

Lessons 27

Factoring Trinomials, Perfect Square Trinomials, Difference of Squares

TRINOMIALS (leading coefficient not a 1)

Form: $ax^2 + bx + c$ **Always write terms in descending order!**

Notice: $(3x - 5)(2x + 3) = 6x^2 + 9x - 10x - 15 = 6x^2 - x - 15$

$6x^2$ is the product of the first two terms, -15 is the product of the last two terms, $-x$ is the sum of the outer and inner terms

There are two methods that can be used to factor trinomials of the form, $ax^2 + bx + c$. The first method is a ‘trial-and-error’ process or ‘reversing FOIL’. The second method is called the ‘Grouping Method’ or Product/Sum Method. I will show both methods and you may choose which method works for you. I recommend the Grouping Method.

Trial-and-Error Method:

1. Make your first terms have a product of ax^2 .
2. Make your last terms have a product of c .
3. Find the sum of the inner and outer terms and check if it equals bx . If not, go back to steps 1 and 2 and try a different combination, until step 3 checks.

OR

Grouping Method (Product/Sum Method)

Follow these steps:

1. If the trinomial is in the form $ax^2 + bx + c$, find a pair of numbers whose product is ac and whose sum is b . Call these numbers r and s .
2. Write the polynomial of the form $ax^2 + rx + sx + c$. Use the ‘grouping’ method to factor. (This method will be demonstrated in class.)

Factor:

1) $6x^2 - 5x - 25$

2) $9a^2 + 18a + 8$

3) $6x^2 + 5x - 6$

4) $8x^2 - 2x - 15$

5) $12x^2 + 23x + 10$

6) $8x^2 - 18xy + 9y^2$

7) $14x + 12x^2 + 4$

Examine the following polynomials and the factors of those polynomials.

$$x^2 + 10x + 25 = (x + 5)(x + 5) \text{ or } (x + 5)^2$$

$$x^2 - 6x + 9 = (x - 3)(x - 3) \text{ or } (x - 3)^2$$

$$4x^2 + 4x + 1 = (2x + 1)(2x + 1) = (2x + 1)^2$$

A Trinomial that when factored equals a binomial squared is called a PERFECT SQUARE TRINOMIAL.

Recognizing a Perfect Square Trinomial:

$A^2 + 2AB + B^2$ or $A^2 - 2AB + B^2$ represents a perfect square trinomial pattern.

1. The first term A^2 is a perfect square.
2. The 3rd term B^2 is a perfect square.
3. The middle terms is double the product of the square roots of the first and third terms.

OR

If the product/sum method for factoring a trinomial is used and the pair of numbers is the same number, this indicates a perfect square trinomial.

Factor the following, if possible. Identify which are perfect square trinomials.

1) $x^2 - 20x + 100$

2) $9x^2 + 6x + 1$

3) $4x^2 - 15x + 9$

4) $a^2 - 22a + 121$

5) $b^2 + b + \frac{1}{4}$

6) $2x^2 - 12xy + 18y^2$

The following binomial pattern is called the DIFFERENCE OF SQUARES. It factors as two binomials; one a sum and the other a difference.

$$A^2 - B^2 = (A + B)(A - B)$$

Factor the following. Identify any differences of squares.

1) $m^2 - 36$

2) $a^2b^2 - 144$

3) $2n^2 - 32$

4) $4x^2 - 9x$

5) $5x^8 - 125y^4$

6) $25y^6 - 36$

7) $98 - 8a^4b^2$