EXAMPLE 9 *Home Range.* The *home range* of an animal is defined as the region to which the animal confines its movements. It has been hypothesized in statistical studies* that the area H of that region can be approximated by the function

$$H = W^{1.41},$$

where W is the weight of the animal. Graph the function.

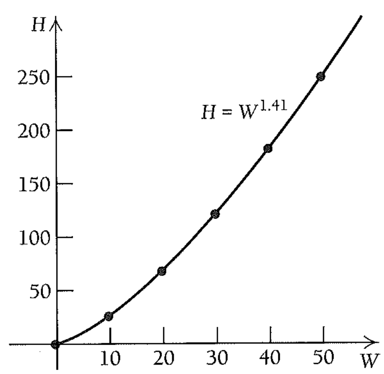
Solution We can approximate function values using a power key $\boxed{y^x}$ on a calculator.

W	0	10	20	30	40	50
H	0	26	68	121	182	249

We see that

$$H = W^{1.41} = W^{141/100} = \sqrt[100]{W^{141}}.$$

The graph is shown below. Note that the function values increase from left to right. As body weight increases, the area over which the animal moves increases.



*J. M. Emlen, *Ecology: An Evolutionary Approach,* p. 200 (Addison-Wesley, Reading, MA, 1973).

Exercise Set 1.3

Using the same set of axes, graph the pair of equations.

1. $y = |x|$ and $y = |x + 3|$
2. $y = |x|$ and $y = |x + 1|$
3. $y = \sqrt{x}$ and $y = \sqrt{x + 1}$
4. $y = \sqrt{x}$ and $y = \sqrt{x - 2}$

Graph.

5. $y = \dfrac{2}{x}$
6. $y = \dfrac{3}{x}$
7. $y = \dfrac{-2}{x}$
8. $y = \dfrac{-3}{x}$

9. $y = \dfrac{1}{x^2}$

10. $y = \dfrac{1}{x - 1}$

11. $y = \sqrt[3]{x}$

12. $y = \dfrac{1}{|x|}$

13. $f(x) = \dfrac{x^2 - 9}{x + 3}$

14. $g(x) = \dfrac{x^2 - 4}{x - 2}$

15. $f(x) = \dfrac{x^2 - 1}{x - 1}$

16. $g(x) = \dfrac{x^2 - 25}{x + 5}$

Convert to expressions with rational exponents.

17. $\sqrt{x^3}$

18. $\sqrt{x^5}$

19. $\sqrt[5]{a^3}$

20. $\sqrt[4]{b^2}$

21. $\sqrt[7]{t}$

22. $\sqrt[8]{c}$

23. $\dfrac{1}{\sqrt[3]{t^4}}$

24. $\dfrac{1}{\sqrt[5]{b^6}}$

25. $\dfrac{1}{\sqrt{t}}$

26. $\dfrac{1}{\sqrt{m}}$

27. $\dfrac{1}{\sqrt{x^2 + 7}}$

28. $\sqrt{x^3 + 4}$

Convert to radical notation.

29. $x^{1/5}$

30. $t^{1/7}$

31. $y^{2/3}$

32. $t^{2/5}$

33. $t^{-2/5}$

34. $y^{-2/3}$

35. $b^{-1/3}$

36. $b^{-1/5}$

37. $e^{-17/6}$

38. $m^{-19/6}$

39. $(x^2 - 3)^{-1/2}$

40. $(y^2 + 7)^{-1/4}$

41. $\dfrac{1}{t^{2/3}}$

42. $\dfrac{1}{w^{-4/5}}$

Simplify.

43. $9^{3/2}$

44. $16^{5/2}$

45. $64^{2/3}$

46. $8^{2/3}$

47. $16^{3/4}$

48. $25^{5/2}$

Determine the domain of the function.

49. $f(x) = \dfrac{x^2 - 25}{x - 5}$

50. $f(x) = \dfrac{x^2 - 4}{x + 2}$

51. $f(x) = \dfrac{x^3}{x^2 - 5x + 6}$

52. $f(x) = \dfrac{x^4 + 7}{x^2 + 6x + 5}$

53. $f(x) = \sqrt{5x + 4}$

54. $f(x) = \sqrt{2x - 6}$

APPLICATIONS

55. *Territorial Area.* Refer to Example 9. The *territorial area* of an animal is defined as its defended region, or exclusive region. For example, a lion has a certain region over which it is ruler. The area T of that region can be approximated by the power function

$$T = W^{1.31},$$

where W is the weight of the animal. Complete the table of approximate function values and graph the function.

W	0	10	20	30	40	50	100	150
T	0	20						

56. *Zipf's Law.* According to Zipf's Law, the number of cities with a population greater than S is inversely proportional to S. In 2000, there were 48 U.S. cities with a population greater than 350,000. Estimate the number of U.S. cities with a population greater than 200,000.[8]

57. *Body Surface Area.* A person whose mass is 75 kg has surface area approximated by

$$f(h) = 0.144h^{1/2},$$

where $f(h)$ is measured in square meters and h is the person's height in centimeters.[9]

a) Find the approximate surface area of a person whose mass is 75 kg and whose height is 180 cm.
b) Find the approximate surface area of a person whose mass is 75 kg and whose height is 170 cm.
c) Graph the function $f(h)$ for $0 \le h \le 200$.

58. *Dinosaurs.* The body mass y (in kilograms) of a theropod dinosaur may be approximated by the function

$$y = 0.73x^{3.63},$$

where x is the total length of the dinosaur (in meters).[10]

a) Find the body mass of *Coelophysis bauri,* which has a total length of 2.7 m.
b) Find the body mass of *Sinraptor dongi,* which has a total length of 7 m.

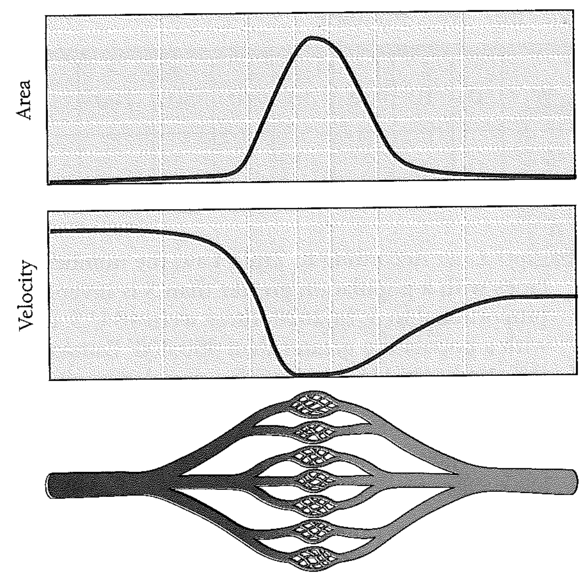
[8] U.S. Bureau of the Census.

[9] U.S. Oncology.

[10] F. Seebacher, "A New Method to Calculate Allometric Length-Mass Relationships of Dinosaurs," *Journal of Vertebrate Paleontology,* Vol. 21, pp. 51–60 (2001).

c) Suppose a therapod has a body mass of 5000 kg. Find its total length.

Capillaries. The velocity of blood in a blood vessel is inversely proportional to the cross-sectional area of the blood vessel. This relationship is called the *law of continuity.*[11]



59. Suppose, in an adult male, blood leaves the aorta at 30 cm/sec, and the cross-sectional area of the aorta is 3 cm^2. Given that blood travels in the capillaries at 0.026 cm/sec, find the total cross-sectional area of his capillaries.[12]

60. Suppose, in an adult female, blood leaves the aorta at 28 cm/sec, and the cross-sectional area of the aorta is 2.8 cm^2. Given that blood travels in the capillaries at 0.025 cm/sec, find the total cross-sectional area of her capillaries.

SYNTHESIS

Solve.

61. $x + 7 + \dfrac{9}{x} = 0$ (*Hint:* Multiply both sides by x.)

62. $1 - \dfrac{1}{w} = \dfrac{1}{w^2}$

63. *Pollution Control.* Pollution control has become a very important concern in all countries. If controls are not put in place, it has been predicted that the function

$$P = 1000t^{5/4} + 14{,}000$$

will describe the average pollution, in particles of pollution per cubic centimeter, in most cities at time t, in years, where $t = 0$ corresponds to 1970 and $t = 37$ corresponds to 2007.

a) Predict the pollution in 2007, 2010, and 2020.
b) Graph the function over the interval $[0, 50]$.

tw 64. At most, how many y-intercepts can a function have? Explain.

tw 65. Explain the difference between a rational function and a polynomial function. Is every polynomial function a rational function?

Technology Connection

Use the **ZERO** feature or the **INTERSECT** feature to approximate the zeros of the function to three decimal places.

66. $f(x) = \dfrac{1}{2}(|x - 4| + |x - 7|) - 4$

67. $f(x) = \sqrt{7 - x^2}$

68. $f(x) = |x + 1| + |x - 2| - 5$

69. $f(x) = |x + 1| + |x - 2|$

70. $f(x) = |x + 1| + |x - 2| - 3$

[11] N.A. Campbell and J.B. Reece, *Biology* (Benjamin Cummings, New York, 2002).

[12] Notice this is the total cross-sectional area of all capillaries, not the cross-sectional area of a single capillary.

Technology Connection

Determining Angles

Let's determine the acute angle whose sine is 0.2. On the TI-83 and many other graphers, this is done using the $\boxed{\text{SIN}^{-1}}$ key. If the grapher is in degree mode, then the answer is approximately 11.537°. If the calculator is in radian mode, then the answer is approximately 0.20136 radians. The $\boxed{\text{COS}^{-1}}$ or $\boxed{\text{TAN}^{-1}}$ key can be used to find an angle if we know its cosine or tangent, respectively.

EXERCISES

Use a grapher to approximate the acute angle in degrees.

1. Find t if $\sin t = 0.12$.
2. Find t if $\cos t = 0.73$.
3. Find t if $\tan t = 1.24$.

Use a grapher to approximate the acute angle in radians.

4. Find t if $\sin t = 0.85$.
5. Find t if $\cos t = 0.62$.
6. Find t if $\tan t = 0.45$.

At times we wish to know an angle in a right triangle when we are given the lengths of two sides. We can find the angle by using the definition of a trigonometric function.

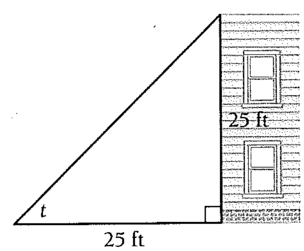
EXAMPLE 8 Find the angle of elevation of the sun if a building 25 ft high casts a

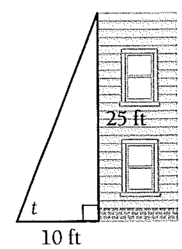
a) 25-ft shadow.
b) 10-ft shadow.

Solution

a) We can see from the figure that $\tan t = \dfrac{25}{25} = 1$. The angle must be 45° since $\tan 45° = 1$.

b) In this case, $\tan t = \dfrac{25}{10} = 2.5$. We have not seen any special angle whose tangent is 2.5. To find the angle we use the $\boxed{\text{TAN}^{-1}}$ key on a calculator to find $t \approx 68.20°$.





Exercise Set 1.4

In Exercises 1–6, convert from degrees into radians. Draw a picture of each angle on the xy-plane.

1. 120° 2. 150°
3. 240° 4. 300°
5. 540° 6. −450°

In Exercises 7–12, convert from radians into degrees. Draw a picture of each angle on the xy-plane.

7. $3\pi/4$ 8. $7\pi/6$
9. $3\pi/2$ 10. 3π
11. $-\pi/3$ 12. $-11\pi/15$

Determine if the following pairs of angles are coterminal.

13. 15° and 395° 14. 225° and −135°
15. 107° and −107° 16. 140° and 440°
17. $\pi/2$ and $3\pi/2$ 18. $\pi/2$ and $-3\pi/2$
19. $7\pi/6$ and $-5\pi/6$ 20. $3\pi/4$ and $-\pi/4$

Use a calculator to find the values of the following trigonometric functions.

21. sin 34° 22. sin 82°
23. cos 12° 24. cos 41°

25. tan 5°
26. tan 68°
27. cot 34°
28. cot 56°
29. sec 23°
30. csc 72°
31. $\sin(\pi/5)$
32. $\cos(2\pi/5)$
33. $\tan(\pi/7)$
34. $\cot(3\pi/11)$
35. $\sec(3\pi/8)$
36. $\csc(4\pi/13)$
37. $\sin(2.3)$
38. $\cos(0.81)$

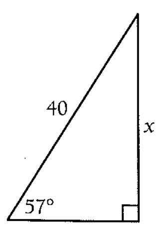
Use a calculator to find the degree measure of an acute angle whose trigonometric function is given.

39. $\sin t = 0.45$
40. $\sin t = 0.87$
41. $\cos t = 0.34$
42. $\cos t = 0.72$
43. $\tan t = 2.34$
44. $\tan t = 0.84$

Use a calculator to find the radian measure of an acute angle whose trigonometric function is given.

45. $\sin t = 0.59$
46. $\sin t = 0.26$
47. $\cos t = 0.60$
48. $\cos t = 0.78$
49. $\tan t = 0.11$
50. $\tan t = 1.26$

Solve for the missing side x.

51.


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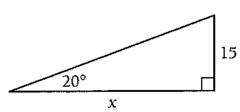

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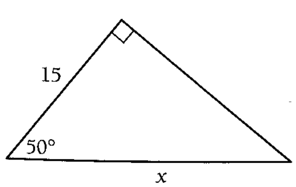

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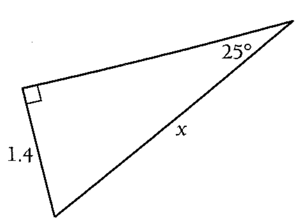

Solve for the missing angle t. Express your answer in degrees.

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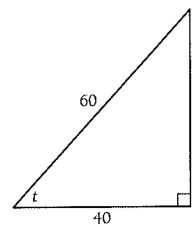

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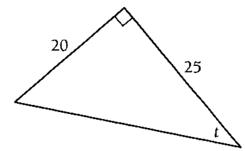

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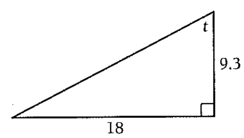

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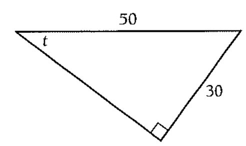
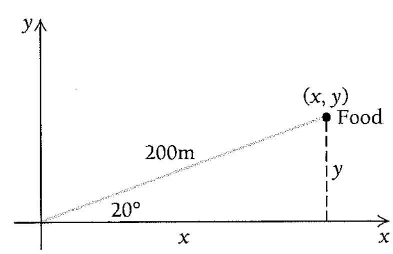


59. Use a Sum Identity to find cos 75°.

APPLICATIONS

60. *Honeybees.* Honeybees communicate the location of food sources to other bees in a hive through an elaborate dance.[15] Through such a dance, the hive learns that a food source is 200 m away at an angle 20° north of the sun, which is rising due east. Find the x- and y-coordinates of the food source. (Think of east as the positive x-direction and north as the positive y-direction.)

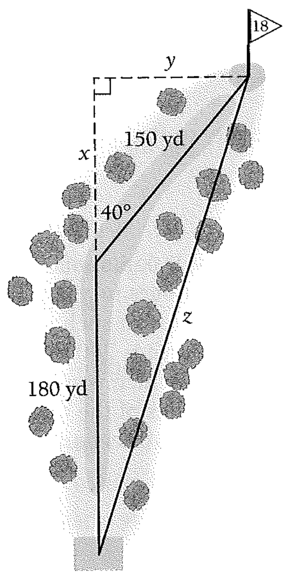


61. *Grade of a Road.* On a 5-mi stretch of highway, the road decreases in elevation at an angle of 4°. How much lower is a car after traveling on this part of the highway? (Remember that there are 5280 feet in a mile.)

[15] Karl von Frisch, *The Dance Language and Orientation of Bees*, Harvard University Press, 1971.

62. *Grade of a Road.* The tangent of a road's angle of elevation t is called the *grade* of the road; the grade is often expressed as a percentage. Suppose a highway through a mountain pass has a grade of 5% and is 6 mi long from the base to the top of the pass. How much higher is the pass than the base?

63. *Golf.* A certain hole on a golf course is 330 yd long with a 40° dogleg, as illustrated in the figure. The distance from the tee to the center of the dogleg is 180 yd, while the distance from the center of the dogleg to the green is 150 yd.
    a) Find x.
    b) Find y.
    c) Find z, the straight-line tee-to-green distance.



Blood Velocity. Ultrasound measures the velocity of blood (in cm/s) through a blood vessel using

$$v = \frac{77{,}000d \sec t}{f}.$$

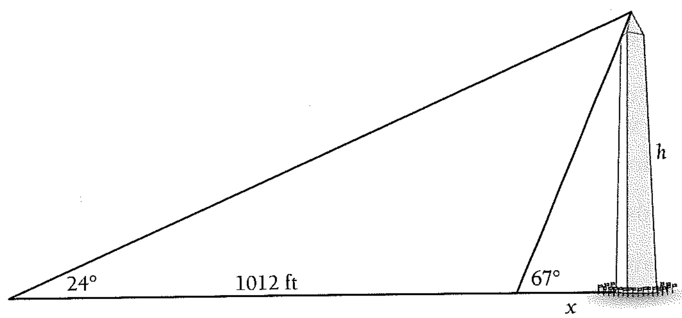
In this formula, f is the emitted ultrasound beam frequency, d is the Doppler shift (or the difference between the emitted and received beam frequencies), and t is the angle between the ultrasound beam and the blood vessel.[16]

64. Suppose $f = 5{,}000{,}000$ Hz, $d = 200$ Hz, and $t = 60°$. Determine the blood velocity through the vessel.

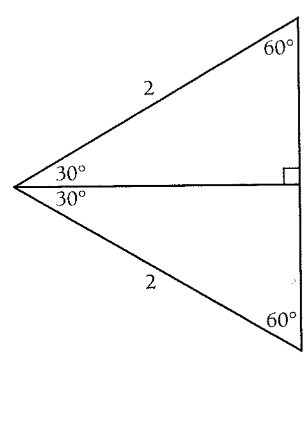
65. Suppose $f = 4{,}000{,}000$ Hz, $d = 100$ Hz, and $t = 65°$. Determine the blood velocity through the vessel.

SYNTHESIS

66. *Washington Monument.* While standing in the Mall in Washington, D.C., a tourist observes the angle of elevation to the top of the Washington Monument to be 67°. After moving 1012 ft farther away from the Washington Monument, the angle of elevation changes to 24°.
    a) Use the small triangle to find x in terms of h.
    b) Use the large triangle to find the height of the Washington Monument.



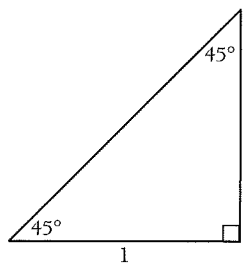
67. *Proportions of 30–60–90 Triangles.* Consider the adjacent 30–60–90 right triangles, each with hypotenuse of length 2, shown in the figure.
    a) Explain why the two triangles form one equilateral triangle.
    b) Explain why the short leg of each triangle has length 1.
    c) Use the Pythagorean theorem to find the length of the long leg.
    d) Explain how this figure gives the trigonometric functions of $\pi/6$ and $\pi/3$.



[16]Triton Technology, Inc.

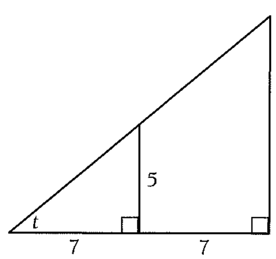
68. *Proportions of 45–45–90 Triangles.* Consider the 45–45–90 right triangle shown in the figure, with a leg of length 1.

   a) Explain why the other leg also has length 1.
   b) Use the Pythagorean theorem to find the length of the hypotenuse.
   c) Explain how this figure gives the trigonometric functions of $\pi/4$.



69. Refer to the figure below.

   a) Use the small right triangle to show that $\tan t = 5/7$.
   b) Use the large right triangle to show that $\tan t = 10/14$.
   tw c) Why don't the trigonometric functions depend on the size of the triangle?



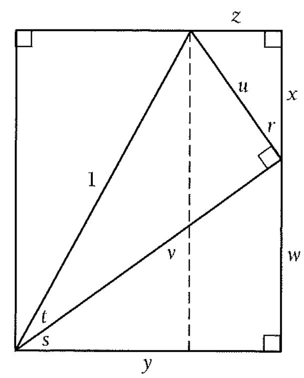
Trigonometric Identities. Many interrelationships between the trigonometric functions can be found, as shown in the following exercises.

70. *Reciprocal.* Use the definitions of the trigonometric functions to derive the Reciprocal Identities.

71. *Ratio.* Use the definitions of the trigonometric functions to derive the Ratio Identities.

72. *Sum Identity for Sine.* Refer to the figure below to answer these questions.[17]

   a) Show that $u = \sin t$ and $v = \cos t$.
   b) Use geometry to show that $r = s$.
   c) Show that $w = \sin s \cos t$.
   d) Show that $x = \cos s \sin t$.
   e) Conclude that $\sin(s + t) = (w + x)/1 = \sin s \cos t + \cos s \sin t$.

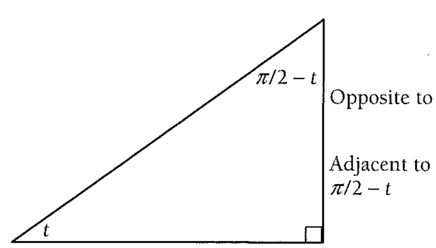


73. *Sum Identity for Cosine.* Use the figure from the previous exercise.

   a) Show that $u = \sin t$ and $v = \cos t$.
   b) Use geometry to show that $r = s$.
   c) Show that $y = \cos s \cos t$.
   d) Show that $z = \sin s \sin t$.
   e) Conclude that $\cos(s + t) = (y - z)/1 = \cos s \cos t - \sin s \sin t$.

74. *Cofunction.*

   a) Use the figure to show that
   $$\cos\left(\frac{\pi}{2} - t\right) = \sin t.$$
   b) Show that $\sin\left(\frac{\pi}{2} - t\right) = \cos t$.



75. *Pythagorean.* Show that $1 + \tan^2 t = \sec^2 t$. (*Hint:* Begin with the Pythagorean Identity of Theorem 6 and divide both sides by $\cos^2 t$.)

76. *Pythagorean.* Show that $1 + \cot^2 t = \csc^2 t$.

77. *Double-Angle for Sine.* Show that $\sin 2t = 2 \sin t \cos t$. (*Hint:* Let $s = t$ and use a Sum Identity.)

[17] R. B. Nelson, *Proofs Without Words II* (Mathematical Association of America, Washington, DC, 2000).

Exercise Set 1.5

Sketch the following angles.

1. $5\pi/4$
2. $-5\pi/6$
3. $-\pi$
4. 2π
5. $13\pi/6$
6. $-7\pi/4$

Use a unit circle to compute the following trigonometric functions.

7. $\cos(9\pi/2)$
8. $\sin(5\pi/4)$
9. $\sin(-5\pi/6)$
10. $\cos(-5\pi/4)$
11. $\cos 5\pi$
12. $\sin 6\pi$
13. $\tan(-4\pi/3)$
14. $\tan(-7\pi/3)$

Use a calculator to evaluate the following trigonometric functions.

15. $\cos 125°$
16. $\sin 164°$
17. $\tan(-220°)$
18. $\cos(-253°)$
19. $\sec 286°$
20. $\csc 312°$
21. $\sin(1.2\pi)$
22. $\tan(-2.3\pi)$
23. $\cos(-1.91)$
24. $\sin(-2.04)$

Find all solutions of the given equation.

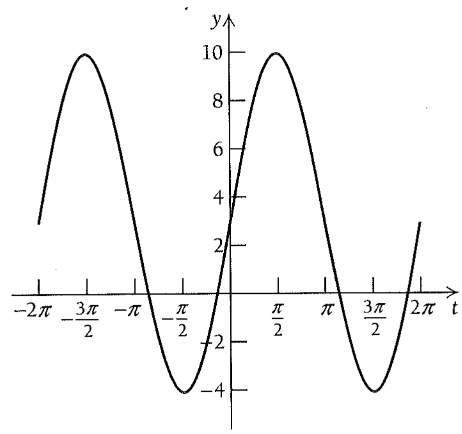
25. $\sin t = \dfrac{1}{2}$
26. $\sin t = -1$
27. $\sin 2t = 0$
28. $2\sin\left(t + \dfrac{\pi}{3}\right) = -\sqrt{3}$
29. $\cos\left(3t + \dfrac{\pi}{4}\right) = -\dfrac{1}{2}$
30. $\cos(2t) = 0$
31. $\cos(3t) = 1$
32. $2\cos\left(\dfrac{t}{2}\right) = -\sqrt{3}$
33. $2\sin^2 t - 5\sin t - 3 = 0$
34. $\cos^2 x + 5\cos x = 6$
35. $\cos^2 x + 5\cos x = -6$
36. $\sin^2 t - 2\sin t - 3 = 0$

For the following functions, find the amplitude, period, and mid-line. Also, find the maximum and minimum.

37. $y = 2\sin 2t + 4$
38. $y = 3\cos 2t - 3$
39. $y = 5\cos(t/2) + 1$
40. $y = 3\sin(t/3) + 2$
41. $y = \dfrac{1}{2}\sin(3t) - 3$
42. $y = \dfrac{1}{2}\cos(4t) + 2$
43. $y = 4\sin(\pi t) + 2$
44. $y = 3\cos(3\pi t) - 2$

For each of the following graphs, determine if the function should be modeled by either $y = a \sin bt + k$ or $y = a \cos bt + k$. Then find a, b, and k.

45.



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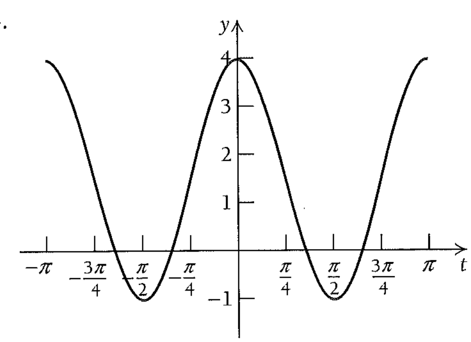

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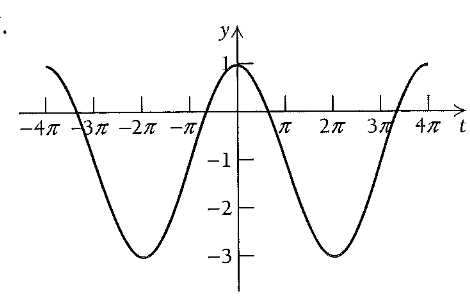
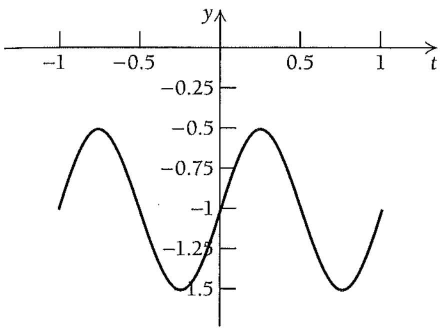


48.



APPLICATIONS

Solar Radiation. The annual radiation (in megajoules per square centimeter) for certain land areas of the northern hemisphere may be modeled with the equation[19]

$$R = 0.339 + 0.808 \cos l \cos s - 0.196 \sin l \sin s - 0.482 \cos a \sin s.$$

In this equation, l is the latitude (between 30° and 60°) and s is the slope of the ground (between 0° and 60°). Also, a is the aspect, or the direction that the slope faces. For a slope facing due north, $a = 0°$, and for a slope facing south, $a = 180°$. For a slope facing either east or west, $a = 90°$.

49. Find the annual radiation of north-facing land at 40° north latitude with a 30° slope.

50. Find the annual radiation of south-facing land at 30° north latitude with a 20° slope.

51. Find the annual radiation of southeast-facing land at 50° north latitude with a 55° slope.

52. Find the annual radiation of flat land at 50° north latitude.

Lung Capacity. As we breathe, our lungs increase and decrease in volume. The volume of air that we inhale and exhale with each breath is called *tidal volume.* The maximum possible tidal volume is called the *vital capacity,* normally approached during strenuous physical activity. Even at vital capacity, the lungs are never completely drained of air; the minimum volume of the lungs is called the *residual volume.*[20]

53. Suppose a man watching television breathes once every 5 sec. His average lung capacity is 2500 mL, and his tidal volume is 500 mL. Express the volume of his lungs using the model $V(t) = a \cos bt + k$, where time 0 corresponds to the lungs at their largest capacity.

54. A woman undergoes her ordinary strenuous workout, breathing once every 2 sec. Her tidal volume is 3400 mL, and her residual volume is 1100 mL. Express the volume of her lungs using the model $V(t) = a \cos bt + k$, where time 0 corresponds to the lungs at their largest capacity.

tw 55. Explain why a periodic model like the cosine function may be reasonable for describing lung capacity.

56. *Body Temperature.* In a laboratory experiment, the body temperature T of rats was measured.[21] The body temperatures of the rats varied between 35.33°C and 36.87°C during the course of the day. Assuming that the peak body temperature occurred at $t = 0$, model the body temperature with a function of the form $T(t) = a \cos bt + k$.

Sound Waves. The pitch of a sound wave is measured by its frequency. Humans can hear sounds in the range from 20 to 20,000 Hz, while dogs can hear sounds as high as 40,000 Hz. The loudness of the sound is determined by the amplitude.[22]

57. The note A above middle C on a piano generates a sound modeled by the function $g(t) = 4 \sin(880\pi t)$, where t is in seconds. Find the frequency of A above middle C.

58. The note A below middle C on a piano generates a sound modeled by the function $g(t) = 4 \sin(440\pi t)$, where t is in seconds. Find the frequency of A below middle C.

59. *Blood Pressure.* During a period of controlled breathing, the systolic blood pressure p of a volunteer averaged 143 mmHg with an amplitude of 5.3 mmHg and a frequency of 0.172 Hz. Assuming that the blood pressure was highest when $t = 0$, find a model $p(t) = a \cos bt + k$ for blood pressure as a function of time.

60. *Blood Pressure.* During an episode of sleep apnea, the systolic blood pressure averaged 137 mmHg with an amplitude of 6.7 mmHg and a frequency of 0.079 Hz. Assuming that the blood pressure was highest when $t = 0$, find a model

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[19] B. McCune and D. Keon, "Equations for potential annual direct incident radiation and heat load," *Journal of Vegetation Science,* Vol. 13, pp. 603–606 (2002).

[20] G.J. Borden, K.S. Harris, and L.J. Raphael, *Speech Science Primer,* 4th ed. (Lippincott Williams & Wilkins, Philadelphia, 2003).

[21] H. Takeuchi, A. Enzo, and H. Minamitani, "Circadian rhythm changes in heart rate variability during chronic sound stress," *Medical and Biological Engineering and Computing,* Vol. 39, pp. 113–117 (2001).

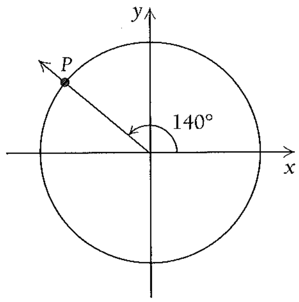
[22] N.A. Campbell and J.B. Reece, *Biology* (Benjamin Cummings, New York, 2002).

$p(t) = a \cos bt + k$ for blood pressure as a function of time.[23]

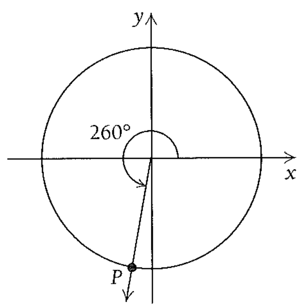
SYNTHESIS

Using a calculator, find the x- and y-coordinates of the following points on the unit circle.

61.



62.



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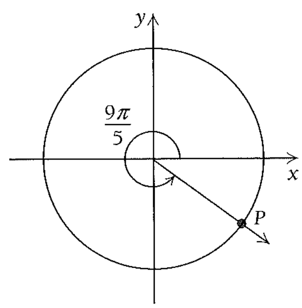

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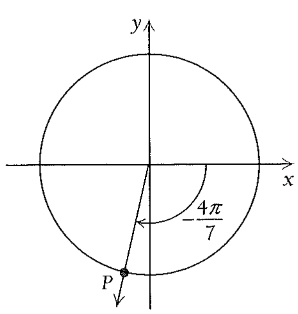
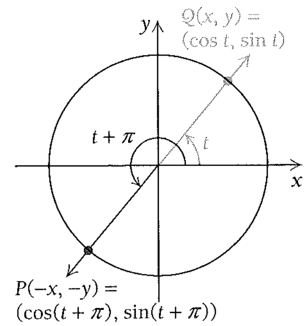


[23]M. Javorka, I. Žila, K. Javorka, and A. Čalkovskă, "Do the oscillations of cardiovascular parameters persist during voluntary apnea in humans?" *Physiological Research,* Vol. 51, pp. 227–238 (2002).

65. Compute $\sin 105°$. (*Hint:* Use a Sum Identity and the fact that $105° = 45° + 60°$.)

66. Compute $\cos 165°$.



67. *Half-Rotations.*
   a) Use the figure to explain why $\sin(t + \pi) = -\sin t$ and $\cos(t + \pi) = -\cos t$.
   b) Rederive the results of part (a) using Sum Identities.
   c) Show that $\tan(t + \pi) = \tan t$.

68. *Amplitude and Mid-Line.* Consider the function $g(t) = a \sin bt + k$, where a and b are positive.
   a) Show that the maximum and minimum of $g(t)$ are $k + a$ and $k - a$, respectively.
   b) Show that the mid-line is the line $y = k$.
   c) Show that the amplitude of $g(t)$ is a.

69. *Period.*
   tw a) Use a unit circle to explain why $\sin t = \sin(t + 2\pi)$ for all numbers t.
   b) Let $g(t) = a \sin bt + k$. Show that $g(t + 2\pi/b) = g(t)$.
   tw c) Why does the result of part (b) imply that the period of $g(t)$ is $2\pi/b$?

Frequency Detection in the Ear. Basilar fibers in the ear detect sound, and they vary in length, tension, and density throughout the basilar membrane. A fiber is affected most by sound frequencies near the *fundamental frequency* f of the fiber, which is approximately

$$f = \frac{1}{2L}\sqrt{\frac{T}{d}}.$$

In this formula, L is the length of the fiber, T is the tension of the fiber, and d is the density of the fiber.[24]

tw 70. At the apex of the basilar membrane, the fibers are long, flexible, and wide. At the apex, the basilar fibers are most affected by what kind of sound frequencies?

[24]N. A. Campbell and J. B. Reece, *Biology* (Benjamin Cummings, New York, 2002).