Lesson 36 Section 7.2

What is a value for $9^{\frac{1}{2}}$?

Consider This.

$9^{\frac{1}{2}} \times 9^{\frac{1}{2}} = 9^{\frac{1}{2}+\frac{1}{2}} = 9^1 = 9$ using the product rule of exponents.

Now, Think!

What other number times itself equals 9? $3 \times 3 = 9$

Since both products equal 9, we can conclude that $9^{\frac{1}{2}} = 3$.

Conclusion: A rational exponent is the same as a root!

Definition of a rational (fractional) exponent: $a^{\frac{1}{n}} = \sqrt[n]{a}$

Evaluate or write an equivalent root for each. Simplify, if possible.

1. $16^{\frac{1}{2}} =$
2. $x^{\frac{1}{3}} =$
3. $(-32)^{\frac{1}{5}} =$
4. $(4c)^{\frac{1}{4}} =$

Write an equivalent expression using exponential notation.

5. $\sqrt[4]{5} =$
6. $\sqrt[3]{ab^2} =$
7. $\sqrt{n} =$

Expanded Definition of a rational (fractional) exponent: $a^{\frac{m}{n}} = \sqrt[n]{a^m}$ or $\left(\sqrt[n]{a}\right)^m$

$$a^{\frac{m}{n}} = \left(a^m\right)^{\frac{1}{n}} = \sqrt[n]{a^m}$$

Proof:

$$a^{\frac{m}{n}} = \left(a^{\frac{1}{n}}\right)^m = \left(\sqrt[n]{a}\right)^m$$

Notice: The exponent *m* can be inside the radical (in the radicand) or outside of the radical. The order does not matter. You can raise to the *m* power first and then take the *n*th root. Or you can take the *n*th root and then raise to the *m* power. **When evaluating (because you do not have the use of a calculator on quizzes or exams), it would be better for you to take the root first, then raise to the exponent power.**

The index of the root comes from the denominator. The exponent is the numerator.

Evaluate or write an equivalent root for each.

8. $27^{\frac{2}{3}} =$

9. $16^{\frac{3}{2}} =$

10. $(mn)^{\frac{3}{4}} =$

Write an equivalent expression using exponential notation.

11. $\sqrt[4]{8^3} =$

12. $\left(\sqrt[3]{(ab)}\right)^5 =$

The rules of exponents we had back at the beginning of the semester also apply with these rational exponents. Here is a summary of those rules.

Product Rule: $a^m \bullet a^n = a^{m+n}$

Quotient Rule: $\frac{a^m}{a^n} = a^{m-n}$

Negative Exponent Rule: $a^{-n} = \frac{1}{a^n}$ or $\frac{1}{a^{-n}} = a^n$ or $\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$

Power Rules: $\left(a^m\right)^n = a^{mn}$

$(ab)^n = a^n b^n$

$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$

Use the rules of exponents to simplify.

13. $8^{\frac{2}{3}} \bullet 8^{\frac{5}{3}} =$

14. $\dfrac{2^{\frac{5}{8}}}{2^{-\frac{1}{8}}} =$

15. $\left(x^{-\frac{1}{4}}\right)\left(x^{\frac{4}{5}}\right) =$

16. $\left(4^{\frac{2}{3}}\right)^{\frac{9}{2}}$

17. $\left(4^{\frac{2}{3}} m^{\frac{3}{4}}\right)^{\frac{1}{2}} =$

18. $\left(x^{-\frac{1}{3}}\right)^{4}$