#### So You Want to Break Codes:

### Careers in Government for Mathematicians

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#### Outline of Talk

- Challenge Problem
- National Security Agency
  - What is NSA?
  - Director's Summer Program
  - Institute for Defense Analyses
  - Center for Communications Research
- What Mathematics Does NSA Do?
  - Shift Cipher
  - ASCII and Unicode
  - Examples
  - Affine Cipher
- 4 Challenge Problem Revisited



#### Challenge Problem!

#### **BANJO** is to **FERNS**

as

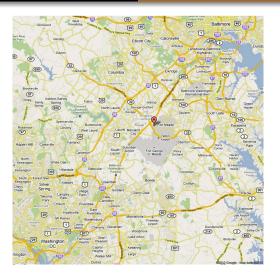
**PECAN** is to [?]

What is NSA? Director's Summer Program Institute for Defense Analyses Center for Communications Research

# Where Are Such Problems Useful?



National Security Agency http://www.nsa.gov/



9800 Savage Road, Ft. George G. Meade, MD http://en.wikipedia.org/wiki/National\_Security\_Agency



Main Building http://en.wikipedia.org/wiki/Fort\_George\_G.\_Meade

What is NSA?
Director's Summer Program
Institute for Defense Analyses
Center for Communications Research



Secure Entrance to Main Building http://en.wikipedia.org/wiki/Fort\_George\_G.\_Meade

#### Facts About NSA

- The National Security Agency/Central Security Service
   (NSA/CSS) is a cryptologic intelligence agency of the United
   States Department of Defense responsible for the collection and
   analysis of foreign communications and foreign signals intelligence,
   as well as protecting U.S. government communications and
   information systems, which involves cryptanalysis and cryptography.
- NSA's mission is to collect information that constitutes "foreign intelligence or counterintelligence" while not acquiring information concerning the domestic activities of United States persons. NSA has declared that it relies on the FBI to collect information on foreign intelligence activities within the borders of the USA, while confining its own activities within the USA to the embassies and missions of foreign nations.
- Largest employer of mathematicians in the world. In fact, NSA has invested many millions of dollars in academic research.

What is NSA?
Director's Summer Program
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To Work for NSA, Must I Live in Maryland?

#### Opportunities at NSA

- Director's Summer Program (DSP)
- Institute for Defense Analyses (IDA)
- Center for Communications Research (CCR)
  - Princeton, NJ
  - La Jolla, CA



#### Director's Summer Program



Institute for Defense Analyses
 https://www.ida.org/



#### $\underline{\mathsf{ID}A}$ Center for Communications Research - Princeton



805 Bunn Drive Princeton, New Jersey 08540

CCR-P is a division of The Institute for Defense Analyses in Alexandria, Virginia.

Center for Communications Research in Princeton, NJ http://www.idaccr.org/

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#### Center for Communications Research

La Jolla



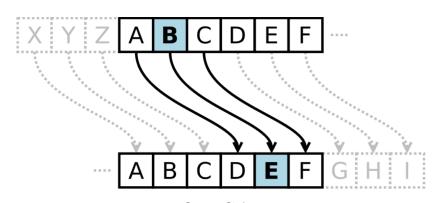




Center for Communications Research in La Jolla, CA http://www.ccrwest.org/

Shift Cipher ASCII and Unicode Examples Affine Cipher

# What Kind of Mathematics Happens at NSA?



Caesar Cipher http://en.wikipedia.org/wiki/Caesar\_Cipher

	ASCII Code Chart															
	ıθ	1	1 2	<sub> </sub> 3	<sub>1</sub> 4	լ 5	<sub>I</sub> 6	7	8	9	ı A	<sub>I</sub> B	С	D	E	<u> </u>
0	NUL	SOH	STX	ETX	E0T	ENQ	ACK	BEL	BS	HT	LF	VT	FF	CR	S0	SI
ī	DLE	DC1	DC2	DC3	DC4	NAK	SYN	ETB	CAN	EM	SUB	ESC	FS	GS	RS	US
2			ıı.	#	\$	%	&	-	(	)	*	+	,	١	٠	/
3	θ	1	2	3	4	5	6	7	8	9	::	;	٧		^	?
4	@	A	В	С	D	Ε	F	G	Н	Ι	J	K	L	М	N	0
5	P	Q	R	S	Т	U	V	W	X	Υ	Z	[	\	]	^	-
6	`	а	b	С	d	е	f	g	h	i	j	k	ι	m	n	0
7	р	q	r	s	t	u	v	W	х	У	z	{		}	~	DEL

American Standard Code for Information Interchange http://en.wikipedia.org/wiki/ASCII

Unicode

http://en.wikipedia.org/wiki/Unicode

#### Example of Caesar Cipher

```
(Local) In[3]:=
        ShiftCipher[Str , Shift Integer] := Module[{lst, n},
            (* Convert input string to a list of Unicode numbers. *)
           lst = ToCharacterCode[Str, "Unicode"];
            (* Determine the number of characters. *)
           n = Length[1st];
           (* Shift by a certain number of places. *)
           new1st = Table [lst[k] + Shift, \{k, 1, n\}];
           (* Return the new string. *)
           Return[FromCharacterCode[newlst]];
          ];
        ShiftCipher["hello", 3]
(Local) Out[4]=
        khoor
```

#### Converting a String to a Number

#### Converting a Number to a String

```
(Local) In[7]:=
        NumberToString[Nmbr Integer] := Module[{n, lst},
          (*Determine the number of "digits" base 2^16. This is the
           number of characters.*)
          n = Ceiling[Log[Nmbr]/Log[2^16]];
          (*Recover list of integers from input.*)
          lst = Table[Mod[Floor[Nmbr/(2^16)^(k-1)], 2^16], \{k, 1, n\}];
          (*Reconstruct string from the list.*)
          Return[FromCharacterCode[lst, "Unicode"]];
        NumberToString[
         85 218 816 667 906 701 223 072 976 692 732 706 460 886 071 310 629 547 154 543 158 \
          539 502 849 555 798 959 641 179 650 415 408 182 058 457 081 319 952 076 636 240 1
(Local) Out[8]=
        Professor Drasin is Great!
```



Mathematical Sciences Research Institute Undergraduate Program MSRI-UP 2010

http://www.msri.org/up/2010/index.html

#### Affine Cipher

#### <u>Plaintext</u>

#### Ciphertext

$$N_{1}, N_{2}, \dots, N_{n}$$

$$\{a_{1}, a_{2}, \dots, a_{n}\}$$

$$N = \sum_{k=1}^{n} a_{k} b^{k-1}$$

$$\lambda, \nu > M_{1}, M_{2}, \dots, M_{n}$$

$$\{c_{1}, c_{2}, \dots, c_{n}\}$$

$$\text{with } c_{k} = \lambda a_{k} + \nu$$

$$\lambda, \nu > M = \sum_{k=1}^{n} c_{k} b^{k-1}$$

$$= \lambda N + \nu \frac{b^{n} - 1}{b - 1}$$

# Solution to Challenge Problem!

## BANJO is to FERNS

as

**PECAN** is to **TIGER** 

#### Mathematica Code

```
(Local) In[17]:=
        ShiftCipher[Str , Shift Integer] := Module[{lst, n},
           (* Convert input string to a list of Unicode numbers. *)
           lst = ToCharacterCode[Str, "Unicode"];
           (* Determine the number of characters. *)
           n = Length[lst];
           (* Shift by a certain number of places. *)
           new1st = Table[lst[k] + Shift, \{k, 1, n\}];
           (* Return the new string. *)
           Return[FromCharacterCode[newlst]];
          1;
        {ShiftCipher["banjo", 4], ShiftCipher["pecan", 4]}
(Local) Out[18]=
        {ferns, tiger}
```

#### Questions?