MA 15200 Lesson 29 Section 4.3

This lesson is on the **properties of logarithms.** Properties of logarithms model the properties of exponents.

I Product Rule

Product Rule of Exponents: $b^m b^n = b^{m+n}$
Notice: When the bases were the same, the **exponents were added** when multiplication was performed. Likewise **logarithms are added** when multiplication is performed in the argument.

Product Rule of Logarithms: $\log_b(MN) = \log_b M + \log_b N$
In words, the logarithm of a product is the sum of the logarithms.

Informal Proof:
$\log 100 = 2 \qquad \log 1000 = 3$
$\log(100 \cdot 1000)$
$= \log(100,000)$
$= 5$
$= 2 + 3$
$= \log 100 + \log 1000$

<u>When a single logarithm is written using this product rule,</u>
<u>we say we are **expanding the logarithmic expression.**</u>

<u>Ex 1:</u> Assume all variables represent positive values.
Use the product rule to expand each expression **and simplify where possible.**

Caution:
$\log_b(x+y) \neq \log_b x + \log_b y$

$a)$ $\log_2(7r) =$

$b)$ $\log_b\left(2x^2 y\right) =$

$c)$ $\log(100ab) =$

$d)$ $\ln(20e^5) =$

II Quotient Rule

Quotient Rule for Exponents: $\dfrac{b^m}{b^n} = b^{m-n}$
Notice: When the bases were the same, the **exponents were subtracted** when division was performed. Likewise, **logarithms are subtracted** when division is performed in the argument.

CAUTION:
$\log_b(M \pm N) \neq \log_b M \pm \log_b N$

$\log_b \frac{M}{N} \neq \frac{\log_b M}{\log_b N}$

Quotient Rule for Logarithms: $\log_b\left(\frac{M}{N}\right) = \log_b M - \log_b N$

In words, the logarithm of a quotient is the difference of the logarithms.

We can also **expand a logarithm** by using the quotient rule.

Ex 2: Assume all variables represent positive values.
Use the quotient rule to expand each logarithm and **simplify where possible.**

$a)$ $\log_3\left(\frac{9}{y}\right)$

$b)$ $\log\left(\frac{x}{1000}\right)$

Note: Our text and online homework does not usually use parenthesis around the argument. However, it would be better to write as in the following.
$\ln\left(\frac{4x^3}{yz^5}\right)$

III Power Rule

Power Rule for Exponents: $(b^m)^n = b^{mn}$

Note: When a power is raised to another power, the **exponents are multiplied**.
Likewise, when a logarithm has an exponent in the argument, **the exponent is multiplied by the logarithm**.

Power Rule for Logarithms: $\log_b M^p = p\log_b M$

In words, the logarithm of a power is the product of the exponent and the logarithm.

We can also **expand a logarithm** by using the power rule.

Ex 3: Assume all variable represent positive values.
Use the power rule to expand each logarithm and **simplify where possible.**

$a)$ $\log x^8 =$

$b) \quad \log_5\left(25^3\right) =$

$c) \quad \ln\sqrt{y} =$

IV Here is a summary of all the properties of logarithms.

Assume all variables represent positive values and that all bases are positive number (not 1).

1. $\log_b(MN) = \log_b M + \log_b N$ Product Rule
2. $\log_b\left(\dfrac{M}{N}\right) = \log_b M - \log_b N$ Quotient Rule
3. $\log_b\left(M^p\right) = p\log_b M$ Power Rule

Ex 4: Use the properties to expand each logarithmic expression. Assume all variables represent positive values.

$a) \quad \log\dfrac{xy}{\sqrt{z}} =$

$b) \quad \ln\dfrac{4x^3}{yz^5} =$

$c) \quad \log_2 3\sqrt{xy} =$

$d) \quad \log_3\left(27x^2\sqrt[3]{y}\right)$

In opposite of expanding a logarithmic expression is **condensing a logarithmic expression**. This is writing a logarithmic expression as a single logarithm.

Ex 5: Condense each expression. In other words, write as a single logarithm. Assume all variables represent positive values.

$a)\quad \log 3 - \log x + 2\log y - \frac{1}{2}\log z =$

$b)\quad \frac{1}{3}\log(x-2) + 2\log x - 2\log 4 =$

$c)\quad \frac{1}{2}(\ln x + 3\ln y) - 3\ln(x+2)$

Ex 6: If $\log_b m = 2.3892, \log_b n = -1.2389, \text{and } \log_b r = 0.8881$, use the properties of logs to find the following values.

$a)\quad \log_b(m^2 n) =$

$b)\quad \log_b\left(\frac{\sqrt{n}}{r}\right) =$

Ex 7: If $\log_b 8 = 1.8928, \log_b 11 = 2.1827, \text{and } \log_b 2 = 0.6309$. Use these values and the properties of logs to find the following values.

$a)\quad \log_b 4 =$

$b)$ $\log_b 88 =$

There may be more than 1 way to determine these values.

$c)$ $\log_b 121 =$

$d)$ $\log_b 44 =$

Ex 8: Let $\log_2 4 = A$ and $\log_2 5 = B$. Write each expression in terms of A and/or B.

$a)$ $\log_2(125) =$

$b)$ $\log_2\left(\frac{5}{4}\right) =$

V Change of Base Formula

Your scientific calculator will approximate or find common logarithms (base 10) or natural logarithms (base e). How can logarithms with other bases be approximated?

$$\log_b M = \frac{\log M}{\log b} \text{ or } \frac{\ln M}{\ln b}$$

Note: $\log\left(\frac{M}{N}\right) \neq \frac{\log M}{\log N}$

The formula above is known as the change of base formula.

Ex 9: Approximate each logarithm to 4 decimal places.

$a)$ $\log_3(22.8) =$

$b)$ $\log_{0.2}(285) =$