Lesson 36   MA 152, Section 3.1

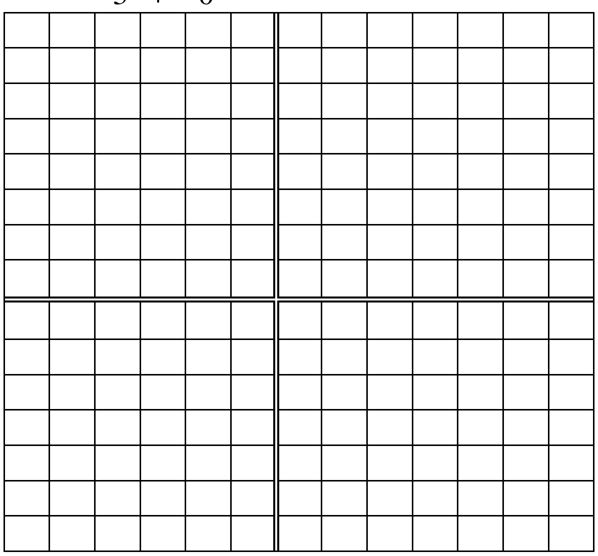
I   Quadratic Functions

A **quadratic function** of the form $y = f(x) = ax^2 + bx + c$, where *a, b,* and *c* are real numbers (general form) has the shape of a **parabola** when graphed. The parabola will open **upward** if the value of *a* is positive and downward is it is negative. The **vertex** is the point or ordered pair where the parabola 'turns'.

Ex 1: Graph the parabola $y = -\frac{1}{2}x^2 - x + \frac{3}{2}$. Find its vertex and direction of opening.

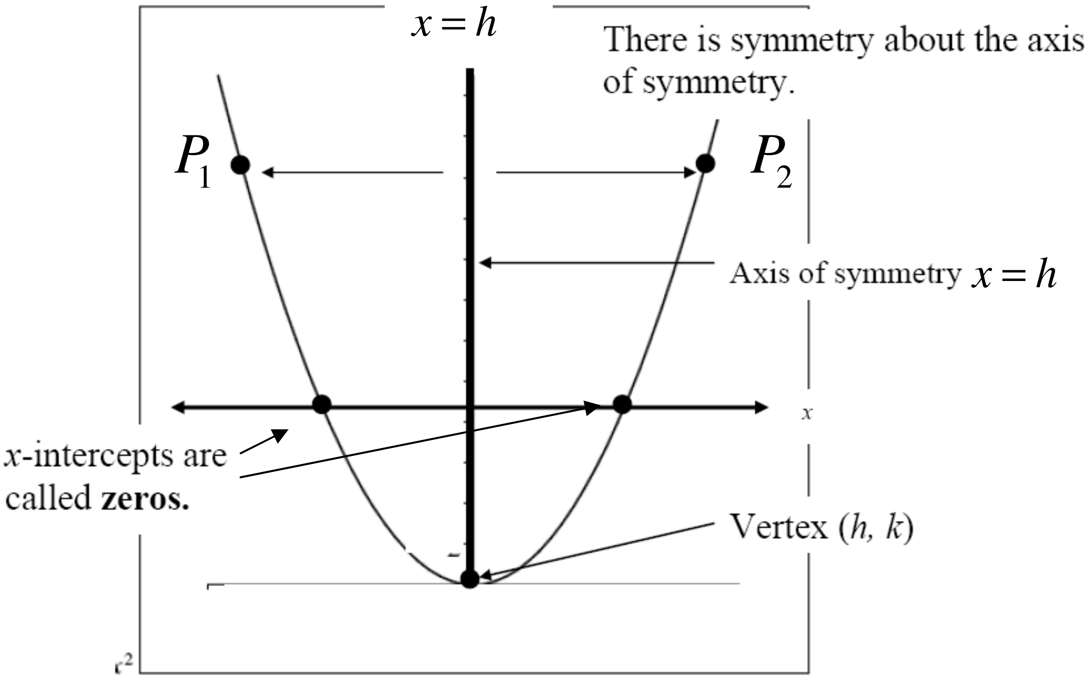
We will use a table of values and plot the points.

x	*y*
0	3/2
1	0
-1	2
2	-5/2
-2	3/2
-3	0



This method is tedious. It will be easier to know how to find the vertex. We could also find intercepts and use symmetry. Notice, the graph is symmetric about a vertical line through the vertex.

The **vertex** will be an ordered pair (*h, k*).
The **axis of symmetry** is a vertical line with through the vertex. Points have symmetry (equal distance) left and right about this vertical line. The equation will be $x = h$.



Look at the graph of the example on page 1 and answer the following questions.

(1) zeros:

(2) axis of symmetry:

(3) a pair of symmetric points

If a 'completing the square' process is used on the general form, equations for h and k of the vertex can be found.

$$y = ax^2 + bx + c$$

$$y = a(x^2 + \frac{b}{a}x) + c$$

$$y = a(x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2) + c - a\left(\frac{b}{2a}\right)^2$$

Let $k = c - a\left(\frac{b}{2a}\right)^2$

$$y = a(x + \frac{b}{2a})^2 + k$$

Let $h = -\frac{b}{2a}$

$$y = a(x-h)^2 + k$$

MA 15300 students are required to know how to complete the square to find a standard quadratic equation from a general quadratic equation.

Students in MA 15200 are not required to know how to do this.

Standard Equation for a Parabola:
If the vertex of a parabola is (*h, k*) and the parabola opens upward or downward, the standard equation of the parabola has the form $y = a(x-h)^2 + k$. If *a* is positive, the parabola opens upward, negative it opens downward.

The standard form of the equation of the parabola that was graphed in Ex. 1 is $y = -\frac{1}{2}(x+1)^2 + 2$.

Ex 2: For each parabola, find :
1) The vertex
2) Equation for the axis of symmetry
3) Direction of opening
4) Domain and range
5) y-intercept
6) A point corresponding to the y-intercept that has the same y-coordinate

$a)\quad f(x) = 2(x+4)^2 + 6$

$b)\quad f(x) = -\frac{1}{4}(x+2)^2$

From the process on page 2, you can see that the coordinates of the vertex can be found from the general form by the following equations.

$$h = -\frac{b}{2a}$$

$$k = c - \frac{b^2}{4a}$$

Rather than finding *k* by using the formula above, it is easier to substitute the value of *h* for *x* in the quadratic equation and solve for *y*. $k = f(h)$

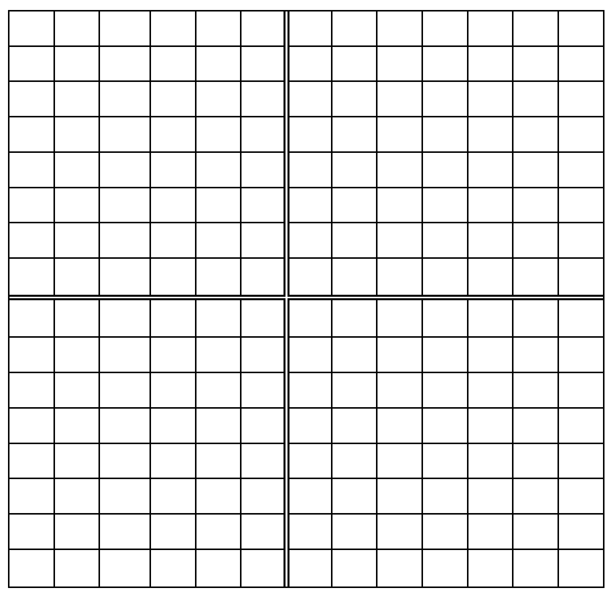
Ex 3: Find the vertex and axis of symmetry of each parabola. Write equations in standard form.

$a)$ $f(x) = 2x^2 - 4x + 7$

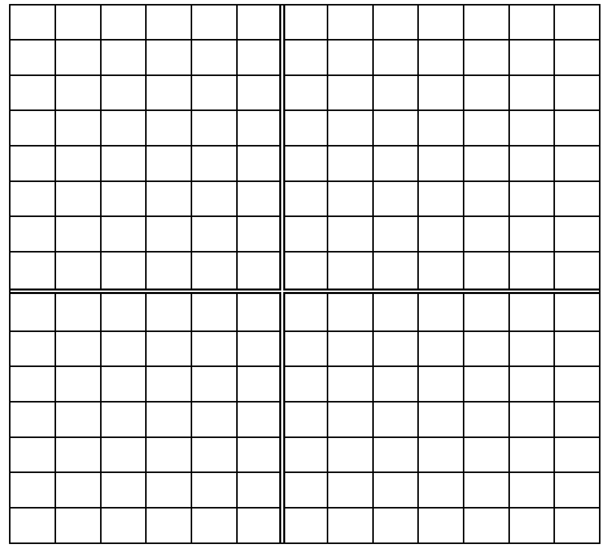
$b)$ $f(x) = -\frac{1}{3}x^2 + 8x - 2$

Ex 4: Graph the parabola by finding the vertex and intercepts. Use symmetry. Describe axis of symmetry, vertex, domain and range (using interval notation).

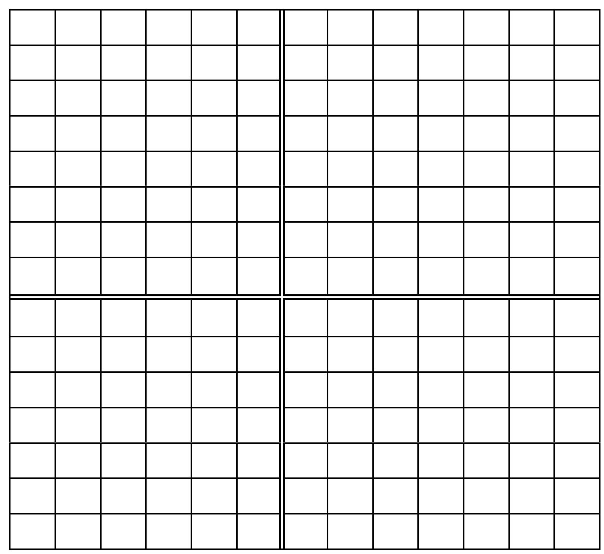
$y = \frac{1}{2}x^2 - 2x + \frac{3}{2}$



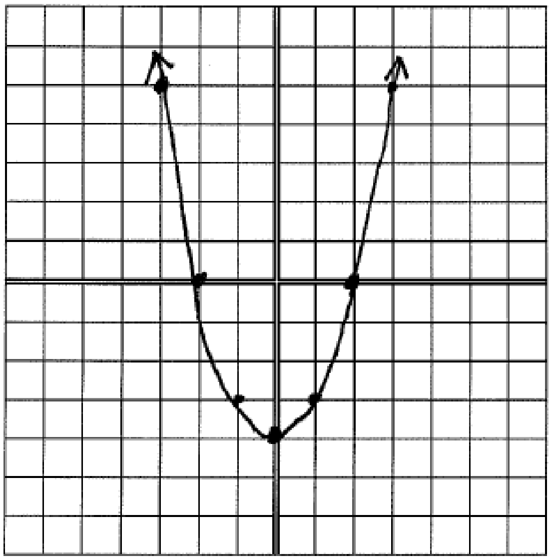
$y = 16 - (x+1)^2$

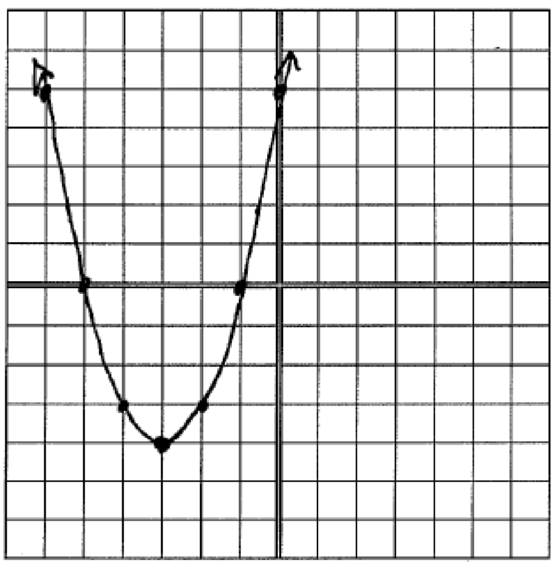


$y = (x-2)^2 - 4$



Ex 5: From each parabola, give the equation in standard form. Assume the value of *a* is 1.





Minimums and Maximums:

If a parabola opens upward, its vertex is the location of a **relative minimum**. If it opens downward, its vertex is the location of a **relative maximum**. If the vertex is (*h, k*), the maximum of minimum value is $k = f(h)$ and it occurs when $x = h$.

Ex 6: For each quadratic function describe if it has a maximum or minimum, what the maximum or minimum value is and where it occurs, and the domain and range of the function (using interval notation).

$a)$ $f(x) = -3x^2 + 12x - 11$

$b)$ $g(x) = \frac{1}{2}x^2 + 4x + 6$

Ex 7: Write an quadratic function (in standard form) that has the same shape as $g(x) = 2x^2$, but with a vertex $(-3, 12)$.

Finding a quadratic function using the vertex and a second point:
The equation for a quadratic function can be found if the vertex and one other point of the parabola is known. Replace x, y, h, and k with the correct values and solve for a in $f(x) = a(x-h)^2 + k$.

Ex 8: Find equation for each in standard form.

$a)$ $V(2, -3)$, $P(1, 4)$

$b)$ $V(9, -1)$, x-intercept is 4

Ex 8: A human cannonball shoots out of a cannon and the parabolic path he travels is given by $f(x) = -0.0125x^2 + x + 5$.

a) What is the maximum height above ground and how far from the beginning (horizontally) does this occur?

b) To the nearest tenth of a foot, if there was no net, how far away (horizontal direction) would he land?

c) How far above the ground was the cannon when it was fired?

Ex 9: Find a pair of number whose sum is 20 and whose product is as large as possible (a maximum). What is this maximum product?