For the multiple choice questions, work will be graded as well as the answers. Show all steps neatly. For full credit, your answer must be derived from your work. If a wrong answer is chosen, partial credit may still be possible.

1) Given ΔABC with $\angle C$ or $\gamma = 53.3^\circ$, $a = 140$, and $c = 115$; approximate the measure(s) of **any possible** $\angle B$ or β.

A $\beta = 77.4^\circ$ *or* 102.6°

B $\beta = 49.3^\circ$ *or* 24.1°

C $\beta = 9.9^\circ$ *or* $63.5^\otimes$

D $\beta = 16.8^\circ$ *or* 56.6°

E None of the above.

$$\frac{\sin 53.3^\circ}{115} = \frac{\sin A}{140}$$

$$\sin A = \frac{140 \sin 53.3^\circ}{115} = 0.976074697$$

A or $\alpha = 77.441549^\circ$ or 102.55845°

B or $\beta = 49.3^\circ$ or 24.1°

There are two possible triangles.

2) An observer at point A, which is 1.3 miles from the base of a mountain, notices the angle of elevation to the peak of the mountain is 23°. The slope of the mountain from its base to the peak is 72°. (See the picture. Picture may not be to scale.) Approximate, to the nearest hundredth of a mile, the **height of the mountain**.

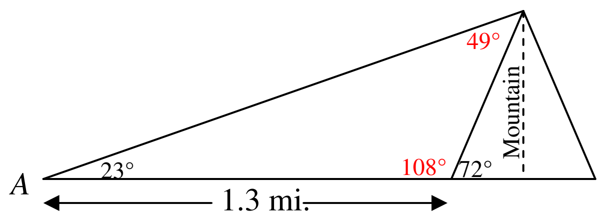
A 1.63 *mi.*

B 0.55 *mi.*

C 0.52 *mi.*

D 0.64 *mi.*

E 0.93 *mi.*



$$\frac{\sin 49^\circ}{1.3} = \frac{\sin 23^\circ}{a}$$

$$a = \frac{1.3 \sin 23^\circ}{\sin 49^\circ} = 0.673040969$$

$$\sin 72^\circ = \frac{height}{a}$$

$$(0.67304)\sin 72^\circ = height$$

$$height = 0.64$$

3) To the nearest tenth of a degree, approximate the measure of the **smallest** angle in a ΔABC with $a = 20$, $b = 18$, and $c = 25$.

A 52.4°

B 7.9°

C 33.2°

D 28.0°

E 45.5°

The smallest angle would be across from the smallest side, $b = 18$.

$b^2 = a^2 + c^2 - 2ac\cos\beta$

$18^2 = 20^2 + 25^2 - 2(20)(25)\cos\beta$

$324 = 400 + 625 - 1000\cos\beta$

$1000\cos\beta = 701$

$\cos\beta = 0.701$

$\beta = 45.5^\circ$

For free response problems, show all work or steps neatly. Circle your final answer(s).

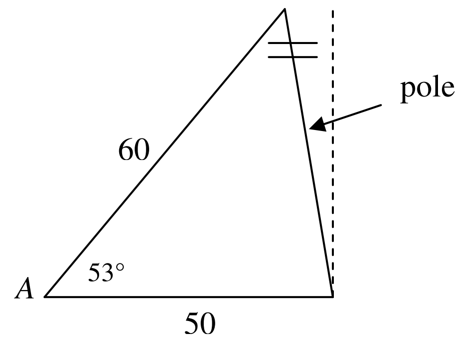
4) A telephone pole is leaning at an angle as seen in the picture below. The distance between point A and the bottom of the pole is 50 feet along level ground and the distance from point A to the top of the telephone pole is 60 feet. If the angle of elevation from A to the top of the telephone pole is 53°, how long is the telephone pole? Round to the nearest tenth of a foot.

$L^2 = 50^2 + 60^2 - 2(50)(60)\cos 53^\circ$

$L^2 = 6100 - 6000\cos 53^\circ$

$L^2 = 2489.10986$

$L = 49.9$ ft.



5) A quarterback releases a football with a speed of 45 feet per second at an angle of 33° with the horizontal. What are the components of the vector that represents this force? Approximate each to two decimal places and write using vector notation ($\langle\ \rangle$).

Use the formulas: $x = \|v\|\cos\theta$ and $y = \|v\|\sin\theta$

$\langle 45\cos 33^\circ, 45\sin 33^\circ \rangle$

$\langle 37.73, 24.51 \rangle$

6) Find a **unit vector** in the **opposite** direction as $\left\langle \frac{18}{5}, \frac{24}{5} \right\rangle$. Which statement is true?

A The x component is $\frac{3}{5}$.

B The y component is $-\frac{3}{5}$.

C The y component is $-\frac{4}{5}$.

D The x component is $-\frac{5}{3}$.

E The x component is $\frac{4}{5}$.

$$\left\|\left\langle \frac{18}{5}, \frac{24}{5} \right\rangle\right\| = \sqrt{\left(\frac{18}{5}\right)^2 + \left(\frac{24}{5}\right)^2} = \sqrt{\frac{324}{25} + \frac{576}{25}} = \sqrt{\frac{900}{25}} = \sqrt{36} = 6$$

$$-u = -\frac{1}{\left\|\left\langle \frac{18}{5}, \frac{24}{5} \right\rangle\right\|}\left\langle \frac{18}{5}, \frac{24}{5} \right\rangle = -\frac{1}{6}\left\langle \frac{18}{5}, \frac{24}{5} \right\rangle = \left\langle -\frac{3}{5}, -\frac{4}{5} \right\rangle$$

$y = -\frac{4}{5}$

7) Given vectors $a = 3i - 2j$ and $b = -6i + j$, in which quadrant would vector $3a + 6b$ be found?

A Q I
B Q II
C Q III
D Q IV
E None of the above.

$$3a + 6b = 3(3i - 2j) + 6(-6i + j)$$
$$= (9i - 6j) + (-36i + 6j)$$
$$= -27i + 0j \text{ or } -27i$$

This vector is found on the negative *y*-axis, so is not in a quadrant.

8) Which of the following vectors is orthogonal with the vector $\langle 3,-7\rangle$?

A $\langle 21,9\rangle$
B $\langle 14,-6\rangle$
C $\langle -7,3\rangle$
D $\langle -21,9\rangle$
E $\left\langle \frac{7}{2},-\frac{3}{2}\right\rangle$

$a \cdot b = 0$

$\langle 14,-6\rangle \cdot \langle 3,-7\rangle = 42 + 42 = 84$ *not*

$\langle -7,3\rangle \cdot \langle 3,-7\rangle = -21 - 21 = -42$ *not*

$\langle -21,9\rangle \cdot \langle 3,-7\rangle = -63 - 63 = -126$ *not*

$\langle 21,9\rangle \cdot \langle 3,-7\rangle = 63 - 63 = 0$ *yes*

$\left\langle \frac{7}{2},-\frac{3}{2}\right\rangle \cdot \langle 3,-7\rangle = \frac{21}{2} + \frac{21}{2} = 21$ *not*

9) Find the angle between the two vectors $r = -4i - 12j$ and $s = 6i - 8j$. Round to the nearest tenth of a degree.

$$r \cdot s = (-4)(6) + (-12)(-8) = -24 + 96 = 72$$
$$\|r\| = \sqrt{(-4)^2 + (-12)^2} = \sqrt{160}$$
$$\|s\| = \sqrt{6^2 + (-8)^2} = \sqrt{100} = 10$$
$$\cos\theta = \frac{72}{10\sqrt{160}} = \frac{9}{5\sqrt{10}} \approx 0.569209979$$
$$\theta = 55.3^\circ$$

10) Which of the following statements is/are true about the graph of the function $g(x)=\dfrac{4x^2-16}{x^2+2x-8}$?

I	A horizontal asymptote is $y=4$.
II	A vertical asymptotes is $x=4$.
III	An x-intercept is 2 or $(2,0)$.

A I and III only
B II and III only
C I only
D I, II, and III
E III only

$$g(x)=\frac{4(x^2-4)}{(x+4)(x-2)}=\frac{4(x+2)(x-2)}{(x+4)(x-2)}$$

$g(x)$ is the same as $f(x)=\dfrac{4(x+2)}{(x+4)}$ except for a hole when $x=2$

$\dfrac{4x}{x}$ horizontal asymptote is $y=4$ I is true.

$x+4=0\rightarrow x=-4$ for vertical asymptote II is false.

Numerator $=0$ $x+2=0\rightarrow x=-2$ $(-2,0)$ is x-intercept. III is false.

11) Find a rational function that satisfies these conditions. Use function notation, $f(x)$, when writing your answer. You may leave the numerator and denominator factored, if you wish.

vertical asymptotes: $x=3$, $x=-2$
horizontal asymptote: $y=0$
x-intercept: 2 or (2, 0)
$f(1)=1$

Vertical asymptotes give factors of the denominator.
The x-intercept gives a factor of the numerator.
The horizontal asymptote means the leading exponent of numerator is less than the leading exponent of denominator.
We do not know if there is a coefficient in the numerator other than 1. Call it a.

$$f(x)=\frac{a(x-2)}{(x-3)(x+2)}$$

$$f(1)=1\rightarrow 1=\frac{a(1-2)}{(1-3)(1+2)}=\frac{-a}{(-2)(3)}$$

$$1=\frac{-a}{-6}\rightarrow a=6$$

$$f(x)=\frac{6(x-2)}{(x-3)(x+2)}$$

12) Find the smallest **positive angle θ from the positive *x*-axis** to the vector v that is represented by $\langle 12,-5\rangle$. Round to the nearest tenth of a degree.

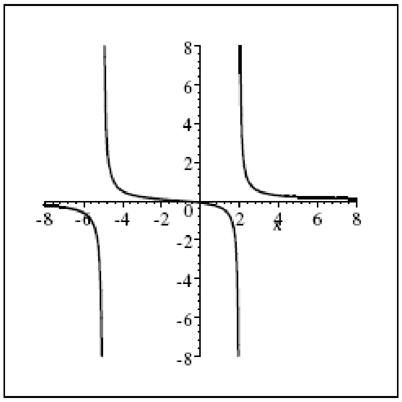
θ is in QIV.

$$\tan\theta=\frac{-5}{12}\approx 0.41666667$$

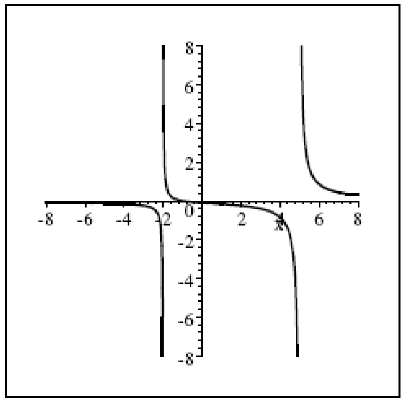
$$\theta_R=22.6199^\circ$$

$$\theta=360^\circ-22.6^\circ=337.4^\circ$$

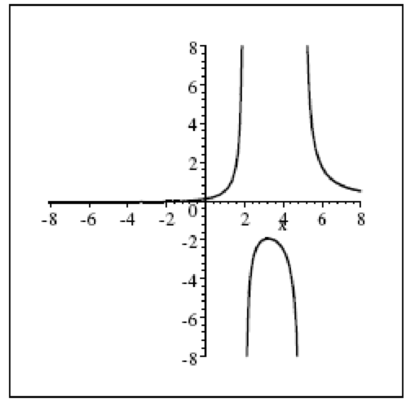
13) Which of the following graphs best approximates the graph for $f(x)=\dfrac{x+1}{x^2+3x-10}$?



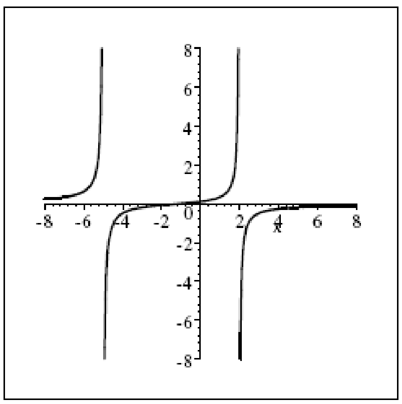
A



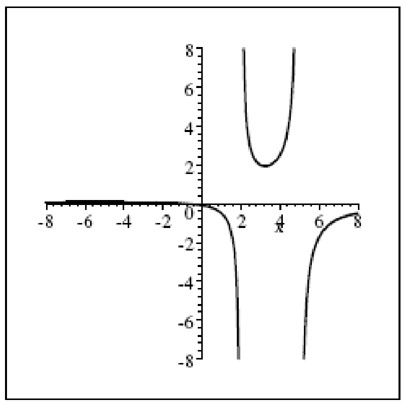
B



C



D



E

$f(x)=\dfrac{x+1}{(x+5)(x-2)}$

horizontal asymptote: $y=0$

vertical asymptotes: $x=-5,\ x=2$

This narrows choices to A and D.

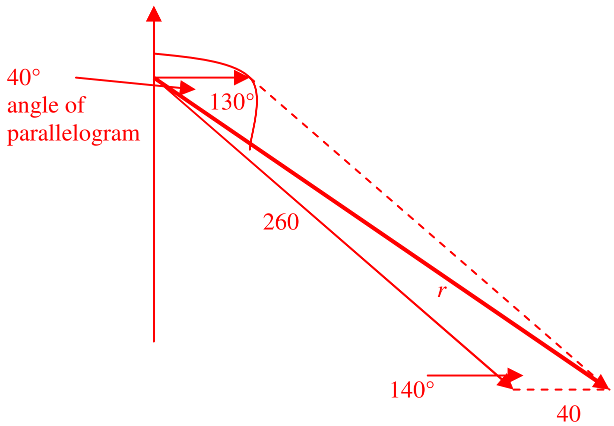
Intercepts are $(-1,0)$ and $(0,-0.1)$.

Those do not help narrowing down the decision.

Find a few points: $\left(-6,-\frac{5}{8}\right)$, $\left(-3,\frac{1}{5}\right)$, $\left(1,-\frac{1}{3}\right)$, and $\left(3,\frac{4}{9}\right)$

These points 'match' choice A.

An airplane with **airspeed** of 260 miles per hour is flying in the direction 130° and a 40 miles per hour wind is blowing directly **from** the west.



14) Find the ground speed of the airplane. Round to the nearest whole number.

Method 1:

$$r^2 = 260^2 + 40^2 - 2(260)(40)\cos 140^\circ$$
$$r^2 = 69200 + 15933.72442$$
$$r^2 = 85133.7244$$
$$r = 291.77684$$

Method 2:

$$p = \langle 260\cos 320^\circ, 260\sin 320^\circ \rangle \quad (40^\circ \text{ 'short' of } 360^\circ)$$
$$w = \langle 40, 0 \rangle$$
$$r = \langle 260\cos 320^\circ + 40, 260\sin 320^\circ \rangle = \langle 239.17156, -169.12478 \rangle$$
$$\|r\| = \sqrt{(239.172^2 + (-167.125)^2} = \sqrt{85133.7248}$$
$$\|r\| = 291.77 \quad \text{ground speed: 292 mph}$$

15) What is the true course of the airplane? Round to the nearest degree.

Let $\alpha =$ the sliver of angle from below the wind vector to the resultant vector.

direction of true course: $90^\circ + \alpha$

$$\frac{\sin\alpha}{260} = \frac{\sin 140^\circ}{291.77684}$$
$$\sin\alpha = \frac{260\sin 140^\circ}{291.77684} = 0.572782879$$
$$\alpha = 34.945^\circ$$

true course: $(90+35)^\circ = 125^\circ$

θ is in Q IV

$$\tan\theta = \frac{-167.12478}{239.17156} = -0.698765286$$
$$\theta_R = 34.9^\circ$$
$$\theta = 90^\circ + 35^\circ$$

true course: 125°