Lesson 38 MA 15200, Appendix I Section 5.5

You are familiar with the simple interest formula, $I = prt$. However, in many accounts the interest is left in the account and earns interest also. We say the account earns **compound interest.**

For example: Suppose Bob invests \$100 at 10% simple interest. At the end of 1 year, Bob has earned $I = 100(.10)(1) = \$10$. He now has \$110.
At the end of the 2nd year, Bob has earned $I = 110(.10)(1) = \$11$. He now has \$121.
At the end of the 3rd year, Bob has earned $I = 121(.10)(1) = \$12.10$. He has a total of \$143.10. I'm sure you get the idea of what is happening.

Formula for Compound Interest with **<u>Annual</u>** compound interest:
$S = P(1+r)^t$, where P is the initial investment (principal), t is the number of years, r is the annual interest rate, and S is the future value or final value.

Ex 1: Assume that \$1500 is deposited in an account in which interest is compounded annually at a rate of 6%. Find the accumulated amount after 5 years.

Ex 2: Assume that \$1500 is deposited in an account in which interest is compounded annually for 5 years. Find the accumulated amount, if the interest rate is 8 ½ %.

Many banks or financial institutions figure interest more often than once a year; quarterly monthly, semiannually, daily, etc. For example, if the annual rate or **nominal rate** is 12% and interest is compounded quarterly, that is equivalent to 3% every 3 months. 3% is called the **periodic rate**.

Formula for Periodic Rate: Periodic Rate = $\dfrac{\text{annual rate}}{\text{number of periods per year}}$

$i = \dfrac{r}{k}$, where r is annual interest rate, k is the number of times interest is paid each year, and i is the periodic rate.

Ex 3: Find the periodic rate in each example.

a) annual rate: 10%, compounded quarterly

b) annual rate: 3.6%, compounded monthly

Compound Interest Formula (Future Value of an Investment):
Let P be principal earning interest compounded k times per year for n years at an annual rate of r. Then, the final or future value will be

$* S = P(1+i)^{kt}$, where $i = \dfrac{r}{k}$

*Earlier in the semester, when we had this formula, it was written $A = P\left(1+\dfrac{r}{n}\right)^{nt}$, where A is the final amount, P is principal or beginning amount, r is annual interest rate, n is number of compounding periods a year, and t is time in years. This lesson the formula is simply written differently.

Ex 4: Assume that $1500 is deposited in an account in which interest is compounded monthly at an annual rate of 6%.

a) Find the accumulated amount after 8 years.

b) How much interest was earned during the 8 years?

Financial institutions are required to provide customers with the **effective rate of interest**, that rate at which, if compounded annually, would provide the same yield as the plan where interest is compounded more frequently.
In other words: For what interest rate is $P(1+r)^n = P(1+i)^{kt}$? If this equation is solved for *r*, we get the following formula.

Effective Rate of Interest: The effective rate of interest *R* for an account paying a nominal or annual interest rate *r*, compounded *k* times per year is….

$$E = (1+i)^k - 1, \text{ where } i \text{ (the periodic rate)} = \frac{r}{k}.$$

<u>Ex 5:</u> Find the effective rate of interest given the annual rate and the compounding frequency.

a) $r = 9\%$, $k = 2$

b) $r = 11\frac{1}{2}\%$, $k = 4$

We studied the continuously compounded formula for an investment earlier in lesson 27. It was given as $A = Pe^{rt}$. For this lesson, it will be written $S = Pe^{rt}$, where *S* is the final amount of the investment.

<u>Ex 6:</u> Jake has the option of investing $1200 at an annual rate of 4.8% compounded quarterly or at an annual rate of 4.6% compounded continuously. Which would result in the best investment in a year's time?

Often people need to know what amount must be invested (principal) in order to end up with a certain future or final value.

$S = P(1+i)^{kt}$

Solve the formula above for *P*.

$S = P(1+i)^{kt}$

Divide both sides by $(1+i)^{kt}$

$\dfrac{S}{(1+i)^{kt}} = P$

Since an exponent is the opposite when moved from denominator to numerator...

$S(1+i)^{-kt} = P$

This is the formula for present value, when you need to **find what principal or investment now** would result in a given final value.

> **Present Value Formula:** The present value *P* that must be deposited now in order to result in a future value *S*, in *t* years is given by…
>
> $P = S(1+i)^{-kt}$, where interest is compounded k times per year at an annual rate r, and $i = \dfrac{r}{k}$

<u>Ex 7:</u> Find the present value of \$15,000 due in 8 years, at the annual rate of 11% and compounding daily. **Note: Compounded daily is counted as 365 times a year.**

Applied Problems

Ex 8: After the birth of their first granddaughter, the Fields deposited $8000 in a savings account paying 6% interest, compounded quarterly. How must will be available for this granddaughter for college, when she turns 18? How much interest was earned during that time?

Ex 9: A financial institution offers two different accounts. The NOW account has a 7.2% annual interest rate, compounded quarterly and the Money Market account is 6.9% annual rate, compounded monthly. Compare the effective interest rates for the two accounts.

Ex 10: A businessman estimates the computer he needs for his business that he plans to buy in 18 months will cost $5500. To meet this cost, how much should he deposit now in an account paying 5.75% compounded monthly?