Lesson 34   Section 6.8, Variation

Examine this table:

# hours worked	Pay
1	\$8
2	\$16
3	\$24
4	\$32
6	\$49
10	\$80

When a relation between pairs of numbers is a <u>constant ratio</u>, such as above; it is called a **Direct Variation.** The ratio above is $\frac{8}{1}$ and we say the pay **varies directly** as hours. The number 8 from the ratio is called the **variation constant**. The **Variation Equation** is $p = 8h$, where p represents pay and h represents hours.

<u>In direct variation, as *x* increases, so does *y*.</u> In the example above, when the number of hours increase, so does the pay.

<u>Basic Direct Variation</u> is of the form $y = kx$. We say *y* ***varies directly as x*** or ***y is directly proportional to x*.** The number *k* is the variation constant or the constant of proportionality.

1) The value of *y* varies directly as *x*. If $y = 5$ when $x = 12$, find the value of the variation constant and the resulting variation equation.

2) The value of *b* is directly proportional to *a*. If $b = 12$ when $a = \frac{1}{3}$, find the constant of proportionality and the resulting variation equation.

Examine this table:

bus speed	time of trip
20 mph	1 hr.
40 mph	½ hr.
60 mph	$\frac{1}{3}$ hr.
80 mph	¼ hr.

When a relation between pairs of numbers is such that the product of the numbers remains constant, such as the table above (product is 20), it represents **Inverse Variation.** The product 20 represents the variation constant and the inverse variation is $t = \frac{20}{r}$, where *t* represents time and *r* represent speed.

In inverse variation, as *x* increases, *y* decreases (or as *x* decreases, *y* increases). In the example above, as the rate of the bus increases, the time decreases.

Basic Inverse Variation is of the form $y = \frac{k}{x}$. We say ***y varies inversely as* x** or ***y is inversely proportional to x***. The number *k* is still the variation constant or the constant of proportionality.

3) Suppose *y* varies inversely as *x*. If $y = 16$ when $x = 4$, find the variation constant and the resulting variation equation.

4) Let *n* be inversely proportional to *m*. Suppose $n = 81$ when $m = 4$. Find the constant of proportionality and the resulting variation equation.

Joint Variation is of the form $y = kxz$.

Combined Variation is of the form $y = k\frac{x}{z}$ or $\frac{kx}{z}$.

5) Suppose y varies jointly as x and z.
   a) If $y = 6$ when $x = 12$ and $z = 4$, find the variation constant and the resulting variation equation.
   b) Use your equation to find the value of y when $x = 3$ and $z = 4$.

6) Let y vary jointly as x and z and inversely as w.
   a) If $y = 20$ when $x = 2$, $z = \frac{1}{2}$, and $w = 10$, find the variation constant and the resulting variation equation.
   b) Find the value of y if $x = 4$, $z = 0.6$, and $w = 0.3$.

7) Suppose y varies directly as the **square** of x and inversely as the **square root** of z.
   a) If $y = 12$ when $x = 3$ and $z = 16$, find the value of the variation constant and the resulting equation.
   b) Find the value of y if $x = 2$ and $z = 9$.

8) The time T required to do a job varies inversely as the number of people P working. It takes 5 hours for 7 volunteers to pick up trash from 1 mile of a highway. How long would it take 10 volunteers to complete the job?

9) The electric current I, in amps, in a circuit varies directly as the voltage V. When 16 volts are applied, the current is 5 amps. What is the current when 20 volts are applied?

10) The stopping distance d of a car (in feet) after the brakes have been applied varies directly as the square of the speed r. Once the brakes are applied, a car traveling 60 miles per hour can stop in 138 feet. What stopping distance corresponds to a speed of 40 mph?