

Lesson 32 Section 6.4
Rational Equations

Remember that a fraction cannot have a zero denominator. Because a rational expression cannot have a zero denominator, you must determine any values of x that would make a zero denominator when solving equations with rational expressions. These value(s) are restricted from the domain.

For example:

$$\frac{3x^2 + 5x}{x - 3}$$

$$\frac{4x}{x^2 - 4}$$

$$\frac{3x - 1}{2x^2 - 6x}$$

$$\frac{3x}{x^4 + 2}$$

$$x - 3 \neq 0$$

$$x^2 - 4 \neq 0$$

$$2x^2 - 6x \neq 0$$

$$x^4 + 2 \neq 0$$

$$x \neq 3$$

$$(x + 2)(x - 2) \neq 0$$

$$2x(x - 3) \neq 0$$

$$x^4 \neq -2$$

$$x + 2 \neq 0 \quad x - 2 \neq 0$$

$$2x \neq 0 \quad x - 3 \neq 0$$

$$x \neq -2 \quad x \neq 2$$

$$x \neq 0 \quad x \neq 3$$

This is impossible.

Caution: Any number that makes a zero denominator in a rational expression or a rational equation cannot be in the domain. In other words, it cannot be x .

To Solve a **Rational Equation**:

1. Factor the denominators and find the LCD.
2. Determine what values of x are restricted from the domain.
3. Multiply each side of the equation (multiply each term of each side) by that LCD. This should 'clear out' the denominators.
4. Solve appropriately.
5. Disregard any possible solution(s) that are restricted from the domain.

Examples:

1) $\frac{x}{5} + \frac{x}{6} = 2$

2) $\frac{1}{2} - \frac{2}{n} = \frac{3}{2n}$

$$3) \quad \frac{4}{10} + \frac{1}{2x} = \frac{2}{5}$$

Examine this example:

$$\frac{3}{2y+10} + \frac{5}{4} = \frac{7}{y+5}$$

$$2(y+5) \quad \text{LCD} = 4(y+5)$$

$$y+5 \neq 0$$

$$y \neq -5$$

$$4(y+5) \left(\frac{3}{2(y+5)} \right) + 4(y+5) \left(\frac{5}{4} \right) = 4(y+5) \left(\frac{7}{y+5} \right)$$

$$2(3) + 5(y+5) = 4(7)$$

$$6 + 5y + 25 = 28$$

$$31 + 5y = 28$$

$$5y = -3$$

$$y = -\frac{3}{5}$$

$$\left\{ -\frac{3}{5} \right\}$$

This problem is started. Finish solving.

$$4) \quad \frac{x+3}{x} - \frac{x+4}{x+5} = \frac{15}{x^2+5x}$$

$$x(x+5)$$

$$\text{LCD} = x(x+5)$$

$$x \neq 0, -5$$

$$x(x+5) \left(\frac{x+3}{x} \right) - x(x+5) \left(\frac{x+4}{x+5} \right) = x(x+5) \left(\frac{15}{x(x+5)} \right)$$

$$(x+5)(x+3) - x(x+4) = 15$$

$$5) \quad x - \frac{6}{x} = -5$$

$$6) \quad \frac{x-2}{x-4} = \frac{2}{x-4}$$

$$7) \quad \frac{3}{x-3} + \frac{5}{x+2} = \frac{5x}{x^2 - x - 6}$$

$$8) \quad \frac{x}{x+1} + \frac{5}{x} = \frac{1}{x^2 + x}$$