MA 15200 Lesson 30 Exponential and Logarithmic Application Problems

In order to solve the applied problems in this lesson, a student must know how to use the log, ln, e^x**,** and power key functions on a scientific calculator.

There are lots of real life problems that have formulas with exponential or logarithmic expressions. The examples found in this lesson are just some.

Ex 1: **Half-Life of an Element**
The **half-life** of an element is the amount of time necessary for the element to decay to half the original amount. Uranium is an example of an element that has a half-life. The half-life of radium is approximately 1600 years. The formula used to find the amount of radioactive material present at time *t*, where A_0 is the initial amount present (at $t = 0$), and h = half-life of the element is $A = A_0 2^{-\frac{t}{h}}$ *or* $A = A_0 2^{-t/h}$.

Tritium, a radioactive isotope of hydrogen, has a half-life of 12.4 years. If an initial sample has 50 grams, how much will remain after 100 years? Round to 4 decimal places.

A subscript of 0, such as A_0, means the amount initially or amount at time 0.

It will be extremely helpful if you know how to store and recall numbers using your TI-30xa calculator.

Work these problems step by step.

Ex 2: **Population Growth**
Another formula represents the population growth of lots of cities, towns, or countries. The formula is $P = P_0 e^{kt}$ where P is the final population, P_0 is the initial population at time 0, t is time in years, and $k = b - d$ (b is birth rate and d is death rate). k is a percent converted to a decimal. Note: In order for a country to grow k must be positive.

The population of a city is 45,000 in 2008. The birth rate is 11 per 1000 and the death rate is 8 per 1000, so the value of k is 0.011 - 0.008 = 0.003. What will be the population in 2018? Round to the nearest whole number.

Ex 3: **Light Intensity**

The intensity of light I (in lumens) at a distance of x meters below the surface of water is represented by $I = I_0 k^x$, where I_0 is the intensity of light above the water and k is a constant that depends on the clarity of the water.

At the center of a certain lake, the intensity of light above the water is 10 lumens and the value of k is 0.8. Find the intensity of light at 3 meters below the surface.

Ex 4: **Population Growth**

The population of Eagle River is growing exponentially according to the model, $P = 375(1.3)^t$, where t is years from the present date. Find the population in 6 years.

Ex 5: **Percent of Alcohol in Bloodstream**

For one individual, the percent of alcohol absorbed into the bloodstream after drinking two shots of whiskey is given by $P = 0.3(1 - e^{-0.05t})$, where t is in minutes. Find the percent of alcohol in the bloodstream after ½ hour.

Ex 6: **Decibel Voltage Gain**

The measure of voltage gain of devices such as amplifiers or the length of a transmission line is measures in decibels. If E_O is the output voltage of a device and E_i is the input voltage, the decibel voltage gain is given by db gain $= 20\log\frac{E_O}{E_i}$.

Find the db gain of an amplifier whose input voltage is 0.71 volt and whose output voltage is 20 volts.

Ex 7: **Earthquake Richter Scale**

The measurement of the intensity of an earthquake is a number from the Richter Scale. If *R* represents the intensity (Richter number), *A* is the amplitude (measured in micrometers), and *P* is the period (time of one oscillation of the Earth's surface, measured in seconds), then $R = \log\frac{A}{P}$. (Notice the Richter scale is based on common logarithms. An earthquake of 4.0 is 10 times greater than one of 3.0.)

Find the intensity of an earthquake with amplitude of 6000 micrometers and a period of 0.08 second. Round to the nearest tenth.

Ex 8: **Charging a Battery**

The time in minutes required to charge a battery depends on how close it is to being fully charged. If *M* is the theoretical maximum charge, *k* is a positive constant that is dependent upon the type of battery and charger, *C* is the given level of *M* (percent or decimal part of M) to which the battery is being charged, then the time *t* required to reach that level is given by $t = -\frac{1}{k}\ln(1 - \frac{C}{M})$. Note: *C* is usually a percentage of *M*. In the question below, $C = 0.4M$.

If $k = 0.201$, how long will it take a battery to reach a 40% charge? Assume that the battery was fully discharged when it began charging.

Ex 9: **Doubling Population**

If a population is growing exponentially at a certain annual rate, then the time required for that population to double is called the doubling time and is given by $t = \frac{\ln 2}{r}$, where *t* is time in years.

A town's population is growing at 9.2% per year. If this growth rate remains constant, how long will it take for the town's population to double?

Ex 10: **Annual Growth Rate**

If an investment is growing continuously for t years, its annual growth rate r is given by the formula $r = \frac{1}{t}\ln\frac{P}{P_0}$, where P is the current value and P_0 is the initial value of the investment.

An investment of $10,400 in America Online in 1992 was worth $10,400,000 in 1999. Find AOL's average annual growth rate during this period. Round to the nearest hundredth of a percent.