* of the circle (side of the angle), and the area of the sector of the circle subtended by the angle is $A = \frac{1}{2}r^2\theta$.

6. There are six trigonometric functions, where an angle is degrees or radians is paired with a ratio (number). These functions are sine (sin), cosine (cos), tangent (tan), cosecant (csc), secant (sec), and cotangent (cot).

7. In a right triangle, the **six trigonometric functions** of an acute angle θ are found by these definitions where the side opposite the angle is opp, the side adjacent to the angle is adj, and the hypotenuse is hyp.

$$\boxed{\begin{array}{lll} \sin\theta = \dfrac{opp}{hyp} & \cos\theta = \dfrac{adj}{hyp} & \tan\theta = \dfrac{opp}{adj} \\ \csc\theta = \dfrac{hyp}{opp} & \sec\theta = \dfrac{hyp}{adj} & \cot\theta = \dfrac{adj}{opp} \end{array}}$$

The ‘word’ SOH-CAH-TOA can be used to help remember the 3 principle trigonometric relationships. Sine is opposite over hypotenuse, cosine is adjacent over hypotenuse, and tangent is opposite over adjacent.

8. Reciprocal Functions:

$$\sin\theta = \frac{1}{\csc\theta} \qquad \cos\theta = \frac{1}{\sec\theta} \qquad \tan\theta = \frac{1}{\cot\theta}$$

$$\csc\theta = \frac{1}{\sin\theta} \qquad \sec\theta = \frac{1}{\cos\theta} \qquad \cot\theta = \frac{1}{\tan\theta}$$

9. Pythagorean and Tangent/Cotangent Identities:

$$\sin^2\theta + \cos^2\theta = 1 \qquad \tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$1 + \cot^2\theta = \csc^2\theta \qquad \cot\theta = \frac{\cos\theta}{\sin\theta}$$

$$1 + \tan^2\theta = \sec^2\theta$$

10. Trigonometric values of commonly used angles:

	30° or $\frac{\pi}{6}$	45° or $\frac{\pi}{4}$	60° or $\frac{\pi}{3}$
$\sin\theta$	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$ or $\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$
$\cos\theta$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$ or $\frac{1}{\sqrt{2}}$	$\frac{1}{2}$
$\tan\theta$	$\frac{1}{\sqrt{3}}$ or $\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$

11. If a central angle θ is in standard position on a circle of radius *r*, the following definitions may be used to find the 6 trigonometric functions. $r = \sqrt{x^2 + y^2}$ and $P(x, y)$ is a point of the terminal side of the angle. (*r* is hyp., *x* is adj., and *y* is opp..

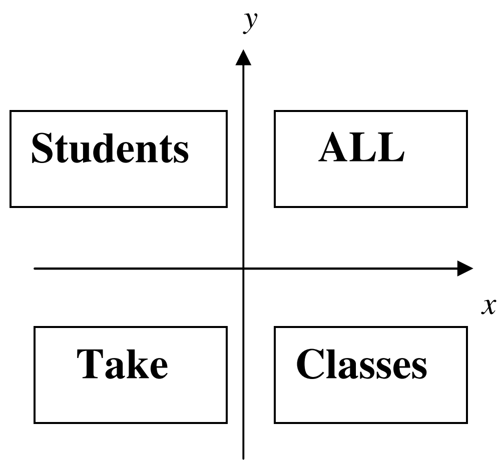
$$\sin\theta = \frac{y}{r} \qquad \cos\theta = \frac{x}{r} \qquad \tan\theta = \frac{y}{x}$$

$$\csc\theta = \frac{r}{y} \qquad \sec\theta = \frac{r}{x} \qquad \cot\theta = \frac{x}{y}$$

12. To find the trigonometric functions of the quadrantal angles, use the following points.

0° or 0	90° or $\frac{\pi}{2}$	180° or π	270° or $\frac{3\pi}{2}$
$(1,0)$	$(0,1)$	$(-1,0)$	$(0,-1)$

13. The following diagram can be used to help ‘remember’ in which quadrants a trigonometric function is positive and in which the function is negative. The saying ‘ALL STUDENTS TAKE CLASSES’ beginning in the first quadrant and continuing counterclockwise is used. ALL means all 6 functions are positive for angles in this quadrant. STUDENTS means the sine and its reciprocal are positive, all other functions are negative in that quadrant. TAKE means the tangent and its reciprocal are positive, all others negative in the quadrant. CLASSES means the cosine and its reciprocal are positive in that quadrant, all others are negative.



14. A **unit circle** is one of radius length 1. For any point on a unit circle, a point $P(x, y)$ that represents a real number *t* can be used to 6 trigonometric values associated with the real number *t*. The following are true: $\sin t = y$ and $\cos t = x$

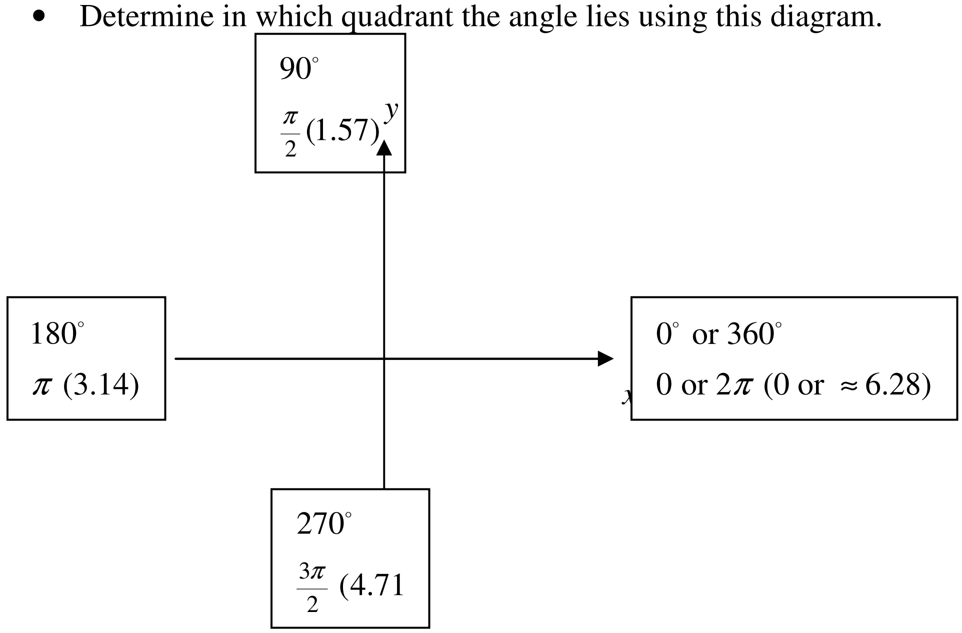
15. Using the *y* values from the points on a unit circle for all real numbers will lead to a graph of $y = \sin x$. Using the *x* values from the points on a unit circle for all real numbers will lead to a graph of $y = \cos x$. These graphs are curves with a period of 2π, domain $(-\infty, \infty)$, and range $[-1,1]$.

16. When finding trigonometric values for angles greater than 90° or π/2, a reference angle will help. A reference angle is the *acute* angle formed in a quadrant between the terminal side of the angle and the *x*-axis.

$QI: \quad \theta_R = \theta \qquad\qquad QII: \quad \theta_R = 180^\circ - \theta \text{ or } \pi - \theta$

$QIII: \quad \theta_R = \theta - 180^\circ \text{ or } \theta - \pi \qquad QIV: \quad \theta_R = 360^\circ - \theta \text{ or } 2\pi - \theta$

17. To find the trigonometric values of any angle θ use the following steps.
    - If necessary, find a coterminal angle between 0° and 360° or between 0 and 2π.
    - Determine in which quadrant the angle lies using this diagram.

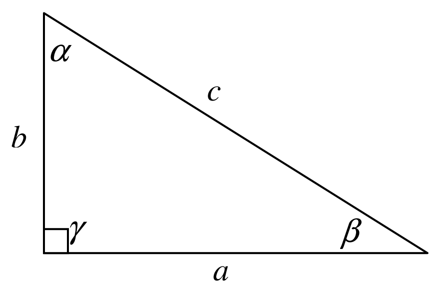


- Find the reference angle.
- Determine (based on the quadrant) if the value is + or -.
- If the function is csc, sec, or cot; write the value using sin, cos, or tan.
- Find an exact value, if the angle is one of the angles in the table on page 2. Otherwise, use a calculator to approximate.

18. To use a calculator to find the trigonometric values: Set the screen mode to either degrees or radians; use the measurement and either the sin, cos, or tan keys. If finding a csc, sec, or cot; use the reciprocal key.

19. Inverse functions: If $\sin\theta = k$, then $\sin^{-1} k = \theta$, and similarly for the remaining 5 trigonometric functions. To find an angle (inverse function) from a calculator: Set the mode in either degrees or radians, enter the number *k* and use the 2nd function key along with the sin, cos, or tan key. Using the sin or tan keys will return an angle in quadrants IV or I. Using the cos key will return an angle in quadrants I or II. To find the inverse of csc, sec, or cot: Find the reciprocal of the number *k* first, then use the 2nd key and a function key. Keep in mind, you might have to find a reference angle to find one of the inverse function values (angles) between 0 and 2π.

20. Solving a right triangle: In a right triangle, γ (gamma) represents the right angle and *c* represents the hypotenuse. The two acute angles are represented by α and β and *a* and *b* represent the sides opposite these angles.



To solve the right triangle for the missing angles and/or sides, use any of the following.

- The Pythagorean Theorem: $a^2 + b^2 = c^2$
- Sum of the angles of a triangle is 180°
- Definitions of $\sin\theta$, $\cos\theta$, or $\tan\theta$

21. Law of Sines: If any triangle ABC has angles α, β, and γ with sides *a*, *b*, and *c*; then----- $\dfrac{\sin\alpha}{a} = \dfrac{\sin\beta}{b} = \dfrac{\sin\gamma}{c}$.

22. Law of Cosines: If any triangle ABC has angles α, β, and γ with sides *a*, *b*, and *c*; then----- $a^2 = b^2 + c^2 - 2bc\cos\alpha$

$$b^2 = a^2 + c^2 - 2ac\cos\beta$$
$$c^2 = a^2 + b^2 - 2ab\cos\gamma$$

23. The law of sines and law of cosines can be used to find the missing parts of an oblique triangle ABC.