MA 15200 Lesson 21 Section 2.2

I Increasing, Decreasing, or Constant Functions

> When describing intervals where function is increasing or decreasing, use from **one *x* to the next *x*.**

A function is **increasing** if in an open interval, whenever $x_1 < x_2$, then $f(x_1) < f(x_2)$. The function values (or points on graph) are always **rising.**

A function is **decreasing** if in an open interval, whenever $x_1 < x_2$, then $f(x_1) > f(x_2)$. The function values (or points on graph) are always **falling**.

A function is **constant** if in an open interval, $f(x_1) = f(x_2)$ for any x_1 and x_2 in the interval. The function values are equal and the graph is **flat.**

When describing the interval where a function is increasing, decreasing, or constant; the intervals are open (no brackets) and you use the ***x*-coordinates**.

II Relative Maxima and Relative Minima

> An endpoint can never be a relative maximum or relative minimum.

A function has a **relative maximum** value $f(a)$ if there is an open interval containing a such that $f(a) > f(x)$ for all $x \neq a$ in the open interval. In other words, the function value $f(a)$ is larger than all other function values in that interval. Graphically, this is the function value of the **highest point** of the interval of the graph of the function.

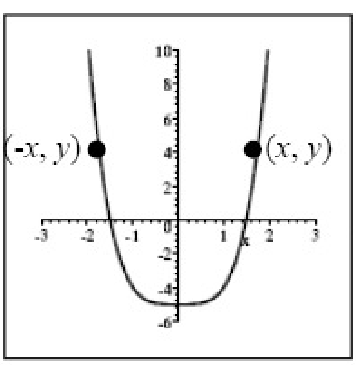
A function has a **relative minimum** value $f(b)$ if there is an open interval containing b such that $f(b) < f(x)$ for all $x \neq b$ in the open interval. In other words, the function value $f(b)$ is smaller than all other function values in that interval. Graphically, this is the function value of the **lowest point** in the interval of the graph of the function.

We say there is a relative maximum or relative minimum value $f(a)$ or $f(b)$ at a certain value of x. The y value of a point is the relative maximum or relative minimum.

III Even or Odd Functions

A function f is an **even function** if $f(-x) = f(x)$ for all x in the domain of f. The ordered pairs $(x, f(x))$ and $(-x, f(x))$ will both be in the function. Graphically, **even functions have symmetry about the *y*-axis or symmetry with respect to the *y*-axis.**

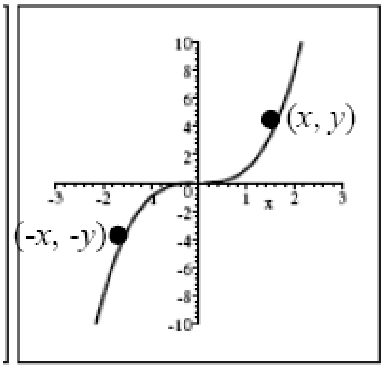
Here is an example of symmetry with respect to the y-axis.



symmetry about the y-axis

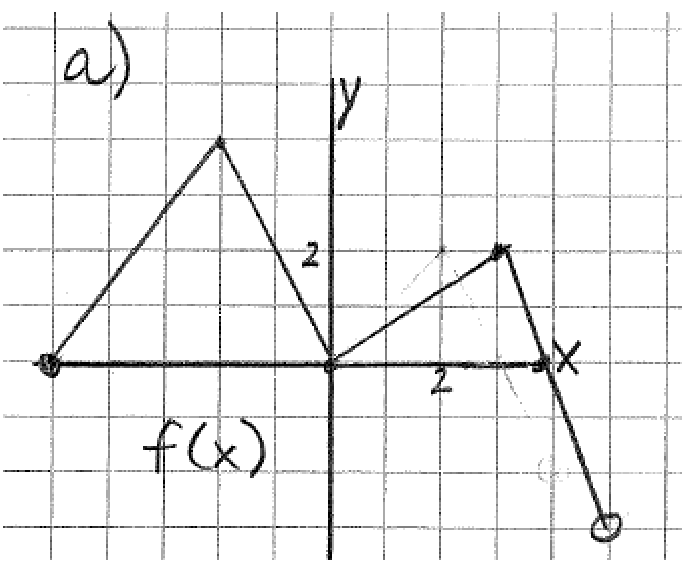
The function *f* is an **odd function** if $f(-x) = -f(x)$ for all x in the domain of *f*. The ordered pairs $(x, f(x))$ and $(-x, -f(x))$ will both be in the function. Graphically, **odd functions have symmetry about the origin or symmetry with respect to the origin.** This means the origin would be the midpoint of the line segment connecting the two points.

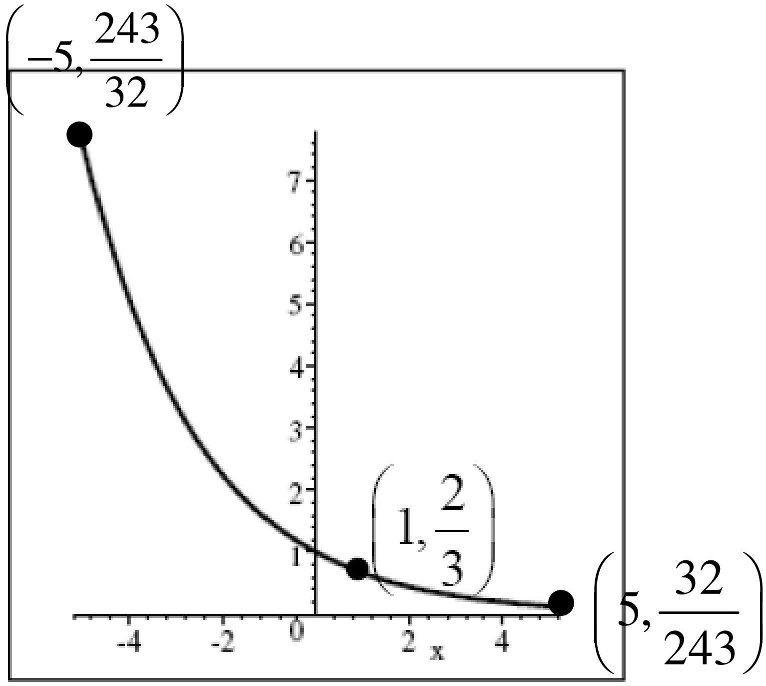
Here is an example of symmetry with respect to the origin.



<u>Ex 1:</u> For each graph determine the following.

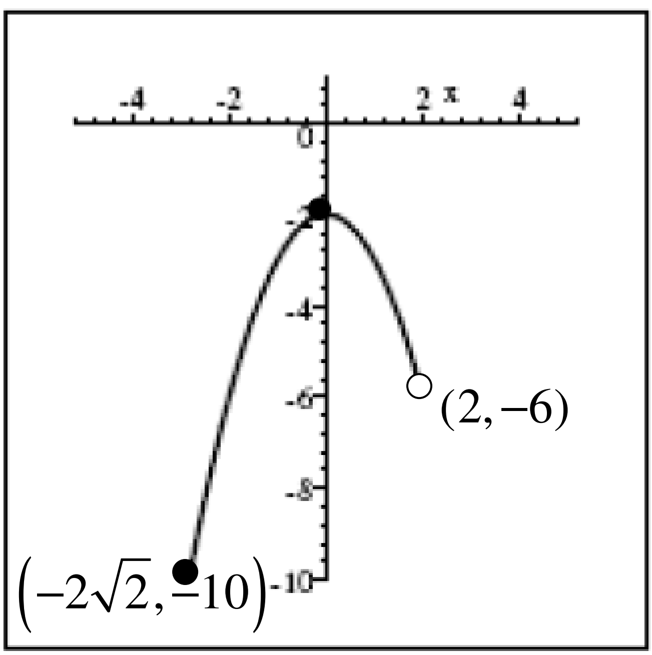
a) The intervals in which the function is increasing, if any.
b) The intervals in which the function is decreasing, if any.
c) The intervals in which the function is constant, if any.
d) The relative maximum(s), if any.
e) The relative minimum(s), if any.
f) Describe if the function as even, odd, or neither (check symmetry).
g) The zeros of the function (zeros are the *x*-intercepts)





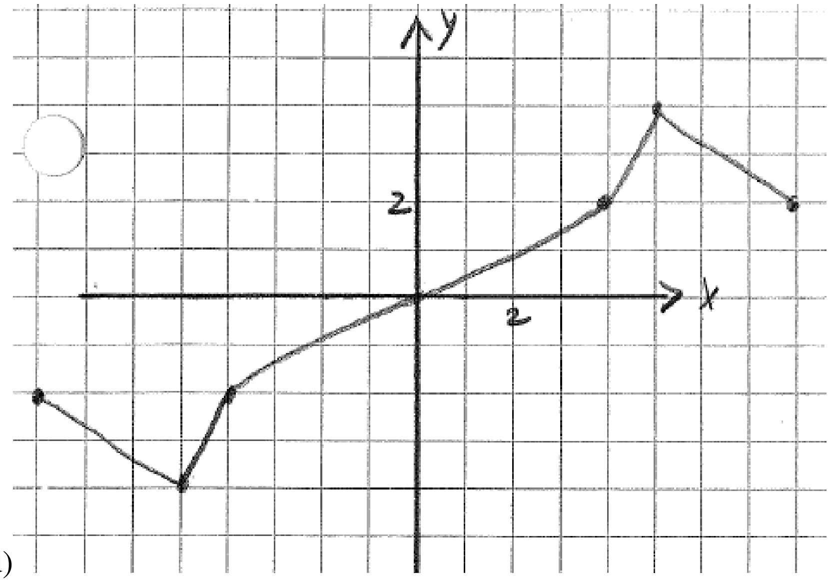
$\left(-5, \frac{243}{32}\right)$
$\left(1, \frac{2}{3}\right)$
$\left(5, \frac{32}{243}\right)$
x

b)



x
$(2, -6)$
$\left(-2\sqrt{2}, -10\right)$

c)

d) 

Ex 2: Describe each function as even, odd, or neither.

$a)\quad f(x) = x^5 - x^3$

$b)\quad g(x) = 4x^4 - 12$

$c)\quad h(x) = 2x^3 - 3x + 2$

$d)\quad f(x) = |x| + 5$

$e)\quad g(x) = 3x^3\sqrt{x^2 + 1}$

IV Piecewise Functions & Evaluating such Functions

A cab driver charges \$4 a ride for a ride 5 miles or less. He charges \$4 plus \$0.50 for every mile over 5 miles, if the ride is greater than 5 miles. This situation could be described by the function $f(x)=\begin{cases} 4 & \text{if } 0<m\le 5 \\ 4+0.5(m-5) & \text{if } m>5 \end{cases}$, where m represents the number of miles. Such a function is called a **piecewise function.** A piecewise function is made up of parts of two or more functions, each with its own domain.

Ex 3: For each piecewise function, find $f(-4),\ f(0),\ and\ f(2)$, if defined.

a) $$f(x)=\begin{cases} 2 & \text{if } x\le 0 \\ x+2 & \text{if } x>0 \end{cases}$$

b) $$f(x)=\begin{cases} 2x+1 \text{ if } x<-4 \\ 3x-9 \text{ if } -4\le x<0 \\ 5x+3 \text{ if } x\ge 0 \end{cases}$$

Ex 4: A cellular phone plan has the function below to describe the total monthly cost where *t* represents the number of calling minutes.

$$C(t)=\begin{cases} 25 & \text{if } 0\le x\le 120 \\ 25+0.30(t-120) & \text{if } x>120 \end{cases}$$

Find the following values and interpret them.

$a)\quad C(100)$

$b)\quad C(120)$

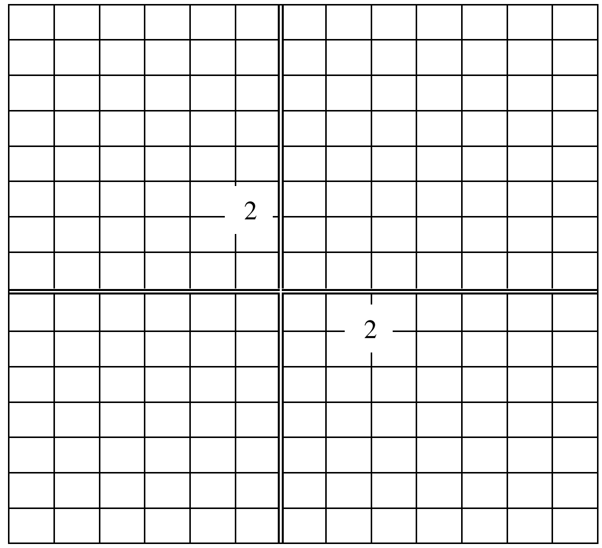
$c)\quad C(140)$

V Graphing Piecewise Functions

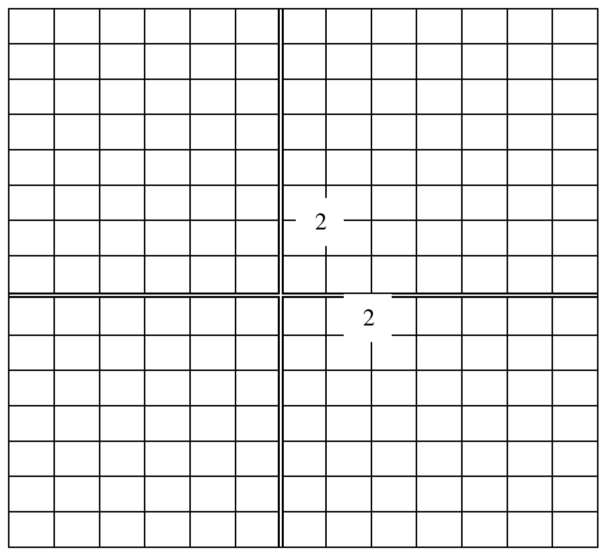
To graph a piecewise function, use a partial table of coordinates to create each piece. For ‘endpoints’, use the appropriate open or closed circle. An **open circle** is used when the *x* value cannot equal the given value, only approach it. A **closed circle** is used when the *x* value can equal the given value.

Ex 5: Graph each piecewise function.

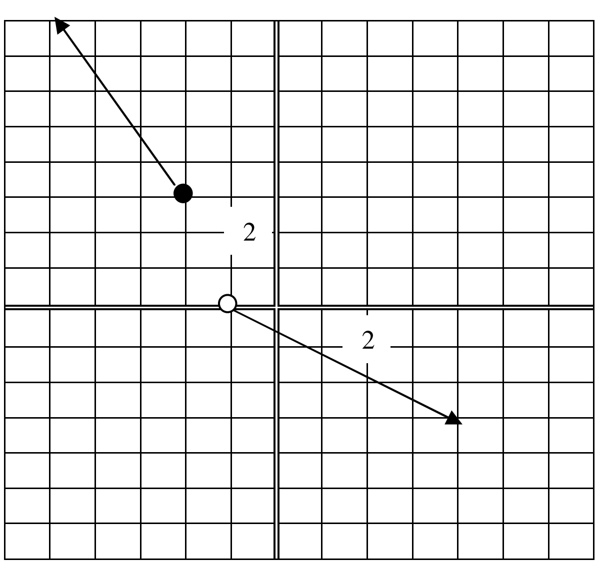
a) $f(x) = \begin{cases} 5 & \text{if } x < 1 \\ x+1 & \text{if } x \geq 1 \end{cases}$



b) $g(x) = \begin{cases} x^2 & \text{if } x \leq -1 \\ -x & \text{if } -1 < x < 2 \\ 2x & \text{if } x \geq 2 \end{cases}$



Ex 6: Describe the domain and range of the piecewise function below.



VI Applied Problems

Ex 7: Use the graph before problem 83 on page 227 of the textbook.

a) What is the **range** for the 'women' graph?

b) At **what age** does the percent of body fat reach a **maximum** for men?

c) What is the difference of the percent of body fat for men and women at age 35?

Ex 8: In a certain city, there is a local income tax that is described by the table below.

If your taxable income is over..	But not over...	The tax you owe is..	Of the amount over..
$0	$10,000	1%	$0
$10,000	$25,000	$100 + 2%	$10,000
$25,000	-	$300 + 3%	$25,000

Write a piecewise function to describe the tax, where x is income.

$$T(x) = \begin{cases} 0.01x & \text{if} \quad x \leq 10,000 \\ 100 + 0.02(x - \square) & \text{if} \quad 10,000 < x \leq 25,000 \\ 300 + \square\,(x - \square) & \text{if} \quad x > 25,000 \end{cases}$$