NAME _____

10-DIGIT PUID _____

REC. INSTR. _____ REC. TIME _____

LECTURER _____

INSTRUCTIONS:

1. There are 7 different test pages (including this cover page). Make sure you have a complete test.
2. Fill in the above items in print. Also write your name at the top of pages 2–7.
3. Do any necessary work for each problem on the space provided or on the back of the pages of this test booklet. Circle your answers in this test booklet. No partial credit will be given, but if you show your work on the test booklet, it may be used in borderline cases.
4. No books, notes, calculators or any electronic devices may be used on this exam.
5. Each problem has its own points assigned. The maximum possible score is 100 points.
6. Using a #2 pencil, fill in each of the following items on your answer sheet:
   (a) On the top left side, write your name (last name, first name), and fill in the little circles.
   (b) On the bottom left side, under SECTION NUMBER, put 0 in the first column and then enter the 3-digit section number. For example, for section 016 write 0016. Fill in the little circles.
   (c) On the bottom, under TEST/QUIZ NUMBER, write 01 and fill in the little circles.
   (d) On the bottom, under STUDENT IDENTIFICATION NUMBER, write in your 10–digit PUID, and fill in the little circles.
   (e) Using a #2 pencil, put your answers to questions 1–12 on your answer sheet by filling in the circle of the letter of your response. Double check that you have filled in the circles you intended. If more than one circle is filled in for any question, your response will be considered incorrect. Use a #2 pencil.
7. After you have finished the exam, hand in your answer sheet and your test booklet to your recitation instructor.

(6 pts) 1. The equation of the sphere that passes through the origin with center (1,2,3) is:

A. $(x-1)^2+(y-2)^2+(z-3)^2=6$

B. $(x-1)^2+(y-2)^2+(z-3)^2=14$

C. $x^2+y^2+z^2=14$

D. $(x+1)^2+(y+2)^2+(z+3)^2=6$

E. $(x+1)^2+(y+2)^2+(z+3)^2=14$

(8 pts) 2. For what values of b are the vectors $\langle -6, b, 2\rangle$ and $\langle b, b^2, b\rangle$ orthogonal?

A. 0,1 and -1

B. 0,3 and -3

C. 0,2 and -2

D. $2\sqrt{2}$ and $-2\sqrt{2}$

E. 1 and 2

(8 pts) 3. The vector projection $\mathbf{proj_a b}$ of a vector $\mathbf{b} = \langle 5, 0\rangle$ onto another vector $\mathbf{a} = \langle 3, -4\rangle$ is:

A. $\frac{3}{5}\langle 5, 0\rangle$

B. $\frac{5}{3}\langle 3, -4\rangle$

C. $\frac{3}{5}\langle 3, -4\rangle$

D. $\frac{3}{25}\langle 5, 0\rangle$

E. $\frac{3}{25}\langle 3, -4\rangle$

(8 pts) 4. A sled is pulled 100 m along a horizontal path by a force of 30 N acting at an angle of 30 degree above the horizontal. The work done by the force is

A. 1500 J

B. 150 L

C. $1500\sqrt{2}$ J

D. $150\sqrt{3}$ J

E. $1500\sqrt{3}$ J

(8 pts) 5. Which of the following is a vector orthogonal to the plane through the points $P(2,1,5)$, $Q(-1,3,4)$, $R(3,0,6)$?

A. $2\mathbf{i}+3\mathbf{j}+2\mathbf{k}$

B. $\mathbf{i}-\mathbf{j}+\mathbf{k}$

C. $-2\mathbf{i}-3\mathbf{j}+\mathbf{k}$

D. $2\mathbf{i}+4\mathbf{j}+2\mathbf{k}$

E. $3\mathbf{i}+\mathbf{j}-3\mathbf{k}$

(8 pts) 6. Find the values of x such that the vectors $\mathbf{a}=\langle 1,0,x\rangle$, $\mathbf{b}=\langle 2,x,1\rangle$, $\mathbf{c}=\langle 6,1,5\rangle$ are coplanar, that is to to say, they lie on a plane.

A. 4 and 1/8

B. 3 and 2

C. 1/6 and 1

D. 1/2 and -1

E. 0 and 2

(8 pts) 7. Find the area of the region enclosed by the curves $x = 4y - y^2$ and $x = 2y - 3$.

A. 64/5

B. 32/3

C. 16

D. 15/7

E. 8/3

(8 pts) 8. Find the formula for the volume of the solid obtained by rotating the region bounded by the curves $y = e^{-x}$, $y = 1$ and $x = 4$ about the line $y = 2$

A. $\int_0^4 e^{-x}dx$

B. $\pi\int_0^4 e^{-x}dx$

C. $\pi\int_0^4 [e^{-2x} - 1]dx$

D. $\pi\int_0^4 [e^{-2x} - 4e^{-x} + 3]dx$

E. $\pi\int_0^4 e^{-2x}dx$

(10 pts) 9. Find the volume of the solid obtained by rotating about the y-axis the region bounded by $y = 3x^2 - x^3$ and $y = 0$.

A. $\pi/6$

B. $8\pi/5$

C. $16\pi/5$

D. $243\pi/20$

E. $243\pi/10$

(10 pts) 10. Consider the following tank in the shape of a triangular prism.
The top is a rectangle with sides $W = 3\text{m}$ and $L = 10\text{m}$. Each vertical cross section is an inverted triangle with height 6m (and with base of length 3m).

It is completely filled with water with density 1000kgm^{-3}. Find the work done necessary to pump all the water just out of the tank.

(Warning: Carry out the computation setting the gravitational acceleration g to be 10ms^{-2}.)

A. $18 \times 10^5\text{J}$

B. $36 \times 10^5\text{J}$

C. $2 \times 10^4\text{J}$

D. $9 \times 10^4\text{J}$

E. $18 \times 10^4\text{J}$

(8 pts) 11. Find the following indefinite integral

$$\int \sin^{-1}(5x)dx$$

A. $\dfrac{x^3 \sin 5x}{15} + C$

B. $\dfrac{x^2 \sin 5x}{5} + \dfrac{2x \cos 5x}{25} + C$

C. $\dfrac{x^2 \sin 5x}{5} + \dfrac{2x \cos 5x}{25} + \dfrac{2}{125} \sin 5x + C$

D. $\dfrac{x^2 \sin 5x}{5} + \dfrac{2x \cos 5x}{25} - \dfrac{2}{125} \sin 5x + C$

E. $\dfrac{x^3 \cos 5x}{3} + \dfrac{x^2 \sin 5x}{5} + C$

(10 pts) 12. Evaluate the following integral

$$\int_1^{\sqrt{3}} \tan^{-1}\left(\frac{1}{x}\right) dx.$$

A. $\left(\dfrac{\sqrt{3}}{6} - \dfrac{1}{4}\right)\pi + \dfrac{1}{2}\ln 2$

B. $\left(\dfrac{\sqrt{3}}{3} - \dfrac{1}{4}\right)\pi + \dfrac{1}{2}\ln 2$

C. $\left(\dfrac{\sqrt{3}}{6} - \dfrac{1}{4}\right)\pi$

D. $\dfrac{\pi}{6} - \dfrac{\pi}{4}$

E. $\dfrac{\pi}{3} - \dfrac{\pi}{4}$