Lesson 33, Section 6.5
Application Problems Using Rational Equations

1. Define a Variable
2. Develop A Plan
3. Write an Equation
4. Solve and Answer the Question

1) The reciprocal of 5, plus the reciprocal of 7, is the reciprocal of what number?

Let x = the number
Reciprocal of 5 plus reciprocal of 7 = reciprocal of x

2) The sum of a number and 21 times its reciprocal is -10. Find the number.

Let n = the number
The number plus 21(reciprocal of number) = -10

Job or Work Problems:

Suppose Joe can paint a house in 9 hours and Clyde can paint the same house in 12 hours. Examine the table below that shows the fractional part of the job that each man does plus the part of the job that is completed together at that time.

TIME	Fraction of the house Painted Joe	Clyde	Together
1 hour	$\frac{1}{9}$	$\frac{1}{12}$	$\frac{1}{9}+\frac{1}{12}$ or $\frac{7}{36}$
2 hours	$\frac{2}{9}$	$\frac{2}{12}$ or $\frac{1}{6}$	$\frac{2}{9}+\frac{1}{6}$ or $\frac{7}{18}$
3 hours	$\frac{3}{9}$ or $\frac{1}{3}$	$\frac{3}{12}$ or $\frac{1}{4}$	$\frac{1}{3}+\frac{1}{4}$ or $\frac{7}{12}$
t hours	$\frac{t}{9}$	$\frac{t}{12}$	$\frac{t}{9}+\frac{t}{12}$

If the job is to be completed by both together, then $\frac{t}{9}+\frac{t}{12}=1$, where t is time both worked together.

This leads to the following summary to model uniform job or work problems.

Let a = time for A to complete the work alone; rate is $\frac{1}{a}$ / unit of time.

Let b = time for B to complete the work alone; rate is $\frac{1}{b}$ / unit of time.

Let t = time both A and B work together.

Plan: rate A(time) + rate B(time) = 1 job

Then, the following equation results: $\left(\frac{1}{a}\right)t+\left(\frac{1}{b}\right)t=1$

$$\frac{t}{a}+\frac{t}{b}=1$$

3) Stan needs 45 minutes to do the dishes, while Bobby can do them in 30 minutes. How long will it take them, if they work together?

4) A community water tank can be filled in 18 hours by the town office well alone and in 22 hours by the high school well alone. How long will it take to fill the tank if both wells are working?

5) Ron can paint his den in 6 hours working alone. If Dee helps him, the job takes 4 hours. Estimate how long it would take Dee alone to paint the den.

6) The HP Laser Jet 9000 works twice as fast as the Laser Jet 2300. (Source: www.hewlettpackard.com) If the machines work together, a university can produce all its staff manuals in 15 hours. Find the time it would take each machine, working alone, to complete the same job.

Let x = time for the 9000 model, $2x$ = time for the 2300 model

7) Jake can cut and split a cord of firewood in 6 fewer hours than Skyler can. When they work together, it takes them 4 hours. How long would it take Jake alone to cut and split a cord of firewood?

Uniform Motion Problems:

8) A canal has a current of 2 miles per hour. Find the speed of Casey's boat in still water, if the boat travels 11 miles down the canal in the same time that it goes 8 miles up the canal.

	Distance	Rate	Time
Down the Canal			
Up the Canal			

9) A local bus travels 7 mph slower than the Express bus. The express travels 45 miles in the same time it takes the local to travel 38 miles. Find the speed of each bus.

	Distance	Rate	Time
Local			
Express			

10) There is a moving sidewalk at the local airport that moves at 1.5 feet per second. Lisa can walk forward 570 feet on the moving sidewalk in the same time she can walk 420 feet on a regular floor of the airport (without the moving sidewalk). How fast does Lisa ordinarily walk without the moving sidewalk?

	Distance	Rate	Time
With moving sidewalk	570	$r + 1.5$	
Without the moving sidewalk	420	r	

11) Allen paddles 55 meters per minute in still water. He paddles 106 meters upstream and 228 meters downstream in a **total time** of 6 minutes. What is the speed (rate) of the current?

	Distance	Rate	Time
Upstream	106	55 - c	$\frac{106}{55-c}$
Downstream	228	55 + c	$\frac{228}{55+c}$

time upstream + time downstream totals 6 min.

$$\frac{106}{55-c}+\frac{228}{55+c}=6$$

$$LCD=(55-c)(55+c)$$

$$(55-c)(55+c)\left(\frac{106}{55-c}\right)+(55-c)(55+c)\left(\frac{228}{55+c}\right)=6(55-c)(55+c)$$

$$106(55+c)+228(55-c)=6(3025-c^2)$$

$$5830+106c+12540-228c=18150-6c^2$$

$$18370-122c=18150-6c^2$$

$$6c^2-122c+220=0$$

$2(3c^2-61c+110)=0$ product = 330, sum = -61

-55 and -6

$$3c^2-6c-55c+110$$

$$3c(c-2)-55(c-2)$$

$$(c-2)(3c-55)=0$$

$c-2=0$ $3c-55=0$

$c=2$ $3c=55$

$c=18\frac{1}{3}$

I showed the product/sum method of factoring the trinomial, since the leading coefficient was not a 1.

Current could be flowing at 2 mps or $18\frac{1}{3}$ mps.