MA 15200 Lesson 26 Section 2.7

This lesson is on **inverse functions**. Examine the temperature formulas below.

$$C = \frac{5}{9}(F-32) \qquad\qquad F = \frac{9}{5}C + 32$$

Replace F in the first equation with 50º.

$$C = \frac{5}{9}(50-32)$$
$$C = \frac{5}{9}(18)$$
$$C = 10$$

Now, replace C in the second with 10º.

$$F = \frac{9}{5}(10) + 32$$
$$F = 18 + 32$$
$$F = 50$$

Notice these functions do opposite things. The first equation turned 50°F to 10°C. The second equation turned 10°C back to 50°F. Such functions are called <u>inverse functions.</u>

Here are two other functions that are inverses of each other.

$$y = 2x + 3 \text{ and } y = \frac{x-3}{2}$$

Notice that in the first function a number is multiplied by 2, then 3 is added to the result. In the second function 3 is subtracted from a number, then divided by 2. Begin with 5 as the number in the first function. Multiply by 2, and then add 3; the result is 13. Now, let 13 be the number in the second function. Subtract 3, divide by 2; the result is 5 (the original number selected for the first function).

Let the first equation above be $f(x) = 2x + 3$ and the second be $g(x) = \frac{x-3}{2}$. Examine:

$$f(g(x)) = f\left(\frac{x-3}{2}\right) = 2\left(\frac{x-3}{2}\right) + 3 = x - 3 + 3 = x$$

$$g(f(x)) = g(2x+3) = \frac{2x+3-3}{2} = \frac{2x}{2} = x$$

The composition of two inverse functions of *x* will always be *x*!

I Verifying Inverse Functions

As demonstrated above, when two functions are inverses of each other, their composition functions (both $f(g(x))$ and $g(f(x))$ equal *x*. To verify (or prove) that two functions are inverses, find both compositions and show they equal *x*.

Ex 1: Determine if the following functions are inverses of each other.

$a)\quad f(x)=4x+5,\ g(x)=\dfrac{x-5}{4}$

$d)\quad f(x)=\dfrac{3}{x}+4,\ g(x)=\dfrac{3}{x-4}$

$b)\quad f(x)=\dfrac{x+1}{x},\ g(x)=\dfrac{1}{x+1}$

$e)\quad f(x)=\sqrt[3]{x-5},\ g(x)=x^3+5$

$c)\quad f(x)=\dfrac{1}{x+1},\quad g(x)=x+1$

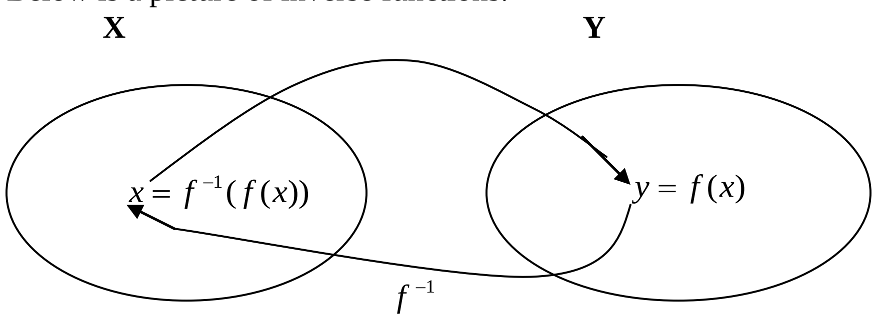
Definition of the Inverse of a Function:
Let *f* and *g* be two functions such that $f(g(x)) = x$ for every *x* in the domain of *g* and $g(f(x)) = x$ for every *x* in the domain of *f*.

The function *g* is called the **inverse of function *f*** and is denoted by f^{-1} (read *f* inverse).

Therefore $f(f^{-1}(x)) = x$ and $f^{-1}(f(x)) = x$. The domain of *f* is equal to the range of f^{-1}, and vice-versa.

Note: The notation f^{-1} does **not** mean $\frac{1}{f}$. The -1 is not an exponent, it is a notation.

Below is a picture of inverse functions.



There is also a good picture in Figure 2.54 on page 285 of the textbook.

II Finding the Inverse of a Function

To find the inverse of a function *f* follow these steps.

1. Replace $f(x)$ with *y* in the equation.
2. Interchange *x* and *y*.
3. Solve for *y*. If this equation does not define *y* as a function of *x*, the function *f* does not have an inverse function and this procedure ends. If this equation does define *y* as a function of *x*, the function *f* has an inverse function.
4. If the function does have an inverse, replace the *y* in step 3 with $f^{-1}(x)$.

Ex 2: Find the inverse of each function or verify that it does not have an inverse.

$a)$ $f(x) = 2x - 3$

$b)$ $f(x) = \dfrac{2}{1+x}$

$c)$ $f(x) = \dfrac{x+2}{x}$

$d)$ $f(x) = \dfrac{1}{x} + 5$

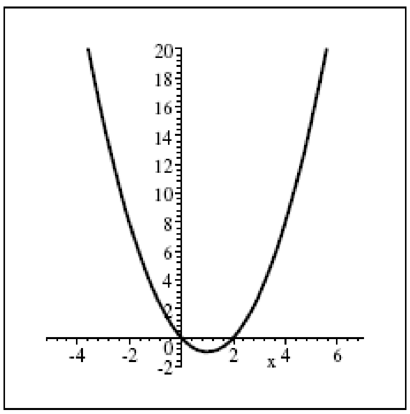
$e)$ $f(x) = x^3 + 2$

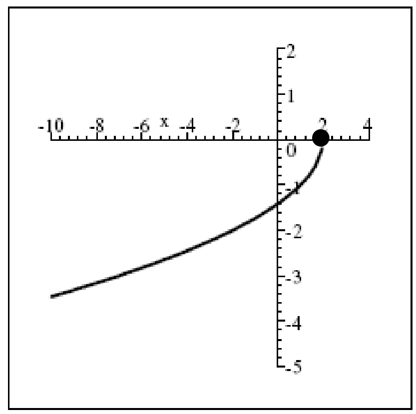
III 1-1 Functions

In order for a function to have an inverse, it must be a **1-to-1 function**, which means for each *x* in the domain, there is only 1 *y* in the range (definition of function) ***and*** for each *y* in the range, there is only 1 *x* in the domain.

One-to-one Functions: A function *f* from a set X to a set Y is called one-to-one if and only if different numbers in the domain of *f* have different outputs in the range of *f*. If the graph of the function *f* is known, the graph must pass not only the vertical line test, but the **horizontal line test** as well.

Ex 3: Determine if these graphs represent 1-to-1 functions.

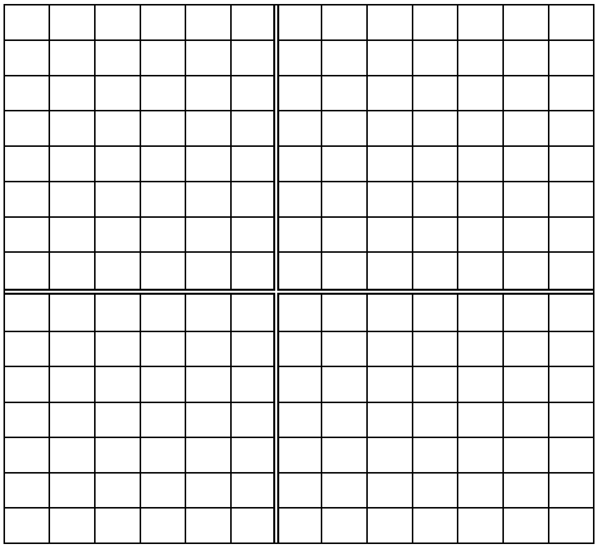
a) 

b) 

IV **Graphs of Inverse Functions**

There is a relationship between the graph of a 1-1 function *f* and its inverse f^{-1}. If the ordered pair (a,b) is on the graph of function f, the ordered pair (b,a) would be on the graph of function f^{-1}. These points would be symmetric with respect to the line $y = x$. The graph of f^{-1} is a reflection of the graph of *f* about the line $y = x$.

Ex 4: Graph the function $f(x) = -2x + 4$ Use key points of that graph to sketch the graph of the inverse of the function. Sketch in the line $y = x$ to show the symmetry about that line.



Ex 5: Use the graph of the function below to sketch its inverse on the same coordinate plane.