Certain formulas are called variation models. These formulas show how one quantity changes in relation to other quantities. There are four types of variation: **Direct, Inverse, Joint, and Combined.** All variation formulas (variation models) involve a constant k.

I   Direct Variation.

$S = 13h$ This is a **direct variation** formula that says the salary of a person is 13 times the hours worked. The constant k is 13 ($13 is the hourly wage). It is easy to understand, that as the number of hours increases, so does the salary earned. The format for direct variation is $y = kx$. In **direct variation** (as long as the constant is positive) as x increases, so does y.   You can also see that this model would have a graph that is a line with a positive slope of k and a y-intercept at 0.

Direct Variation:
If a situation is described by an equation in the form
$$y = kx,$$
where k is a nonzero constant, we say that **y varies directly as x or y is directly proportional to x.** The number k is called the **constant of variation** or the **constant of proportionality.**

Steps for Solving a Direct Variation Problem:
  1. **Write an equation that models the given English statement.**
  2. **Substitute the given values into the equation in step 1 and solve for k, the constant.**
  3. **Substitute the value of k into the equation in step 1.**
  4. **Use the equation form step 3 (the variation formula) to solve and answer the problem's question.**

Ex 1: Find the **constant of proportionality** for each. **Write the resulting variation equation.**
       y is directly proportional to x. If $x = 30$, then $y = 15$.

Ex 2: y is directly proportional to x. If $y = 15$ when $x = 4$, find y when $x = \dfrac{7}{5}$.

<u>Ex 3:</u> Hooke's law states that the force required to stretch a spring a distance is directly proportional to the distance stretched. A force of 3 newtons stretches a spring 15 centimeters. A force of 4 newtons would stretch the spring how far?

<u>Ex 4:</u> a) *b* is directly proportional to the **cube** of *a*. *b* is $\dfrac{81}{2}$ when *a* is 3. Find *b* when *a* is 4.

b) *y* varies directly as the sum of *x* and *r*. When *x* = 5 and *r* = 6, then $y = \dfrac{33}{2}$. Find *x* when *y* = 12 and *r* = 2.

II   Inverse Variation

An example of a model for **inverse variation** is $t = \dfrac{200}{r}$. This says that the time (in hours) to complete a 200 miles trip **varies inversely** as the rate (in miles per hour). The constant *k* is 200. It is easy to understand, that as the rate increases, the time decreases. The format for **inverse variation** is $y = \dfrac{k}{x}$. In inverse variation, as long as the constant *k* is positive, then as *x* increases, *y* decreases and vice-versa. The graph of an inverse function (with a positive *k*) is a curve similar to one shown on page 398 of the textbook.

> **Inverse Variation:**
> If a situation is described by an equation of the form
> $$y = \frac{k}{x},$$
> where k is a nonzero constant, we say that **y varies inversely as x or y is inversely proportional to x.** The number k is called the constant of variation.

Use the same 4 steps to solve an inverse variation problem as for a direct variation problem.

Ex 5:   Find the constant of variation and the variation equation that results.

   R varies inversely as the square of I. If $I = 25$, then $R = 100$.

Ex 6:   The time to drive a certain distance (t) varies inversely to the rate of speed (r).
   Mary drives 47 miles per hour for 4 hours. How long would it take her to make
   the same trip at 55 mph? Round to the nearest tenth.

Ex 7:   The intensity of illumination on a surface varies inversely as the square of the
   distance from the light source. A surface is 12 meters from a light source and has
   an intensity of 2. How far must the surface be from the light source to receive
   twice as intense illumination? Approximate to the nearest tenth of a meter.

III   Joint Variation

Joint variation is 'direct variation' with more than one variable. It has the form $y = kxz$ or $y = kxwz$, depending on how many variables are involved. Of course, some variables could be replaced by powers or roots. The same 4 step process is used to solve a joint variation problem.

Ex 8: q varies jointly with the square of m and the square root of n. If $q = 45$ when $m = 3$ and $n = 25$, find q when $m = 4$ and $n = 36$.

IV   Combined Variation

If a model involves both direction variation and inverse variation, it is called **combined variation**. Both variations occur at the same time. An example of a combined variation model would be the formula $S = \dfrac{kA}{P}$; Sales vary directly as the advertising budget and inversely as the price of the company's product. As the advertising budget goes up the sales go up (direct), as the price of the product goes up, the sales go down (inverse). This model represents putting the models $S = k_1 A$ and $S = \dfrac{k_2}{P}$ together.

Ex 9: A varies directly as the square of a and inversely as b. A equals 24 when a equals 4 and b equals 2. Find the value of A when a is 5 and b is 4.

Ex10:   The power, in watts, dissipated as heat in a resistor varies directly with the square of the voltage and inversely with the resistance. If 20 volts are placed across a 20-ohm resistor, it will dissipate 20 watts. What voltage across a 10-ohm resistor will dissipate 40 watts?

<u>Ex 11:</u> You have heard of IQ, a person's intelligence quotient. The value of IQ varies directly as a person's mental age and inversely as that person's chronological age. A person with a mental age of 30 and a chronological age of 25 has an IQ of 120. What is the chronological age of a person with a mental age of 30 and an IQ of 90? Round to the nearest year, if necessary.