# Lesson 35 MA 152, Section 2.8

## I Circles

**An earlier lesson has the distance and midpoint formulas. You will need to remember those formulas for this lesson.**

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

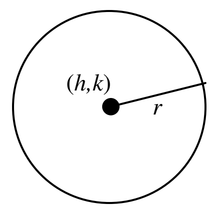
$$M : \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

A **circle** is a set of all points in a plane that are a fixed distance, called the **radius**, and from a point called its **center.**

**Equation of a circle:** The distance formula can be used to find the equation of a circle. Let the center of the circle be represented by the ordered pair (*h, k*) and the radius of the circle by *r*. Every point (*x, y*) is *r* units from (*h, k*).

$$r = \sqrt{(x-h)^2 + (y-k)^2}$$

$$r^2 = (x-h)^2 + (y-k)^2$$




This is the **standard equation for a circle.**

> Standard Equation of a Circle:
> If the center of a circle is (*h, k*) and the radius of the circle is *r* units, the an equation for the circle is $(x-h)^2 + (y-k)^2 = r^2$.

If the center of the circle is the origin, then the standard equation of the circle is $x^2 + y^2 = r^2$.

The **general equation for a circle** is an equation where the parentheses are cleared from the standard form (binomials are squared)

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> General Equation of a Circle:
> $x^2 + y^2 + cx + dy + e = 0$, where $c, d$, and $e$ represent real numbers.

Ex 1: Find the standard equation in (a) and the general form in (b) for each circle described.

$a)$ center: $(2, -3)$, radius: 4 units

b) center: $(-3,6)$, a point of the circle: $(12,2)$

Ex 2: Identify the center and length of radius for each circle.

$a)$ $(x-4)^2 + y^2 = 49$

$b)$ $(x+9)^2 + (y-\frac{1}{2})^2 = 21$

To change an equation of a circle from general form to standard form, a completing the square process must be used. Examine this example.

$2x^2 + 2y^2 - 8x + 12y - 24 = 0$

$x^2 + y^2 - 4x + 6y - 12 = 0$

$(x^2 - 4x \qquad) + (y^2 + 6y \qquad) = 12$ center: (2, -3), radius: 5

$(x^2 - 4x + 4) + (y^2 + 6y + 9) = 12 + 4 + 9$

$(x-2)^2 + (y+3)^2 = 25$

The following steps were used to convert above from general to standard.

1. If necessary, divide so each squared term has coefficients of 1.
2. Arrange the x terms together and the y terms together and move the constant to the other side.
3. Complete the square for the x's and for the y's. Balance the equation by adding the numbers used to complete the square to the other side as well.
4. Write with binomial squared terms and combine the numbers.

Ex 3: Write each circle equation in standard form.

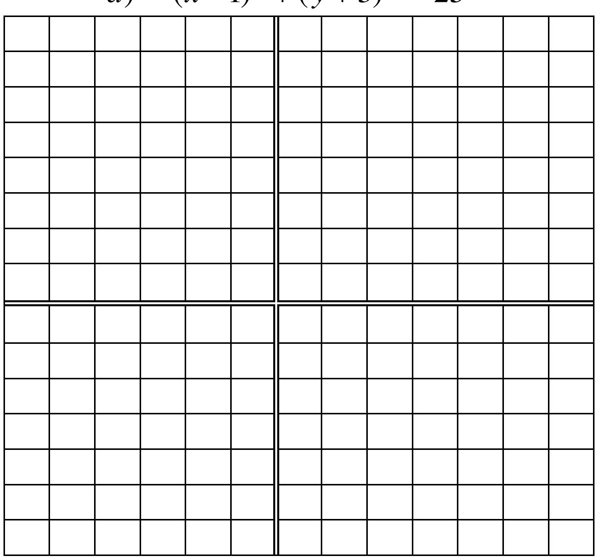
$a)$ $x^2 + y^2 - 10x - 20y - 2 = 0$

$b)$ $4x^2 + 4y^2 - 4x + 8y + 1 = 0$

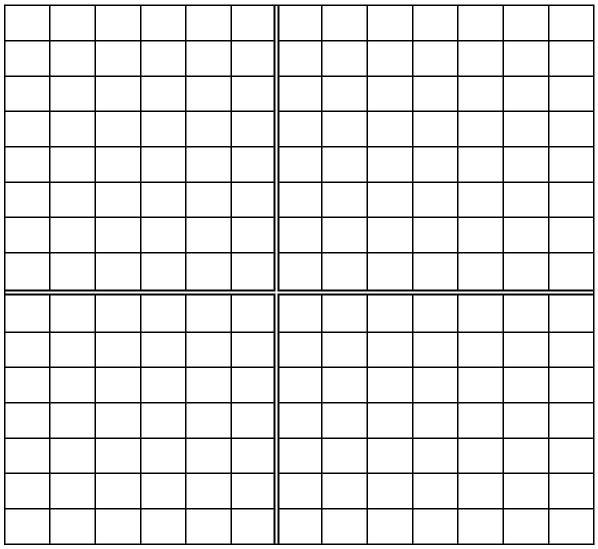
Graphing a circle: To graph circle, locate the center. Find points *r* units up, down, left, and right from the center. (Sometimes it may be valuable to locate other points in a table.) Draw as smooth a circle as possible. Identify the domain and range.

Ex 4: Graph each circle and identify the domain and range:

$a)$ $(x-1)^2 + (y+3)^2 = 25$



$b)\quad x^2 + y^2 + 2x - 4y - 4 = 0$



Ex 5: Find the equation (in standard form) for a circle with endpoints of a diameter of the circle at (3, -2) and (5, 8).

Ex 6: Find the equation of a circle (in general form) if the circle has a radius of 8 units and the center is at the intersection of lines $x + 2y = 8$ and $2x - 3y = -5$ .

<u>Ex7:</u> The picture below represents two gears, a larger gear on the left that has a circle equation $x^2 + y^2 = 16$. The smaller gear has a center at (7, 0) and only touches the larger gear where they meet. Find an equation for the smaller gear. Notice the *x*-axis and *y*-axis and locate the origin.