NAME SOLUTIONS

10-DIGIT PUID ______

REC. INSTR. ______ REC. TIME ______

LECTURER ______

INSTRUCTIONS:

1. There are 8 different test pages (including this cover page). Make sure you have a complete test.
2. Fill in the above items in print. Also write your name at the top of pages 2–8.
3. Do any necessary work for each problem on the space provided or on the back of the pages of this test booklet. Circle your answers in this test booklet. No partial credit will be given.
4. No books, notes, calculators or any electronic devices may be used on this exam.
5. Each problem has 8 points assigned. 4 points are given for taking the exam. The maximum possible score is 96+4=100 points.
6. Using a #2 pencil, fill in each of the following items on your scantron sheet:
   (a) On the top left side, write your name (last name, first name), and fill in the little circles.
   (b) On the bottom left side, under SECTION NUMBER, put 0 in the first column and then enter the 3-digit section number. For example, for section 016 write 0016. Fill in the little circles.
   (c) On the bottom, under TEST/QUIZ NUMBER, write 02 and fill in the little circles.
   (d) On the bottom, under STUDENT IDENTIFICATION NUMBER, write in your 10–digit PUID, and fill in the little circles.
   (e) Using a #2 pencil, put your answers to questions 1–12 on your scantron sheet by filling in the circle of the letter of your response. Double check that you have filled in the circles you intended. If more than one circle is filled in for any question, your response will be considered incorrect. Use a #2 pencil.
7. After you have finished the exam, hand in your scantron sheet and your test booklet to your recitation instructor.

(8 pts) 1. Find the absolute maximum and absolute minimum of the function

$$f(x) = 3x^4 - 4x^3 - 12x^2 + 1$$

on the interval $[-3, 3]$, without specifying the value of x which attains the absolute maximum or absolute minimum.

A. absolute max 33 absolute min -31

B. absolute max 28 absolute min -4

* C. absolute max 244 absolute min -31

D. absolute max 33 absolute min 1

E. NO absolute max absolute min -35

$f'(x) = 12x^3 - 12x^2 - 24x$
$= 12x(x^2 - x - 2)$
$= 12x(x+1)(x-2)$

$f'(x) = 0$: $x = -1, 0, -2$

$f(-1) = -4$

$f(0) = 1$

$f(2) = -31$ ← abs. min

$f(-3) = 3 \cdot 81 + 4 \cdot 27 - 12 \cdot 9 + 1$
$= 243 + 108 - 108 + 1 = 244$ ← abs. max

$f(3) = 28$

(8 pts) 2. If $f(x) = \dfrac{x^2}{x-2}$, then find the values of x at which f has a local maximum or a local minimum.

$f'(x) = \dfrac{(x-2)2x - x^2 \cdot 1}{(x-2)^2} = \dfrac{x^2 - 4x}{(x-2)^2}$

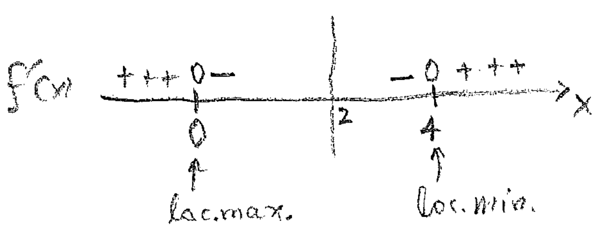
A. local max at $x = 2$ local min at $x = 0$

B. local max at $x = 2$ local min at $x = 4$

* C. local max at $x = 0$ local min at $x = 4$

D. local max at $x = 0$ local min at $x = 2$

E. local max at $x = 0$ NO local min



(8 pts) 3. If $g(x) = 3x^4 - 4x^3$, which of the following statements are true?

(1) g is increasing on $(1, \infty)$.

(2) g has a local minimum at $x = 0$.

(3) The graph of g is concave downward on $\left(0, \frac{2}{3}\right)$.

$g'(x) = 12x^3 - 12x^2 = 12x^2(x-1)$

$g'(x)$: $- - - \; 0 \; - - - \; 0 + + + +$; marked points 0, 1 on the x-axis

(1) is true

(2) is not true

$g''(x) = 36x^2 - 24x = 12x(3x-2)$

$g''(x)$: $+ + + + \; 0 \; - - - \; 0 + + + +$; marked points 0, $\frac{2}{3}$ on the x-axis

(3) is true

A. (1), (2), and (3)

B. (3) only

C. (1) only

D. (1) and (2) only

* E. (1) and (3) only

(8 pts) 4. Suppose that the second derivative of the function $f(x)$ is given by

$$f''(x) = (x+3)^3 \overbrace{(x+1)^2}^{\geq 0} (x-1)^5 \overbrace{(x-3)^6}^{\geq 0} \overbrace{(x-5)^4}^{\geq 0}.$$

How many inflection points does the graph of $y = f(x)$ have ?

$f''(x)$: $+ + 0 - -$ at -3; $- - 0 + +$ at 1 on the x-axis

A. None

B. 1

* C. 2

D. 3

E. 4

(8 pts) 5. If $f'(x) = g'(x)$ for all x on the interval $(0, 9)$ and $f(1) - g(1) = 3$, then determine the value for $f(6) - g(6)$.

A. 5

B. 4

✱ C. 3

D. 2

E. can not be determined from the above information only

Let $F(x) = f(x) - g(x)$

Then $F'(x) = f'(x) - g'(x) = 0$ on $(0, 9)$

$\therefore$ $f(x) - g(x) = \text{const}$ on $(0, 9)$

$f(1) - g(x) = 3$

$\therefore$ $f(6) - g(6) = 3$

(8 pts) 6. Compute the following limit

$$\lim_{x \to 0} \frac{e^{3x} - 1 - x}{x^2}$$

Warning: In the numerator, the power of e is $3x$ and not x.

$$\lim_{x \to 0} \frac{e^{3x} - 1 - x}{x^2} \overset{L'H}{=} \lim_{x \to 0} \frac{3e^{3x} - 1}{2x}$$

$\frac{0}{0}$

$$\lim_{x \to 0^+} \frac{e^{3x} - 1 - x}{x^2} \overset{L'H}{=} \lim_{x \to 0^+} \frac{\overbrace{3e^{3x} - 1}^{\to 2}}{\underbrace{2x}_{\to 0^+}} = \infty$$

$\frac{0}{0}$

$$\lim_{x \to 0^-} \frac{e^{3x} - 1 - x}{x^2} \overset{L'H}{=} \lim_{x \to 0^-} \frac{\overbrace{3e^{3x} - 1}^{\to 2}}{\underbrace{2x}_{\to 0^-}} = -\infty$$

$\frac{0}{0}$

✱ A. DNE

B. $\frac{1}{2}$

C. 1

D. 2

E. ∞

(8 pts) 7. Compute the following limit

$$\lim_{x\to\infty}\left(1+\frac{2}{x}\right)^{x}$$

$\left(1+\frac{2}{x}\right)^x = e^{\ln(1+\frac{2}{x})^x} = e^{x\ln(1+\frac{2}{x})}$

$\lim_{x\to\infty}\left(1+\frac{2}{x}\right)^x = e^{\lim_{x\to 0} x\ln(1+\frac{2}{x})} = e^2$

$\lim_{x\to\infty} x\ln(1+\frac{2}{x}) = \lim_{x\to\infty}\frac{\ln(1+\frac{2}{x})}{\frac{1}{x}} =$ $\frac{0}{0}$

$\overset{L'H}{=} \lim_{x\to\infty}\frac{\frac{1}{1+\frac{2}{x}}\left(-\frac{2}{x^2}\right)}{-\frac{1}{x^2}} = 2\lim_{x\to\infty}\frac{1}{1+\frac{2}{x}} = 2$

A. 0

B. 1

C. e

* D. e^2

E. DNE

(8 pts) 8. Compute the following limit

$$\lim_{x\to 1}\left(\frac{x}{x-1}-\frac{1}{\ln x}\right)$$

$\lim_{x\to 1}\left(\frac{x}{x-1}-\frac{1}{\ln x}\right) = \lim_{x\to 1}\frac{x\ln x - x + 1}{(x-1)\ln x}$

$\infty - \infty$ $\qquad \frac{0}{0}$

$\overset{L'H}{=} \lim_{x\to 1}\frac{x\cdot\frac{1}{x}+\ln x - 1}{(x-1)\frac{1}{x}+\ln x}$

$= \lim_{x\to 1}\frac{\ln x}{1-\frac{1}{x}+\ln x} \overset{L'H}{=} \lim_{x\to 1}\frac{\frac{1}{x}}{\frac{1}{x^2}+\frac{1}{x}}$

$\frac{0}{0}$

$= \lim_{x\to 1}\frac{x}{1+x} = \frac{1}{2}$

A. 0

B. $\frac{1}{4}$

C. $\frac{1}{3}$

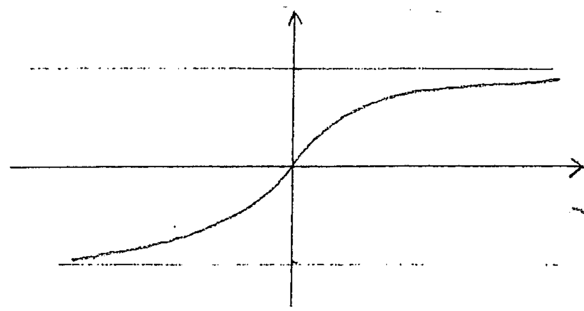
* D. $\frac{1}{2}$

E. 1

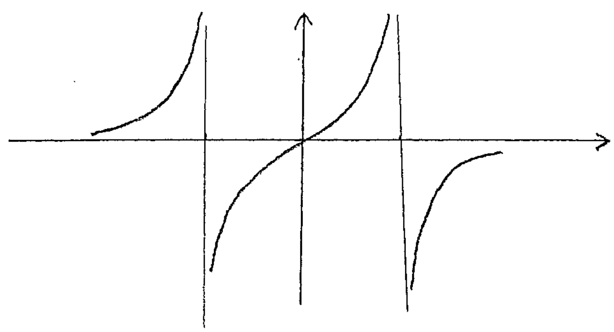
(8 pts) 9. Which of the following is the graph of the function $f(x) = \dfrac{x^2}{x^2+9}$?

symm about y axis
$y \geq 0$
HA $y = 1$

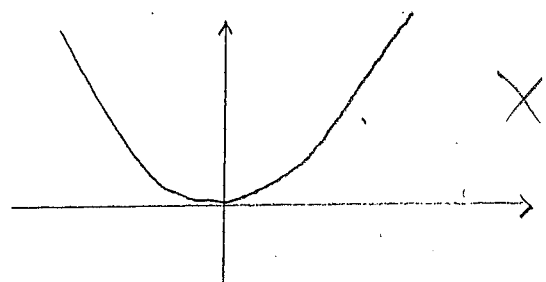
A.



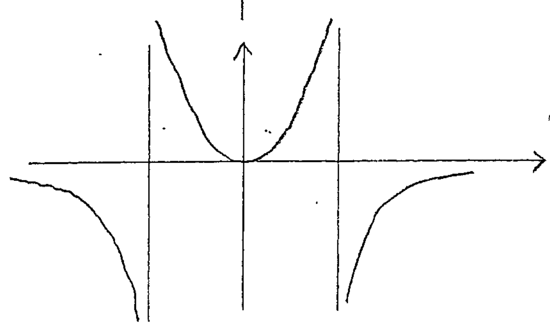
B.



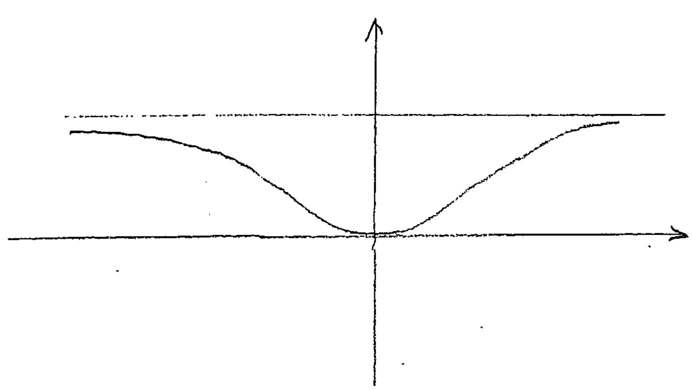
C.



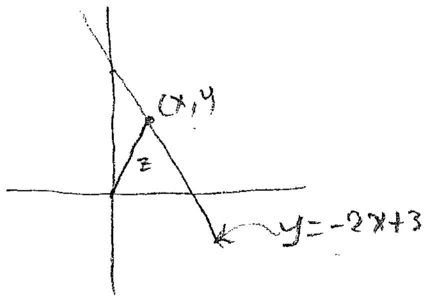
D.



E.



(8 pts) 10. Find the point on the line $y = -2x + 3$ that is closest to the origin.



$$z = \sqrt{x^2 + y^2}$$
$$z = \sqrt{x^2 + (-2x+3)^2}$$
$$z = \sqrt{x^2 + 4x^2 - 12x + 9}$$
$$z = \sqrt{5x^2 - 12x + 9}$$

A. $\left(-\frac{6}{5}, \frac{3}{5}\right)$

* B. $\left(\frac{6}{5}, \frac{3}{5}\right)$

C. $(-3, -3)$

D. $\left(-\frac{5}{6}, \frac{4}{3}\right)$

E. $\left(\frac{5}{6}, \frac{4}{3}\right)$

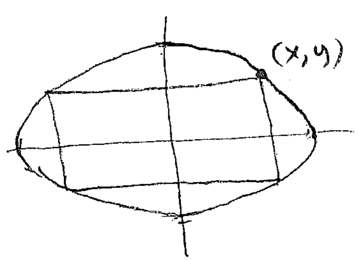
$$\frac{dz}{dx} = \frac{1}{2\sqrt{5x^2 - 12x + 9}}(10x - 12)$$

$\frac{dz}{dx}$ — — — — — 0 + + + + x at $\frac{6}{5}$ (abs. min)

abs min at $x = \frac{6}{5}$ $\quad y = -2\left(\frac{6}{5}\right) + 3 = -\frac{12}{5} + 3 = \frac{3}{5}$

(8 pts) 11. Find the area of the largest rectangle that can be inscribed in the ellipse

$$\frac{x^2}{25} + \frac{y^2}{9} = 1.$$



$$A = 2x\,2y = 4xy$$
$$y^2 = 9\left(1 - \frac{x^2}{25}\right)$$
$$y^2 = \frac{9}{25}(25 - x^2)$$
$$y = \frac{3}{5}\sqrt{25 - x^2}$$

* A. 30

B. 15

C. 12

D. 5

E. 3

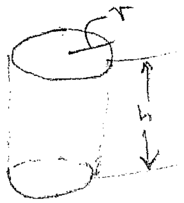
$$A = \frac{12}{5} x \sqrt{25 - x^2}$$
$$\frac{dA}{dx} = \frac{12}{5}\left[x \frac{1}{2\sqrt{25-x^2}}(-2x) + \sqrt{25-x^2}\right]$$
$$= \frac{12}{5}\left[\frac{-x^2}{\sqrt{25-x^2}} + \frac{25-x^2}{\sqrt{25-x^2}}\right] = \frac{12}{5}\,\frac{25-2x^2}{\sqrt{25-x^2}}$$

$$\frac{dA}{dx} = 0 \quad x^2 = \frac{25}{2} \to x = \frac{5}{\sqrt{2}}$$

$\frac{dA}{dx}$ + + + + 0 — — — — x at $\frac{5}{\sqrt{2}}$ (abs. max)

$$\max A = \frac{12}{5}\cdot\frac{5}{\sqrt{2}}\sqrt{25 - \frac{25}{2}}$$
$$= \frac{12}{5}\cdot\frac{5}{\sqrt{2}}\cdot\frac{5}{\sqrt{2}} = 30$$

(8 pts) 12. A cylindrical can (with both top and bottom lids) is to hold 2000cm^3 of oil. Find the radius of the can that will minimize the cost of the metal to manufacture the can.



$$V = \pi r^2 h.$$
$$\pi r^2 h = 2000$$
$$A = 2\pi r^2 + 2\pi r h$$
$$A = 2\pi r^2 + 2\pi r \frac{2000}{\pi r^2}$$
$$A = 2\pi r^2 + \frac{4000}{r}$$
$$\frac{dA}{dr} = 4\pi r - \frac{4000}{r^2}$$
$$= 4\,\frac{\pi r^3 - 1000}{r^2}$$
$$\frac{dA}{dr} = 0 \rightarrow r^3 = \frac{1000}{\pi}$$
$$r = \frac{10}{\sqrt[3]{\pi}}$$

$\frac{dA}{dr}$ $- - - - \; 0 \; + + + + +$ $\rightarrow r$

$\frac{10}{\sqrt[3]{\pi}}$

abs. min.

A. $\sqrt[3]{\frac{500}{\pi}}$

* B. $\frac{10}{\sqrt[3]{\pi}}$

C. $\frac{10}{\pi}$

D. $\frac{20}{\sqrt[3]{\pi}}$

E. $\frac{20}{\pi}$