Matlab assignment 5

Read section 11 of *Matlab for Math 303*.

Consider the heat equation with boundary condition

$$\begin{cases} u_{xx} = u_t \\ u(x,0) = x \\ u(0,t) = u(40,t) = 0 \end{cases}$$

for $0 \le x \le 40$ and $t \ge 0$. Here $u(x,t)$ represents the temperature of a rod at (x,t).

(a) Find the series solution $u(x,t)$ and write an m-file for the partial sum $u(x,t,n)$.

(b) Let $n = 100$. Plot $u(x,t,100)$ versus x, $0 \le x \le 40$, for $t = 0, 10, 20, 50, 100$ and 400 with different color styles (or line styles) for each curve on the same figure.

(c) Let $n = 100$. Plot $u(x,t,100)$ versus t, $0 \le t \le 1000$, for $x = 20$. (Note that the command `fplot` only recognizes `x` as an independent variable. So you need to type $u(20,x,100)$ **not** $u(20,t,100)$ with `fplot` command.)

(d) Let $n = 100$. How long does it take for the entire rod to cool off to a temperature of no more than 2 degrees? (Find the answer correct up to one decimal place.)

(e) Let $n = 100$. Do a 3-d plot of $u(x,t,100)$ over the rectangle

$$0 \le x \le 40, \quad 0 \le t \le 400, \quad \Delta x = 0.1, \quad \Delta y = 1.$$