

MA 15200 Lesson 20 Section 2.1

A **relation** is any set of ordered pairs. The set of all first components of the ordered pairs is called the **domain**. The set of all second components is called the **range**.

Relations can be represented by tables, sets, equations of two variables, or graphs.

Name	% of all Names
Smith	1.006%
Johnson	0.810%
Williams	0.699%
Brown	0.621%
Jones	0.621%

The table at the left would represent a relation where the ordered pairs are of the form (name, %). An example would be (Williams, 0.699%). The domain would be {Smith, Johnson, Williams, Brown, Jones} and the range is {1.006%, 0.810%, 0.699%, 0.621%}.

Ex 1: Find the domain and range of each relation.

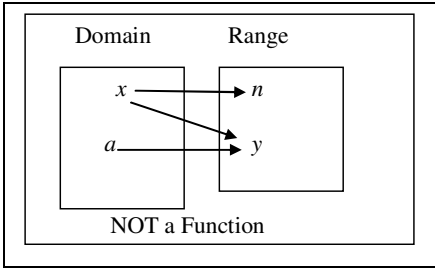
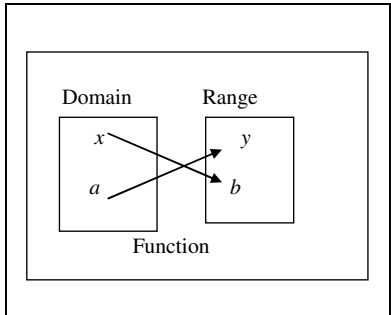
- a) $\{(2, -3), (3, -4), (4, -5), (5, -5), (2, -6)\}$

A relation in which each member of the domain corresponds to exactly one member of the range is a **function**. The table above that pairs a last name with a percent of all names is a function because each last name is paired to exactly one percent. Another way to identify a function is the following, it is a **relation in which no two ordered pairs have the same first component and different second components**.

Definition of a Function
 A **function** is a correspondence from a first set, called the **domain**, to a second set, called the **range**, such that each element in the domain corresponds to *exactly one* element in the range.

If the table was changed as below, the relation is not a function. The percent 0.621% would be paired with both Brown and Jones.

% of all Names	Name
1.006%	Smith
0.810%	Johnson
0.699%	Williams
0.621%	Brown
0.621%	Jones



I Determining Whether a Relation is a Function

Ex 2: Determine if each relation is a function.

a)

x	y
0	1
1	0
-1	0
2	-3
-3	-8
-2	-3
4	-15

b) $\{(2, -3), (3, -4), (4, -5), (5, -5), (2, -6)\}$

II Determining Whether an Equation Represents a Function

Many functions are written as equations of two variables. For example, $R = -0.6x + 94$, where R represents the average number of meals per person Americans ordered from restaurants and x represents the number of years after 1984. The x is called the **independent variable**, because any number of years after 1984 can be selected. The y is called the **dependent variable**, because its value depends upon the value of x .

Not all equations represent functions. If an equation is solved for y and more than one value of y can be obtained for a given x , then the equation does not define a function.

Ex 3: Determine if each equation is a function or not. Find the domain in interval notation.

a) $y = x^2 + 2$

c) $x = y^2 + 2$

b) $y = \sqrt{x-3}$

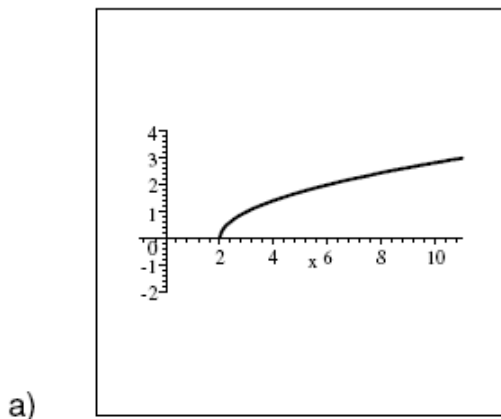
d) $xy - y = -1$

e) $y = \frac{2x}{x-5}$

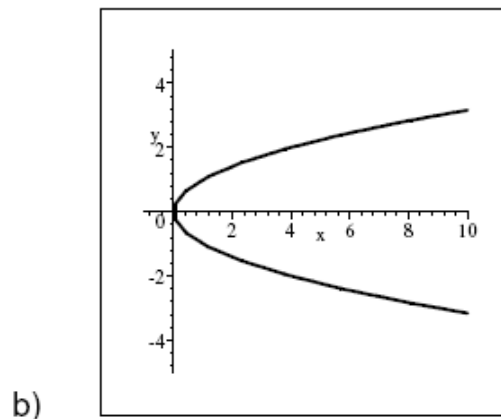
III Determining if a Graph Represents a Function

Graphs of Functions: A function can be graphed by determining the set of all ordered pairs (points) where x is in the domain and y is in the range. Because each x can only be paired to one y , the **vertical line test** can be used to determine if a graph represents a function. If every possible vertical line would intersect the graph only once, then the graph represents a function.

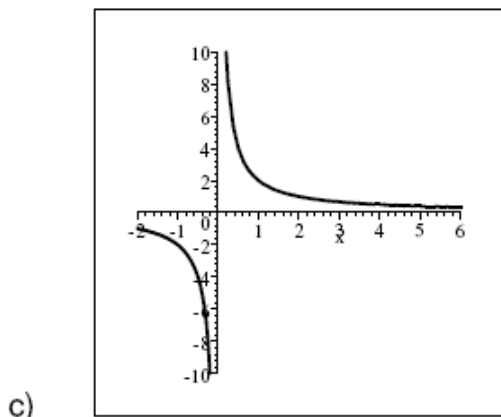
Ex 2: Determine which graphs are functions.



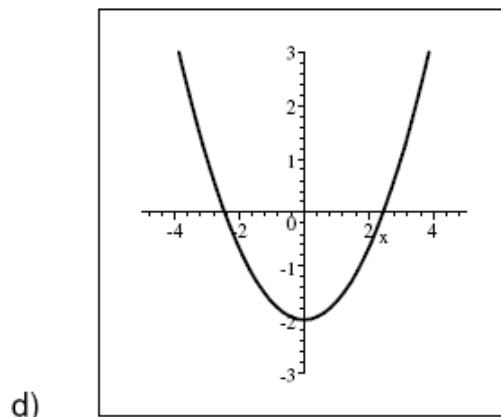
$y = \sqrt{x-2}$



$y^2 = x$



$y = \frac{2}{x}$



$y = \frac{1}{3}x^2 - 2$

The Vertical Line Test for Functions

If any vertical line intersects a graph in more than one point, the graph does not define y as a function of x .

IV Function Notation

Functions are often named using letters such as f , g , h , F , G , or p . The input of the function is x . The output of the function is y , which can be represented as $f(x)$, read f of x or function of x . This notation means the **value of the function at the number x** and is known as **function notation**.

The notation $f(2)$ means replace a 2 for the value of x (or the independent variable) in the function. Some substitutions for x do not have to be numbers. $f(x-4)$ says to find the **function value** when the original x is replaced with $x - 4$. Remember $f(x)$ means the same thing as y . The function value is the y or $f(x)$ value.

Ex 3: If $f(x) = 3x^2 - 2x$ and $g(x) = 4x + 2$, find the following.

a) $f(-1) =$

b) $g(-4) =$

c) $f(a) =$

d) $g(2k) =$

e) $g(m-2) =$

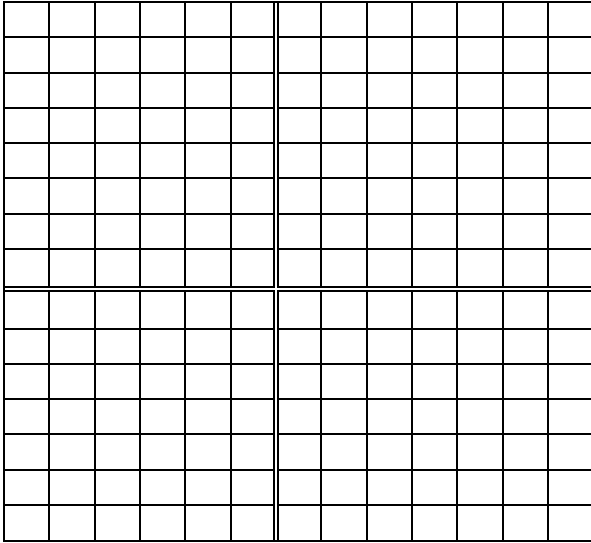
f) $f(n^2 + 1) =$

g) $g(x+5) =$

V Graphs of Functions

The graph of a function is the graph of its ordered pairs. We have already graphed some equations, but will graph one function here.

Ex 4: Graph $g(x) = 2x - 3$

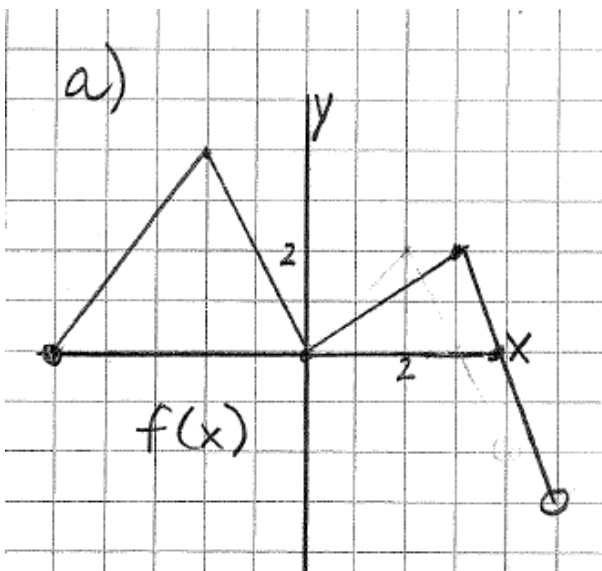


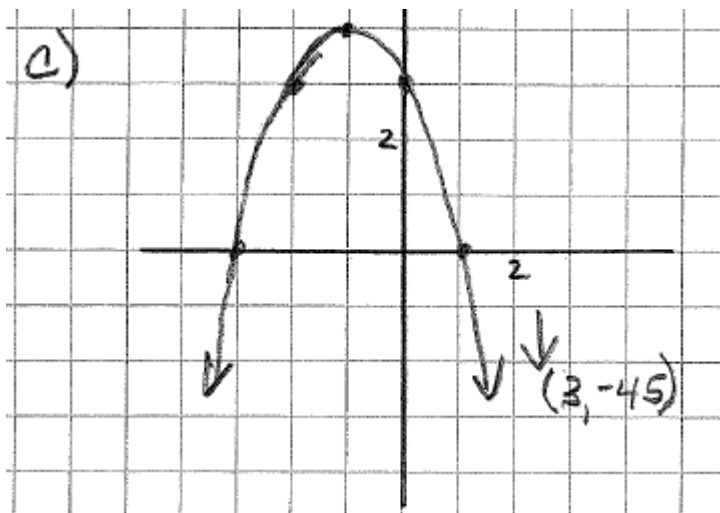
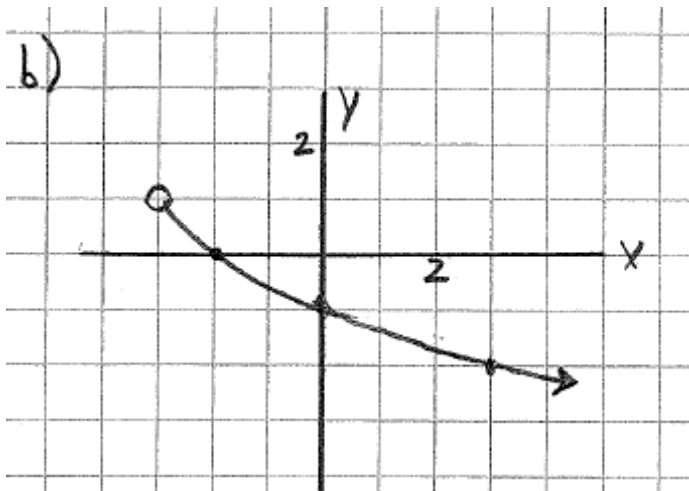
The graph above is a line. Any function of the form $f(x) = mx + b$ has a straight line graph and is called a **linear function**.

VI Analyzing the Graph of a Function

Ex 5: For each graph shown, find the following.

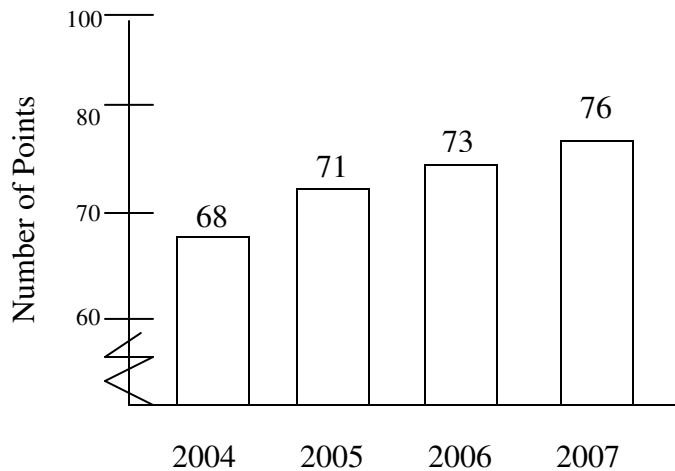
- The Domain in set-builder notation and interval notation.
- The Range in set-builder notation and interval notation.
- $f(3)$
- For what value of x is $f(x) = 4$?
- Any x -intercepts
- Any y -intercept





VII Applied Function Problems

The bar graph below shows the average final exam score for 4 consecutive years.



The functions $f(x) = 3x + 56$ and $g(x) = \frac{2}{5}x^2 + 56$ model the average exam score where x is the number of years after 2000.

- Ex 6:
- a) According to the graph, what was the average exam score in 2007?
 - b) Using the function f model, what was the average exam score in 2007?
 - c) Does function f underestimate or overestimate the actual exam score according to the graph? By how much?
 - d) Using the function g model, what was the average exam score in 2007?
 - e) Does the function g overestimate or underestimate the actual average given in the graph? By how much?