

Lesson 24, Section 5.1 Polynomials

Definition: A **term** is a number, a variable, a power of a variable, or the product of any of these. A term is also commonly called a **monomial**.

$$\text{Examples: } \begin{cases} -3 \\ w \\ x^4 \\ 4xy^2 \end{cases}$$

Definition: A **Polynomial** is a sum (or difference) of Terms (monomials).

$$\text{Examples: } \begin{cases} 4x + y \\ 2x^3 + 3x^2 - 4x + 2 \\ 4a^2 - 4ab + 3b^3 \end{cases}$$

Definition: A **Binomial** is a polynomial of two terms. A **Trinomial** is a polynomial of three terms.

$$\text{Binomial Examples: } \begin{cases} 3xy + 2x^2 \\ 4 - 3a \end{cases}$$

$$\text{Trinomial Examples: } \begin{cases} 4r^2 - 3r + 5 \\ ab - b - a \end{cases}$$

Definition: The **Degree** of a monomial (or term) is the sum of its exponents of the variables.

$$\text{Examples with degrees: } \begin{cases} 4x^5 & \text{degree 5} \\ 7x^2y^2 & \text{degree 4} \\ -3xyz^5 & \text{degree 7} \\ 4 & \text{degree 0} \end{cases}$$

Definition: The **Coefficient** of a monomial (or term) is the number factor.

$$\text{Examples and coefficients: } \begin{cases} 5xy & \text{coefficient } 5 \\ -x^2 & \text{coefficient } -1 \\ \frac{3\pi}{2}xy & \text{coefficient } \frac{3\pi}{2} \end{cases}$$

The **Leading Term** of a Polynomial is the term of the highest degree. Its coefficient is called the **Leading Coefficient**. The degree of the polynomial is the degree of the leading term. The degree of a constant (number) is zero.

- 1) List the terms, the degree of each term, the coefficient of each term, the leading term, and the degree of the polynomial.

$$4x^5 + 6x^3y^3 - 7xy^2 - 12x$$

Terms: $4x^5$ $6x^3y^3$ $-7xy^2$ $-12x$

Degrees:

Coefficients:

Leading Term:

Degree of the Polynomial:

Generally a polynomial of one variable is written in **descending order of powers**. (Occasionally, directions may state to write in ascending order.)

- 2) Write in descending order of powers.

$$4x^5 - 3x^2 + x - 2x^4 + 3 - \frac{1}{2}x^3$$

Definition: A Polynomial Function is of the form $P(x) = a$ polynomial.

Examples: $\begin{cases} f(x) = 3x^2 - 4x + 1 \\ P(t) = 3t^4 - 2t^2 \\ G(r) = 5r^5 - 4r^3 + 2 \end{cases}$

- 3) Evaluate this polynomial function for $x = -2$.

$$P(x) = 4x^3 - 3x^2 + 2x$$

- 4) Evaluate $g(-1)$ if $g(x) = -4x^3 + 2x^2 + 5x - 7$.

To add or subtract polynomials, combine 'like' terms.

5) Add $3x^3 - 4x + 6$ and $4x^3 - 7x^2 - 2x + 10$.

6) Subtract $3x^3 - 4x^2 - x$ from $2x^3 + x - 4$.

7) $\left(\frac{2}{3}xy + \frac{5}{6}x + 3.1x^2\right) + \left(-\frac{4}{5}x + \frac{3}{4}xy - 4.2x^2\right)$

8) $(5x^2 + 19x - 23) + (7x^2 - 2x + 1) + (-2x^2 - x + 8)$

9) $(9r - 5w - t) - (7r + 4w - 2t) - (6r - 8t)$

10) $(10xy - 4x^2y - 3y^2) - (9x^2y + 4y^2 - 7xy)$

Application Examples:

- 11) The number of milligrams ($M(t)$) of ibuprofen in the bloodstream for t hours after swallowing a 400 mg tablet is given by $M(t) = 0.5t^4 + 3.45t^3 - 96.65t^2 + 347.7t$ for $0 \leq t \leq 6$. (Source: Dr. P. Carey, Burlington, VT)

How many milligrams are in the bloodstream in 2 hours after a 400 mg tablet is swallowed?

- 12) The speed $v(t)$ in miles per hour at which a diver enters the water after diving is approximated by $v(t) = 21.82t$ where t is number of seconds of falling. (Source: www.guinnessworldrecords.com)

A diver in Acapulco, Mexico is in the air for 5 seconds. What is the speed at which he enters the water?

In business, total **profit** is total **revenue** minus total **cost**. If functions can be determined for profit, revenue, and cost; then $P(x) = R(x) - C(x)$.

- 13) If the revenue of a company selling x futons is $R(x) = 280x - 0.7x^2$ and the cost of making x futons is $C(x) = 8000 + 0.5x^2$, find the profit function and the profit from the sale of 200 futons.