Ma 15200 Lesson 18 Section 1.7

I Representing an Inequality

There are 3 ways to represent an inequality. (1) Using the inequality symbol (sometime within set-builder notation), (2) using interval notation, and (3) using a number line graph.

The following table illustrates all three ways. Notice that interval notation looks like an ordered pair, sometimes with brackets. When writing the ordered pair, always write the lesser value to the left of the greater value. A parenthesis next to a number illustrates that *x* gets very, very close to that number, but never equals the number. A bracket next to a number means it can equal that number. With ∞ or $-\infty$ a parenthesis is always used, since there is not an exact number equal to ∞ or $-\infty$.

INEQUALITIES

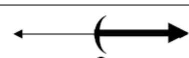
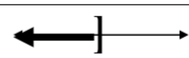
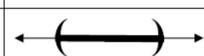
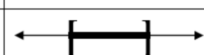
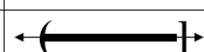
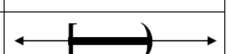
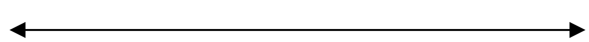
Inequality	Interval Notation	Graph
$x > 3$	$(3, \infty)$	
$x \le 0$	$(-\infty, 0]$	
$-2 < x < 6$	$(-2, 6)$	
$6 \le x \le 7$	$[6, 7]$	
$-10 < x \le 1$	$(-10, 1]$	
$5 > x \ge 1$	$[1, 5)$	

Table 1.4 on page 174 of the textbook also illustrates the 3 ways to represent an inequality where $a < b$.

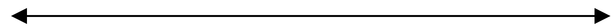
<u>Ex 1:</u> Write this inequality in interval notation and graph on a number line.

$\{x \mid x > 1\}$

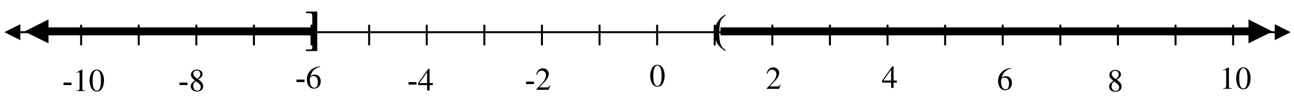


Ex 2: Write the set of numbers represented on this number line as an inequality and in interval notation.

Ex 3: Write the following as an inequality and graph on a number line. $(-\infty, 5]$



Examine the number line below.



In set-builder notation, it would be represented as $\{x \mid x \leq -6 \ or \ x > 1\}$ and in interval notation, it would be represented as $(-\infty, -6] \cup (1, \infty)$.

II Solving a Linear Inequality in One Variable

Solving linear inequalities is similar to solving linear equation, with one exception. Examine the following.

$5 < 10$

Add -6 to both sides: $-1 < 4$ True

Multiply both sides by 2: $10 < 20$ True

Divide both by 5: $1 < 2$ True

However, try multiplying by -2: $-10 < -20$ False

Divide by -5: $-1 < -2$ False.

This leads to the following properties of Inequalities

1) **The Addition Property of Inequality**

If $a < b$, then $a + \boldsymbol{c} < b + \boldsymbol{c}$

$$a - \boldsymbol{c} < b - \boldsymbol{c}$$

2) **The Positive Multiplication Property of Inequality**

If $a < b$ and c is positive, then $a\boldsymbol{c} < b\boldsymbol{c}$

$$\frac{a}{\boldsymbol{c}} < \frac{b}{\boldsymbol{c}}$$

3) **The Negative Multiplication Property of Inequality**

If $a < b$ and c is negative, then $a\boldsymbol{c} > b\boldsymbol{c}$

$$\frac{a}{\boldsymbol{c}} > \frac{b}{\boldsymbol{c}}$$

Ex 4: Solve each inequality. Write the solutions with the inequality symbol, in interval notation, and graph the solutions on a number line.

$a)$ $3(x+2) \le -4(5-x)$

$b)$ $\dfrac{6(x-4)}{5} > \dfrac{3(x+2)}{4}$

$c)$ $\dfrac{5}{9}(a+3) - a \ge \dfrac{4}{3}(a-3) - 1$

It the variables 'drop out' of a linear inequality, the solution is either all real numbers (except for any that may not be in the domain) or there is no solution.

$3(x+2) < 3x+7$

$3x+6 < 3x+7$

$6 < 7$

The result above is always true, 6 is less than 7. The solution is $\{x \mid x \text{ is a real number}\}$ or $\mathbb{R}$ or $(-\infty, \infty)$.

$3(x+2) < 3x+2$

$3x+6 < 3x+2$

$6 < 2$

Six is never less than 2. The result is false. The solution is $\varnothing$ or no solution.

III Solving Compound Inequalities

When solving an inequality such as $-12 < 3x + 3 < 2$, the goal is to **isolate the *x* in the middle.** Such an inequality is called a compound inequality and means the same as $-12 < 3x + 3$ and $3x + 3 < 2$. The solution will be the numbers that, when substituted in $3x + 3$, yield between -12 and 2.

Example:

$-4 < 2x - 1 < 5$

Begin by adding 1 to the left, middle, and right.

$-3 < 2x < 6$

Divide the left, middle, and right by 2.

$-\frac{3}{2} < x < 3$

Any number between -1 ½ and 3 makes the inequality statement true.

<u>Ex 5:</u> Solve each compound inequality. Write the solutions using the inequality symbols, in interval notation, and graph the solutions on a number line.

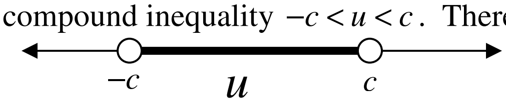
a) $1 < 3x - 2 < 12$

b) $-2 < 6 - 3x < 3$

IV Solving Absolute Value Inequalities

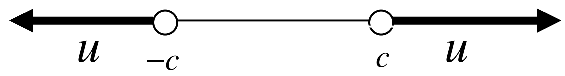
Absolute Value Inequalities: $|u| < c$ or $|u| \le c$, if $c \ge 0$

The inequality $|u| < c$ indicates all values less than c units from the origin. Therefore $|u| < c$ is equivalent to the compound inequality $-c < u < c$. There is a similar statement for $|u| \le c$.



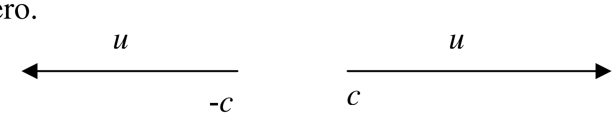
Absolute Value Inequalities: $|u| > c$, $|u| \geq c$, if $c > 0$

The inequality $|u| > c$ indicates all values <u>more than</u> c units from the origin. Therefore $|u| > c$ is equivalent to the inequality statement $u < -c$ or $u > c$. There is a similar statement for $|u| \geq c$.

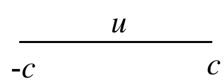


To help you keep the two cases straight in your head, I recommend thinking of a number line.

If the absolute value is greater than a positive number *c*, it is greater than that many units away from zero.



If the absolute value is less than a positive number *c*, it is within that many units of zero.



<u>Ex 6:</u> Solve each. Write solutions using interval notation and graph the solutions on a number line.

Hint: Always isolate the absolute value before writing an inequality without the absolute value.

$a)$ $|x+4| < 6$

$b)$ $|3x+5| + 1 \leq 9$

$c)$ $\left|\dfrac{5x+2}{3}\right| < 1$

Ex 7: Solve each inequality. Write the solutions using interval notation and graph the solutions on a number line.

$a)\quad |2-x|>5$

$b)\quad \left|\frac{x}{2}+3\right|\geq 7$

$c)\quad |3x-4|+2>7$

V Applied Problems

Ex 8: Mary wants to spend **less than** \$600 for a DVD recorder and some DVDs. If the recorder of her choice costs \$425 and DVDs cost \$7.50 each, how many DVDs could Mary buy?

Ex 9: The percentage, P, of US voters who used punch cards or lever machines in national elections can be modeled by the formula $P = -2.5x + 63.1$ where x is the number of years after 1994. In which years will fewer than 35.7% of US voters use punch cards of lever machines?

Ex 10: A college provides its employees with a choice of two medical plans shown in the following table.

Plan 1:	\$100 deductible payment	30% of the remaining payments
Plan 2:	\$200 deductible payment	20% of the remaining payments

For what size hospital bills is plan 2 better for the employee than plan 1? (Assume the bill is over \$200.)

Ex 11: The room temperature in a public courthouse during a year satisfies the inequality $|T - 71| < 3$ where T is in degrees F. Express the range of temperatures without the absolute value symbol.