We have discussed powers where the exponents are integers or rational numbers. There also exists powers such as $2^{\sqrt{3}}$. You can approximate powers on your calculator using the power key. On most one-liner scientific calculators, the power key looks like $\boxed{y^x}$

Enter the base into the calculator first, press the power key, enter the exponent, and press enter or equal.

Ex 1: Approximate the following powers to 4 decimal places.

$a)$ $2^{3\sqrt{2}} =$ ___________________

$b)$ $(2.3)^{4.8} =$ __________________

I Exponential Functions

A Basic **exponential function *f* with base *b*** is defined by $f(x) = b^x$ or $y = b^x$, where b is a positive constant other than 1 and x is any real number.

> The following are **not** exponential functions. Why?
>
> $f(x) = x^3$ $f(x) = 1^x$
>
> $f(x) = (-4)^x$ $f(x) = x^x$

A calculator may be needed to evaluate some function values of exponential functions. (See example 1 above.)

Many real life situations model exponential functions. One example given in your textbook models the average amount spent (to the nearest dollar) at a shopping mall after *x* hours and is $f(x) = 42.2(1.56)^x$. The base of this function is 1.56. Notice there is also a 'constant' (42.2) multiplied by the power. Be sure to follow the order of operations; find the exponent power first, then multiply that answer by the 42.2.

Suppose you wanted to find the amount spent in a mall after browsing for 3 hours.
Let $x = 3$.

$$f(3) = 42.2(1.56)^3$$
$$= 42.2(3.796416)$$
$$= 160.2087552$$

To the nearest dollar, a person on average would spend $160.

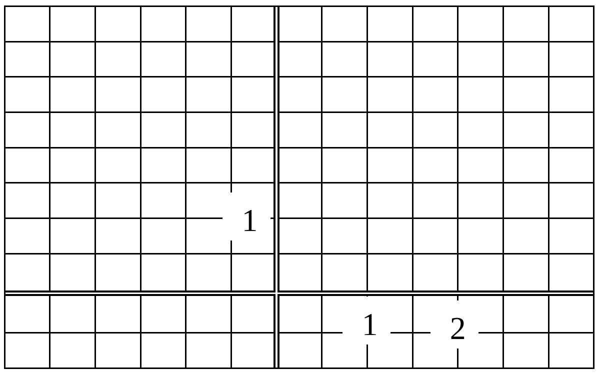
II Graphing Exponential Functions

To graph an exponential function, make a table of ordered pairs as you have for other types of graphs. Notice: If $x = 0$ for b^x, the value is 1 (zero power is 1). For a basic exponential function, the y-intercept is 1.

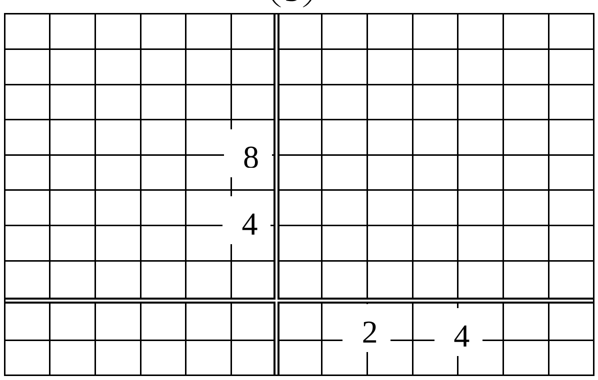
Also, notice that y values will always be positive, so the graph always lies above the x-axis.

Ex 2: Graph each exponential function.

$a)$ $y = \left(\frac{3}{2}\right)^x$



$b)$ $f(x) = \left(\frac{1}{3}\right)^x$



There are several exponential graphs shown in figure 4.4 on page 415 of the text. After examining several graphs, the following characteristics can be found.

Characteristics of Exponential Functions of the form $f(x) = b^x$ (basic)

1. The domain of the function is all real numbers $(-\infty, \infty)$ and the range is all positive real numbers $(0, \infty)$ (graph always lies above the x-axis).
2. Such a graph will always pass through the point (0, 1) and the y-intercept is 1. There will be no x-intercept.
3. If the base b is greater than 1 $(b > 1)$, the graph goes up to the right and is an increasing function. The greater the value of b, the steeper the increase (exponential growth).
4. If the base is between 0 and 1 $(0 < x < 1)$, the graph goes down to the right and is a decreasing function (exponential decay). The smaller the value of b, the steeper the decrease.
5. The graph represents a 1-1 function and therefore will have an inverse.
6. The graph approaches but does not touch the x-axis. The x-axis is known as an **asymptote**.

III The Natural Base *e* and the Natural Exponential Function

There is an irrational number, whose symbol is *e*, that is used quite often as a base for an exponential function. This number is the value of $\left(1+\frac{1}{n}\right)^n$ as *n* becomes very, very large or goes toward infinity. An approximation of this number is $e = 2.718281827$ and the number *e* is called the **natural base**. The function $f(x) = e^x$ is called the **natural exponential function.** To approximate the powers of *e*, use these steps on your TI-30XA calculator.

1. Enter the exponent in your calculator.
2. Because the *e* power is above the *ln* key, you must press the [2nd] key first and then the [ln] key.
3. The result is approximately that power.

The number *e* is similar to the irrational number π. Your calculator will only give approximations of these numbers or their powers.

Ex 3: Approximate each power to 4 decimal places.

a) $e^3 =$

b) $e^{0.024} =$

c) $e^{-\frac{2}{3}} =$

Another life model that uses an exponential function is $f(x) = 1.26e^{0.247x}$, which approximates the gray wolf population of the Northern Rocky Mountains *x* years after 1978. (Notice: Multiply 0.247 by *x*, find the number *e* to that power, then multiply the result by 1.26.)

Ex 4: Use the model above the approximate the gray wolf population in 2008.
2008 is 30 years after 1978. Let *x* = 30.

$$\begin{aligned} f(30) &= 1.26e^{0.247(30)} \\ &= 1.26e^{7.41} \\ &= 1.26(1652.426347) \\ &\approx 2082 \end{aligned}$$

2082 gray wolves

IV Compound Interest

One of the most common models of exponential functions used in life are the models of compound interest. You know that the Simple Interest Formula is $I = Prt$ and the amount accumulated with simple interest is $A = P + Prt$. However, in this model, interest is only figured at the very end of the time period. In most situations, interest is determined more often; sometimes annually, monthly, quarterly, etc. Then the amount accumulated in the account can be determined by the formula below.

> Compound Interest Formula:
> If an account has interest compounded *n* times per year for *t* years with principal *P* and an annual interest rate *r* (in decimal form), the amount of money in the account is found by
> $A = P\left(1+\frac{r}{n}\right)^{nt}$

Some banks or financial institutions may **compound interest continuously**. If that happens, the formula above becomes the following that uses the number *e*.

> Compound Continuously Formula:
> If an account is compounded continuously for *t* years with principal *P* at an annual intrest rate *r* (in decimal form), the amount of money in the account is found by
> $A = Pe^{rt}$.

Ex 5: Suppose $8000 is invested for 5 years at 4.5% annual interest. Find the amount in the account at the end of the 5 years if...

a) interest is compounded quarterly

> Always convert percent rates to decimals in these types of formulas. We are also assuming no additional deposits were made.

b) interest is compounded monthly

c) interest is compounded continuously

Ex 6: Lily's parents deposited an amount in her account on her day of birth. The account earned 6% annual interest compounded continuously and on her 18th birthday the account was $40,000. How much was the initial deposit by her parents?

Ex 7: Which investment would yield the greatest amount of money for an initial investment of $500 over a period of 6 years; 7% compounded quarterly or 6% compounded continuously?

V Other Applied Problems

Ex 8: The population of a city is 45,000 in 2000. The population growth is represented by $P = 45e^{0.011t}$ in thousands for *t* years after 2000. What will be the population in 2010?

Ex 9: The formula $S = C(1+r)^t$ models an inflation value for *t* years from now, where *C* is the current price, *r* is the inflation rate, and *S* is the inflated value. If a house currently is worth $89,000 and the inflation rate is 1.2%, what would the house be worth in 15 years from now?

Ex 10: Sometimes more than one function model could be used for some life situations. Suppose the Purdue Mathematics department determines that the percentage of mathematics remembered x weeks after learning the math can be described by the linear model below or the exponential model below.

$$f(x) = -3.6x + 87$$

$$g(x) = 78e^{-0.1x} + 22$$

a) Determine the percentage of math remembered 4 weeks after learning the math using the linear model.

b) Determine the percentage of math remembered 4 weeks after learning the math using the exponential model.

c) If statistics show that, on average, a math student remembered 75% of what they learned 4 weeks after learning, which model best approximated the percentage after 4 weeks?