Assignment 2

Read sections 3, 4, and 5 of *MATLAB for Math303* if you are not familiar with Matlab. Turn in the printouts of your command window, M-files, and the graphs, if any, for each problem. Also label all the graphs.

1. Consider the initial value problem:

$$\begin{cases} y'' + 4y = 0 \\ y(0) = 1 \\ y'(0) = -2 \end{cases}$$

We have a power series solution

$$y(x) = a_0 + a_1 x + a_2 x^2 + \cdots$$

where $a_0 = y(0) = 1$ and $a_1 = y'(0) = -2$. The exact solution is

$$y(x) = \cos(2x) - \sin(2x).$$

(a) Compute a_2 and a_3 by hand.

(b) Write M-files for the following three functions

$$\begin{cases} y_2(x) = a_0 + a_1 x + a_2 x^2 \\ y_3(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3 \\ y(x) = \cos(2x) - \sin(2x) \end{cases}$$

(c) Plot the graphs of $y_2(x), y_3(x)$ and $y(x)$ on the same figure. Use different curve styles for each curve, and label the figure appropriately.

2. Consider the initial value problem (cf. 6.4. #16 of the textbook)

$$y'' + \frac{1}{4}y' + y = kg(t), \quad y(0) = 0, \quad y'(0) = 0$$

where $g(t) = u_{3/2}(t) - u_{5/2}(t)$ and $k > 0$ is a parameter. Then the solution is

$$y(t) = 4ku_{3/2}(t)h\left(t - \frac{3}{2}\right) - 4ku_{5/2}(t)h\left(t - \frac{5}{2}\right)$$

where

$$h(t) = \frac{1}{4} - \frac{\sqrt{7}}{84}e^{-t/8}\sin\left(\frac{3\sqrt{7}t}{8}\right) - \frac{1}{4}e^{-t/8}\cos\left(\frac{3\sqrt{7}t}{8}\right).$$

Write appropriate M-files and plot the solutions for $k = 2, k = 2.5$ and $k = 3$ in the same figure using different styles for each curve.