CHAPTER 1
Functions, Graphs, and Limits

CHAPTER 1
Functions, Graphs, and Limits

Section 1.1 The Cartesian Plane and the Distance Formula

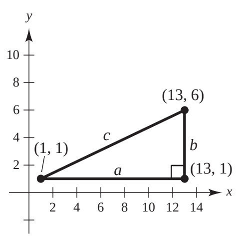
Solutions to Even-Numbered Exercises

2. (a) $a = \sqrt{(13 - 1)^2 + (1 - 1)^2} = 12$

$b = \sqrt{(13 - 13)^2 + (6 - 1)^2} = 5$

$c = \sqrt{(13 - 1)^2 + (6 - 1)^2} = 13$

(b) $a^2 + b^2 = (12)^2 + (5)^2 = 169 = c^2$

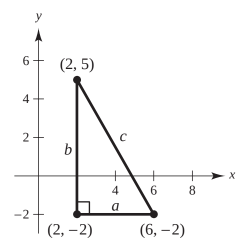


4. (a) $a = \sqrt{(6 - 2)^2 + (-2 + 2)^2} = 4$

$b = \sqrt{(2 - 2)^2 + (5 + 2)^2} = 7$

$c = \sqrt{(2 - 6)^2 + (5 + 2)^2} = \sqrt{65}$

(b) $a^2 + b^2 = (4)^2 + (7)^2 = 65 = c^2$

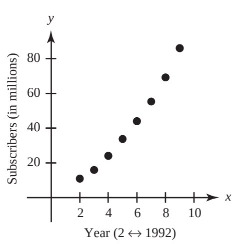


6. (a) $a = 3 - 1 = 2$

$b = 1 - (-4) = 5$

$c = \sqrt{(1 - (-4))^2 + (1 - 3)^2} = \sqrt{5^2 + 2^2} = \sqrt{29}$

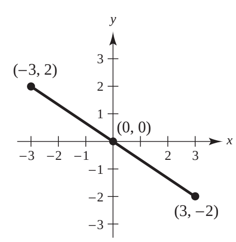
(b) $a^2 + b^2 = 2^2 + 5^2 = 29 = c^2$



8. (a) See graph.

(b) $d = \sqrt{(-3 - 3)^2 + (2 + 2)^2} = \sqrt{52} = 2\sqrt{13}$

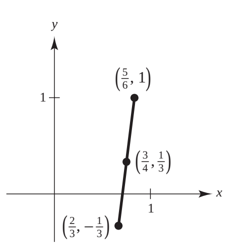
(c) Midpoint $= \left(\dfrac{-3 + 3}{2}, \dfrac{2 + (-2)}{2}\right) = (0, 0)$



10. (a) See graph.

(b) $d = \sqrt{\left(\dfrac{5}{6} - \dfrac{2}{3}\right)^2 + \left(1 + \dfrac{1}{3}\right)^2} = \sqrt{\dfrac{1}{36} + \dfrac{16}{9}} = \dfrac{\sqrt{65}}{6}$

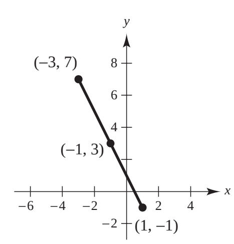
(c) Midpoint $= \left(\dfrac{(5/6) + (2/3)}{2}, \dfrac{1 - (1/3)}{2}\right) = \left(\dfrac{3}{4}, \dfrac{1}{3}\right)$



12. (a) See graph.

(b) $d = \sqrt{(-3 - 1)^2 + (7 + 1)^2} = \sqrt{16 + 64} = 4\sqrt{5}$

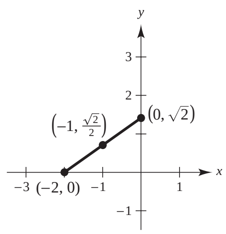
(c) Midpoint $= \left(\dfrac{-3 + 1}{2}, \dfrac{7 - 1}{2}\right) = (-1, 3)$



14. (a) See graph.

(b) $d = \sqrt{(-2-0)^2 + \left(0 - \sqrt{2}\right)^2} = \sqrt{6}$

(c) Midpoint $= \left(\dfrac{-2+0}{2}, \dfrac{0+\sqrt{2}}{2}\right) = \left(-1, \dfrac{\sqrt{2}}{2}\right)$



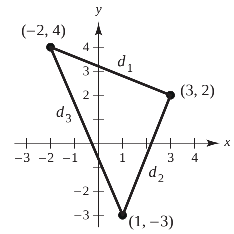
16. $a = \sqrt{(-2-3)^2 + (4-2)^2} = \sqrt{29}$

$b = \sqrt{(3-1)^2 + (2+3)^2} = \sqrt{29}$

$c = \sqrt{(-2-1)^2 + (4+3)^2} = \sqrt{58}$

Since $a = b$ the figure is an isosceles triangle.

[**Note:** It is also a right triangle since $a^2 + b^2 = c^2$.]



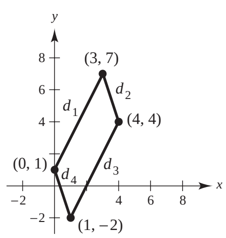
18. $a = \sqrt{(3-0)^2 + (7-1)^2} = 3\sqrt{5}$

$b = \sqrt{(3-4)^2 + (7-4)^2} = \sqrt{10}$

$c = \sqrt{(4-1)^2 + (4+2)^2} = 3\sqrt{5}$

$d = \sqrt{(1-0)^2 + (-2-1)^2} = \sqrt{10}$

Since $a = c$ and $b = d$, the figure is a parallelogram.



20. $d_1 = \sqrt{(-5-0)^2 + (11-4)^2} = \sqrt{74}$

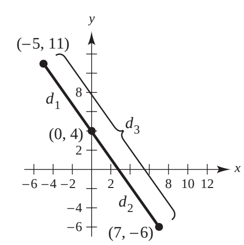
$d_2 = \sqrt{(0-7)^2 + (4+6)^2} = \sqrt{149}$

$d_3 = \sqrt{(-5-7)^2 + (11+6)^2} = \sqrt{433}$

$d_1 + d_2 \approx 20.80888$

$d_3 \approx 20.80865$

Since $d_1 + d_2 \neq d_3$, the points are not collinear.



22. $d_1 = \sqrt{(-1-3)^2 + (1-3)^2} = 2\sqrt{5}$

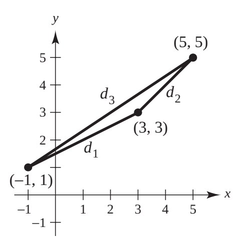
$d_2 = \sqrt{(3-5)^2 + (3-5)^2} = 2\sqrt{2}$

$d_3 = \sqrt{(-1-5)^2 + (1-5)^2} = 2\sqrt{13}$

$d_1 + d_2 \approx 7.30056$

$d_3 \approx 7.21110$

Since $d_1 + d_2 \neq 3$, the points are not collinear.



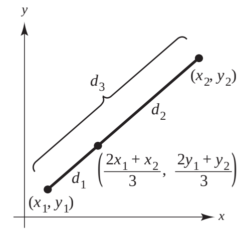
24. $d = \sqrt{(x-2)^2 + (2+1)^2} = 5$

$$\sqrt{x^2 - 4x + 13} = 5$$
$$x^2 - 4x + 13 = 25$$
$$x^2 - 4x - 12 = 0$$
$$(x+2)(x-6) = 0$$
$$x = -2, 6$$

26. $d = \sqrt{(5-5)^2 + (y-1)^2} = 8$

$$\sqrt{(y-1)^2} = 8$$
$$(y-1)^2 = 64$$
$$y - 1 = \pm 8$$
$$y = 1 \pm 8$$
$$y = -7, 9$$

28. To show that $\left(\frac{2x_1 + x_2}{3}, \frac{2y_1 + y_2}{3}\right)$ is a point of trisection of the line segment joining (x_1, y_1) and (x_2, y_2), we must show that $d_1 = \frac{1}{2}d_2$ and $d_1 + d_2 = d_3$.



$$d_1 = \sqrt{\left(\frac{2x_1 + x_2}{3} - x_1\right)^2 + \left(\frac{2y_1 + y_2}{3} - y_1\right)^2}$$

$$= \sqrt{\left(\frac{x_2 - x_1}{3}\right)^2 + \left(\frac{y_2 - y_1}{3}\right)^2} = \frac{1}{3}\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$d_2 = \sqrt{\left(x_2 - \frac{2x_1 + x_2}{3}\right)^2 + \left(y_2 - \frac{2y_1 + y_2}{3}\right)^2}$$

$$= \sqrt{\left(\frac{2x_2 - 2x_1}{3}\right)^2 + \left(\frac{2y_2 - 2y_1}{3}\right)^2} = \frac{2}{3}\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$d_3 = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Therefore, $d_1 = \frac{1}{2}d_2$ and $d_1 + d_2 = d_3$. The midpoint of the line segment joining $\left(\frac{2x_1 + x_2}{3}, \frac{2y_1 + y_2}{3}\right)$ and (x_2, y_2) is

$$\text{Midpoint} = \left(\frac{\frac{2x_1 + x_2}{3} + x_2}{2}, \frac{\frac{2y_1 + y_2}{3} + y_2}{2}\right) = \left(\frac{x_1 + 2x_2}{3}, \frac{y_1 + 2y_2}{3}\right).$$

30. (a) $\left(\frac{2(1) + 4}{3}, \frac{2(-2) + 1}{3}\right) = (2, -1)$

$\left(\frac{1 + 2(4)}{3}, \frac{-2 + 2(1)}{3}\right) = (3, 0)$

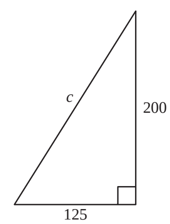
(b) $\left(\frac{2(-2) + 0}{3}, \frac{2(-3) + 0}{3}\right) = \left(-\frac{4}{3}, -2\right)$

$\left(\frac{-2 + 2(0)}{3}, \frac{-3 + 2(0)}{3}\right) = \left(-\frac{2}{3}, -1\right)$

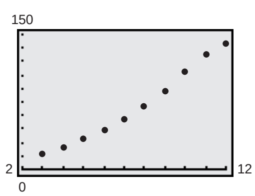
32. $c^2 = 200^2 + 125^2$

$c^2 = 55{,}625$

$c \approx 235.8495$ feet



34.



Let $t = 3$ correspond to 1993. Answers will vary. The number of subscribers appears to be increasing rapidly (not linearly).

36. (a) $\frac{8550 - 10{,}400}{10{,}400} \approx -0.178 \approx -17.8\%$

(b) $\frac{10{,}700 - 8{,}900}{8{,}900} \approx 0.202 \approx 20.2\%$

38. (a) $\frac{107 - 103}{103} \approx 0.039 \approx 3.9\%$

(b) $\frac{159 - 148}{148} \approx 0.074 \approx 7.4\%$

40. (a) Revenue midpoint $= \left(\frac{1999 + 2003}{2}, \frac{256.6 + 508.6}{2}\right)$

$= (2001, 382.6)$

Revenue estimate for 2001: \$382.6 million

Profit midpoint $= \left(\frac{1999 + 2003}{2}, \frac{34.3 + 74.8}{2}\right)$

$= (2001, 54.55)$

Profit estimate for 2001: \$54.55 million

(b) Actual 2001 revenue: \$379.8 million

Actual 2001 profit: \$48.2 million

(c) The revenue increased in a linear pattern (382.6 is close to 379.8). The profit is somewhat linear (54.55 is close to 48.2).

(d) 1999 Expenses: $256.6 - 34.3 = \$222.3$ million

2001

2003 Expenses: $508.6 - 74.8 = \$433.8$ million

(e) Answers will vary.

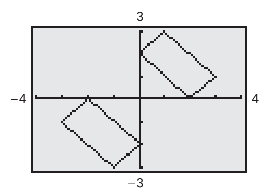
42. (a) $(0, 2)$ is translated to $(0 - 3, 2 - 3) = (-3, -1)$

$(1, 3)$ is translated to $(1 - 3, 3 - 3) = (-2, 0)$

$(3, 1)$ is translated to $(3 - 3, 1 - 3) = (0, -2)$

$(2, 0)$ is translated to $(2 - 3, 0 - 3) = (-1, -3)$

(b)



Section 1.2 Graphs of Equations

2. (a) This is a solution point since $7(6) + 4(-9) - 6 = 0$

(b) This is not a solution point since $7(-5) + 4(10) - 6 = -1 \neq 0$

(c) This is a solution point since $7\left(\frac{1}{2}\right) + 4\left(\frac{5}{8}\right) - 6 = 0$

4. (a) This is not a solution point since $x^2y + x^2 - 5y = 0^2\left(\frac{1}{5}\right) + 0^2 - 5\left(\frac{1}{5}\right) = -1 \neq 0$.

(b) This is a solution point since $x^2y + x^2 - 5y = 2^2(4) + 2^2 - 5(4) = 0$.

(c) This is not a solution point since $x^2y + x^2 - 5y = (-2)^2(-4) + (-2)^2 - 5(-4) = 8 \neq 0$.

6. (a) This is not a solution point since $3(-5) + 2(-7)(-5) - (-7)^2 = -14 \neq 5$

(b) This is a solution point since $3(6) + 2(-1)(6) - (-1)^2 = 5$

(c) This is a solution point since $3\left(\frac{6}{5}\right) + 2(1)\left(\frac{6}{5}\right) - 1^2 = 5$

8. The graph of $y = -\frac{1}{2}x + 2$ is a straight line with y-intercept at $(0, 2)$. Thus, it matches (b).

10. The graph of $y = \sqrt{9 - x^2}$ is a semicircle with intercepts $(0, 3)$, $(3, 0)$, and $(-3, 0)$. Thus, it matches (f).

12. The graph of $y = x^3 - x$ has intercepts at $(0, 0)$, $(1, 0)$, and $(-1, 0)$. Thus, it matches (d).

14. Let $y = 0$: $4x - 5 = 0$

$$x = \tfrac{5}{4}$$

x-intercept: $\left(\tfrac{5}{4}, 0\right)$

Let $x = 0$: $-2y - 5 = 0$

$$y = -\tfrac{5}{2}$$

y–intercept: $\left(0, -\tfrac{5}{2}\right)$

16. Let $x = 0$: $y = 3$

y-intercept: $(0, 3)$

Let $y = 0$: $x^2 - 4x + 3 = 0$

$$(x - 3)(x - 1) = 0$$

$$x = 3, 1$$

x-intercepts: $(3, 0)$, $(1, 0)$

18. Let $x = 0$: $y^2 = 0$

$$y = 0$$

y-intercept: $(0, 0)$

Let $y = 0$: $x^3 - 4x = 0$

$$x(x - 2)(x + 2) = 0$$

$$x = 0, 2, -2$$

x-intercepts: $(0, 0)$, $(2, 0)$, $(-2, 0)$

20. The y-intercept is $(0, 0)$. To find the x-intercepts, let $y = 0$ to obtain

$$0 = \frac{x^2 + 3x}{(3x + 1)^2}$$

$$0 = x^2 + 3x$$

$$0 = x(x + 3)$$

$$x = 0, -3.$$

Thus, the x-intercepts are $(0, 0)$ and $(-3, 0)$.

22. Let $x = 0$: $8y = 1$

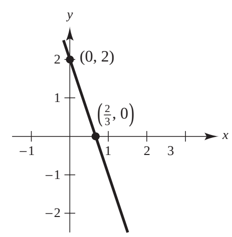
$$y = \tfrac{1}{8}$$

y-intercept: $\left(0, \tfrac{1}{8}\right)$

Let $y = 0$: $-x^2 = 1$

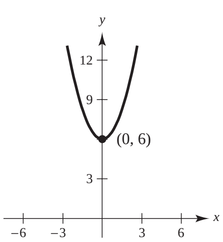
No x-intercepts

24. The graph of $y = -3x + 2$ is a straight line with slope -3 and $y =$ intercept $(0, 2)$.

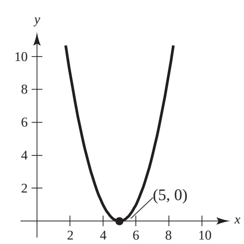


26. The graph of $y = x^2 + 6$ is a parabola with vertex at $(0, 6)$, which is also the only intercept.

x	0	±1	±2
y	6	7	10

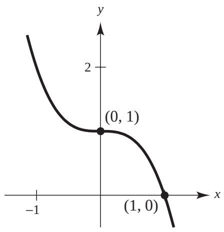


28. The graph of $y = (5 - x)^2$ is a parabola with vertex at $(5, 0)$.



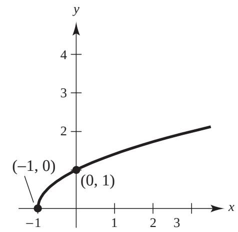
30. Intercepts: $(0, 1)$ and $(1, 0)$

x	0	1	-1	2
y	1	0	2	-7



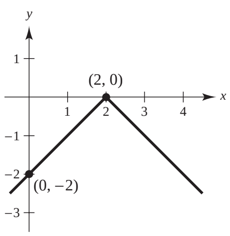
32. The graph of $y = \sqrt{x + 1}$ is a translation of $y = \sqrt{x}$ one unit to the left.

Intercepts: $(-1, 0)$, $(0, 1)$



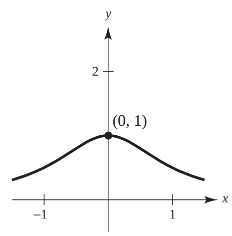
34. Intercepts: $(2, 0)$ and $(0, -2)$

x	2	0	1	3	4
y	0	-2	-1	-1	-2



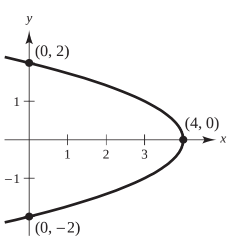
36. Intercept: $(0, 1)$

x	0	± 1	± 2	± 3
y	1	$\frac{1}{2}$	$\frac{1}{5}$	$\frac{1}{10}$



38. Intercepts: $(0, 2)$, $(0, -2)$, $(4, 0)$

x	0	3	4
y	± 2	± 1	0



40.
$$(x - 0)^2 + (y - 0)^2 = 5^2$$
$$x^2 + y^2 = 25$$
$$x^2 + y^2 - 25 = 0$$

42.
$$(x + 4)^2 + (y - 3)^2 = 3^2$$
$$x^2 + 8x + 16 + y^2 - 6y + 9 = 9$$
$$x^2 + y^2 + 8x - 6y + 16 = 0$$

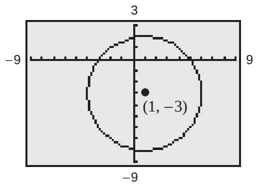
44. Radius $= \sqrt{(-1 - 3)^2 + (1 + 2)^2} = 5$
$$(x - 3)^2 + (y + 2)^2 = 5^2$$
$$x^2 - 6x + 9 + y^2 + 4y + 4 = 25$$
$$x^2 + y^2 - 6x + 4y - 12 = 0$$

46. Center $=$ midpoint $= \left(\dfrac{-4 + 4}{2}, \dfrac{-1 + 1}{2}\right) = (0, 0)$

Radius $=$ distance from the center to an endpoint $= \sqrt{(4 - 0)^2 + (1 - 0)^2} = \sqrt{17}$
$$(x - 0)^2 + (y - 0)^2 = \left(\sqrt{17}\right)^2$$
$$x^2 + y^2 - 17 = 0$$

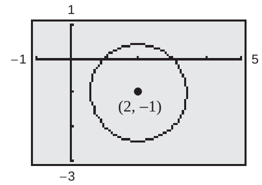
48. $(x^2 - 2x + 1) + (y^2 + 6y + 9) = 15 + 1 + 9$

$$(x - 1)^2 + (y + 3)^2 = 25$$



50. $(x^2 - 4x + 4) + (y^2 + 2y + 1) = -3 + 4 + 1$

$$(x - 2)^2 + (y + 1)^2 = 2$$

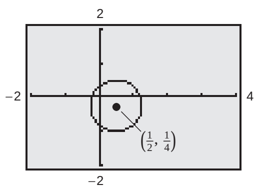


52.

$$x^2 + y^2 - x + \tfrac{1}{2}y - \tfrac{1}{4} = 0$$

$$\left(x^2 - x + \tfrac{1}{4}\right) + \left(y^2 + \tfrac{1}{2}y + \tfrac{1}{16}\right) = \tfrac{1}{4} + \tfrac{1}{4} + \tfrac{1}{16}$$

$$\left(x - \tfrac{1}{2}\right)^2 + \left(y + \tfrac{1}{4}\right)^2 = \tfrac{9}{16}$$

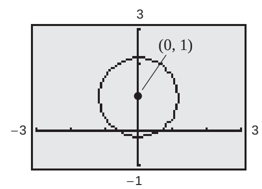


54.

$$x^2 + y^2 - 2y - \tfrac{1}{3} = 0$$

$$x^2 + (y^2 - 2y + 1) = \tfrac{1}{3} + 1$$

$$x^2 + (y - 1)^2 = \tfrac{4}{3}$$



56. The first equation gives $y = 7 - x$. Hence,

$$3x - 2(7 - x) = 5x - 14 = 11$$

$$5x = 25$$

$$x = 5, y = 2$$

58. Solving for y in the second equation yields $y = 2x - 1$ and substituting this value into the first equation gives us the following.

$$x^2 + (2x - 1) = 4$$

$$x^2 + 2x - 5 = 0$$

$$x = -1 \pm \sqrt{6} \text{ by the Quadratic Formula}$$

The corresponding y-values are $y = -3 \pm 2\sqrt{6}$, so the points of intersection are $\left(-1 + \sqrt{6}, -3 + 2\sqrt{6}\right)$ and $\left(-1 - \sqrt{6}, -3 - 2\sqrt{6}\right)$.

60. By equating the y-values for the two equations, we have

$$\sqrt{x} = x$$

$$x = x^2$$

$$0 = x(x - 1)$$

$$x = 0, 1.$$

The corresponding y-values are $y = 0, 1$, so the points of intersection are $(0, 0)$ and $(1, 1)$.

62. By equating the y-values for the two equations, we have

$$x^3 - 2x^2 + x - 1 = -x^2 + 3x - 1$$

$$x^3 - x^2 - 2x = 0$$

$$x(x + 1)(x - 2) = 0$$

$$x = 0, -1, 2.$$

The corresponding y-values are $y = -1, -5$, and 1, so the points of intersection are $(0, -1)$, $(-1, -5)$, and $(2, 1)$.

64. (a) $C_g = \text{(initial price)} + \text{(cost per mile)}$

$$= 20{,}930 + \frac{1.759x}{16}$$

$$C_h = 22{,}052 + \frac{1.759x}{35}$$

(b)

$$C_g = C_h$$

$$20{,}930 + \frac{1.759x}{16} = 22{,}052 + \frac{1.759x}{35}$$

$$\left(\frac{1.759}{16} - \frac{1.759}{35}\right)x = 1122$$

$$0.0596804x = 1122$$

$$x \approx 18{,}800 \text{ miles}$$

66. $R = C$

$35x = 6x + 500{,}000$

$29x = 500{,}000$

$x = 500{,}000/29 \approx 17{,}242$ units

68.
$$R = C$$
$$3.29x = 5.5\sqrt{x} + 10{,}000$$
$$(3.29x - 10{,}000)^2 = (5.5\sqrt{x})^2$$
$$10.8241x^2 - 65{,}800x + 100{,}000{,}000 = 30.25x$$
$$10.8241x^2 - 65{,}830.25x + 100{,}000{,}000 = 0$$

By using the Quadratic Formula, we have $x = \dfrac{65{,}830.25 \pm \sqrt{3{,}981{,}815.062}}{21.6482}$

$$x \approx 3133 \text{ units.}$$

[**Note:** $x = 2949$ units is an extraneous solution.]
You can also solve this problem with a graphing utility by determining the point of intersection of the two equations $y_1 = R = 3.29x$ and $y_2 = C = 5.5\sqrt{x} + 10{,}000$.

70. $p = 190 - 15x = 75 + 8x$

$115 = 23x$

$x = 5$ (Thousand)

Equilibrium point $(x, p) = (5, 115)$.

72. Model: $y = \dfrac{-4.97 + 0.021t}{1 - 0.025t}$

($t = 55$ corresponds to 1955)

(a)

t	55	60	65	70	75	80	85	90	95	100
Model	10.2	7.4	5.8	4.7	3.9	3.3	2.8	2.5	2.2	1.9
Exact	9.9	7.8	5.9	4.2	3.6	3.1	2.8	2.6	2.6	1.7

The model is a good fit.

(b) For 2010, $t = 110$ and $y \approx 1.5\%$.

(c) Answers will vary.

74. Model: $y = 60.64t^2 - 544.0t + 12{,}624$

($t = 8$ corresponds to 1998)

(a)

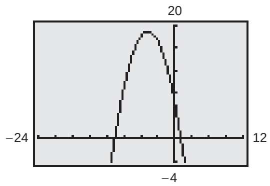
Year	1998	1999	2000	2001	2002
Transplants	12,153	12,640	13,248	13,977	14,824

(b) 1998: 12,244 transplants.
2002: 14,741 transplants.
The model seems accurate.

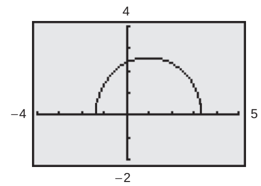
(c) For 2008, $t = 18$ and $y \approx 22{,}479$ transplants. Answers will vary.

76. If C and R represent the cost and revenue for a business, the break-even point is that value of x for which $C = R$. For example, if $C = 100{,}000 + 10x$ and $R = 20x$, then the break-even point is $x = 10{,}000$ units.

78. Intercepts: $(0, 6.25)$, $(1.0539, 0)$, $(-10.5896, 0)$

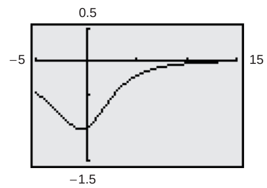


80.



Intercepts: $(3.3256, 0)$, $(-1.3917, 0)$, $(0, 2.3664)$

82.



Intercepts: $(0, -1)$, $(13.25, 0)$

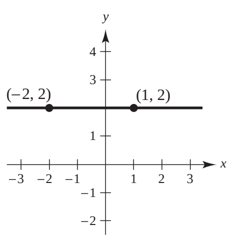
Section 1.3 Lines in the Plane and Slope

2. The slope is 2 since the line rises two units vertically for each unit of horizontal change from left to right.

4. The slope is -1 since the line falls one unit vertically for each unit of horizontal change from left to right.

6. The points are plotted in the accompanying graph and the slope is

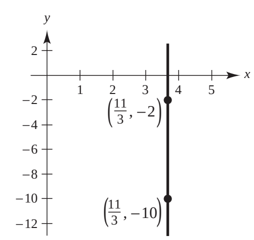
$$m = \frac{2 - 2}{1 - (-2)} = 0.$$



8. The points are plotted in the accompanying graph and the slope is

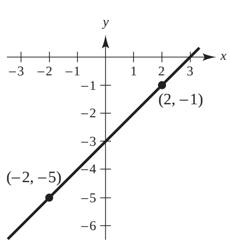
$$m = \frac{-10 - (-2)}{\frac{11}{3} - \frac{11}{3}} = \frac{-8}{0}. \quad \text{Undefined}$$

The line is vertical.



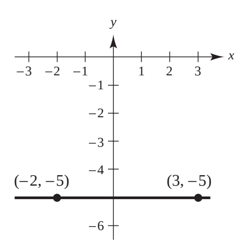
10. The points are plotted in the accompanying graph and the slope is

$$m = \frac{-1 - (-5)}{2 - (-2)} = \frac{4}{4} = 1.$$



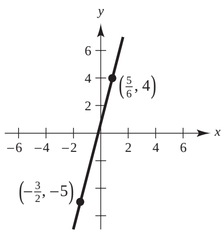
12. The points are plotted in the accompanying graph and the slope is

$$m = \frac{-5 - (-5)}{-2 - 3} = 0.$$



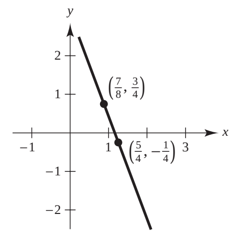
14. The points are plotted in the accompanying graph and the slope is

$$m = \frac{4 + 5}{(5/6) + (3/2)} = \frac{27}{7}.$$



16. The points are plotted in the accompanying graph and the slope is

$$m = \frac{(-1/4) - (3/4)}{(5/4) - (7/8)} = -\frac{8}{3}.$$



18. The equation of this horizontal line is $y = -1$. Therefore, three additional points are $(0, -1)$, $(1, -1)$, and $(2, -1)$.

20. The equation of this line is

$$y + 2 = \tfrac{5}{2}(x + 2)$$
$$y = \tfrac{5}{2}x + 3$$

Therefore, three additional points are $(0, 3)$, $(2, 8)$, $(4, 13)$.

22. The equation of this line is

$$y + 6 = -1(x - 10)$$
$$y = -x + 4.$$

Therefore, three additional points are $(11, -7)$, $(9, -5)$, and $(8, -4)$.

24. The equation of this vertical line is $x = -3$. Therefore, three additional points are $(-3, 0)$, $(-3, 1)$, and $(-3, 2)$.

26. $2x + y = 40$

$$y = -2x + 40$$

Therefore, the slope is $m = -2$, and the y-intercept is $(0, 40)$.

28. $6x - 5y = 15$

$$y = \tfrac{6}{5}x - 3$$

Therefore, the slope is $m = \frac{6}{5}$, and the y-intercept is $(0, -3)$.

30. $2x - 3y = 24$

$$y = \tfrac{1}{3}(2x - 24) = \tfrac{2}{3}x - 8$$

Slope is $m = \frac{2}{3}$, y-intercept is $(0, -8)$

32. $x + 5 = 0$

$$x = -5$$

The line is vertical. Slope is undefined and there is no y-intercept.

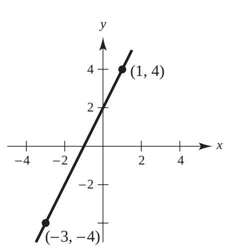
34. Since the line is horizontal, the slope is $m = 0$, and the y-intercept is $(0, -1)$.

36. The slope of the line is

$$m = \frac{4 - (-4)}{1 - (-3)} = 2.$$

Using the point-slope form, we have

$$y - 4 = 2(x - 1)$$
$$y = 2x + 2$$
$$0 = 2x - y + 2.$$

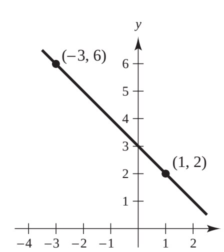


38. The slope of the line is

$$m = \frac{2 - 6}{1 - (-3)} = -1.$$

Using the point-slope form, we have

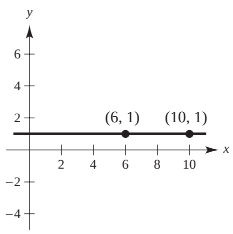
$$y - 2 = -1(x - 1)$$
$$y = -x + 3$$
$$x + y - 3 = 0.$$



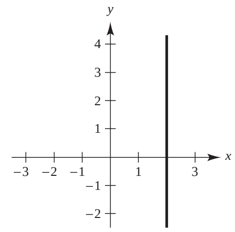
40. The slope of the line is $m = \dfrac{1 - 1}{10 - 6} = 0.$

The line is horizontal and its equation is

$$y = 1$$
$$y - 1 = 0.$$



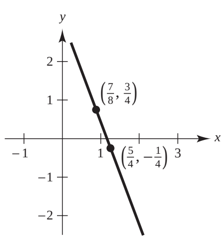
42. Slope is undefined. Line is vertical: $x = 2$



44. The slope of the line is $m = \dfrac{(-1/4) - (3/4)}{(5/4) - (7/8)} = -\dfrac{8}{3}.$

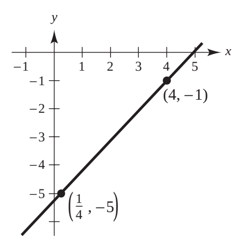
Using the point-slope form, we have

$$y - \frac{3}{4} = -\frac{8}{3}\left(x - \frac{7}{8}\right)$$
$$y - \frac{3}{4} = -\frac{8}{3}x + \frac{7}{3}$$
$$12y - 9 = -32x + 28$$
$$32x + 12y - 37 = 0.$$



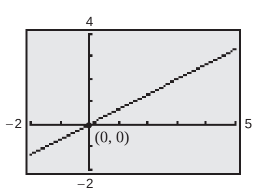
46. The slope is $m = \dfrac{-1 - (-5)}{4 - (1/4)} = \dfrac{4}{(15/4)} = \dfrac{16}{15}.$

$$y + 1 = \frac{16}{15}(x - 4)$$
$$15y + 15 = 16x - 64$$
$$15y - 16x + 79 = 0$$

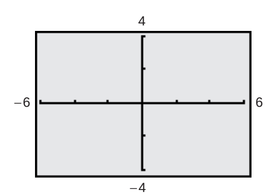


49. Using the slope-intercept form, we have

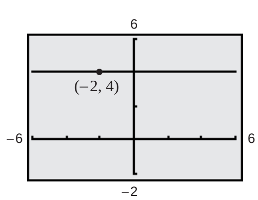
$$y = \tfrac{2}{3}x + 0$$
$$2x - 3y = 0.$$



50. Since the slope is undefined, the line is vertical and its equation is $x = 0.$

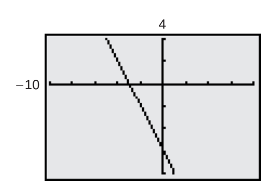


52. Since the slope is 0, the line is horizontal and its equation is $y = 4.$



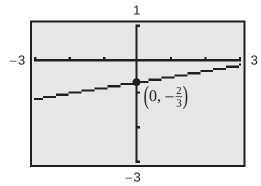
54. Using the point-slope form we have

$$y + 4 = -2(x + 1)$$
$$y = -2x - 6$$
$$2x + y + 6 = 0$$



56. Using the point-slope form, we have

$$y + \frac{2}{3} = \frac{1}{6}(x - 0)$$
$$6y + 4 = x$$
$$0 = x - 6y - 4.$$



58. The slope of the line joining $(-5, 11)$ and $(0, 4)$ is $\frac{11 - 4}{-5 - 0} = \frac{7}{-5} = -\frac{7}{5}$.

The slope of the line joining $(0, 4)$ and $(7, -6)$ is $\frac{4 - (-6)}{0 - 7} = -\frac{10}{7}$.

Since the slopes are different, the points are not collinear.

$$d_1 = \sqrt{(-5 - 0)^2 + (11 - 4)^2} = \sqrt{25 + 49} = \sqrt{74} \approx 8.60233$$
$$d_2 = \sqrt{(7 - 0)^2 + (-6 - 4)^2} = \sqrt{49 + 100} = \sqrt{149} \approx 12.20656$$
$$d_3 = \sqrt{(-5 - 7)^2 + [11 - (-6)]^2} = \sqrt{144 + 289} = \sqrt{433} \approx 20.80865$$

Since $d_1 + d_2 \neq d_3$, the points are not collinear.

60. Since the line is horizontal, it has a slope of $m = 0$, and its equation is

$$y = 0x + (-5)$$
$$y = -5.$$

62. The line is vertical: $x = -5$

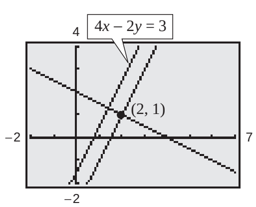
64. Given line: $y = 2x - \frac{3}{2}$

(a) Parallel: $m_1 = 2$

$$y - 1 = 2(x - 2)$$
$$0 = 2x - y - 3$$

(b) Perpendicular: $m_2 = -\frac{1}{2}$

$$y - 1 = -\frac{1}{2}(x - 2)$$
$$2y - 2 = -x + 2$$
$$x + 2y - 4 = 0$$



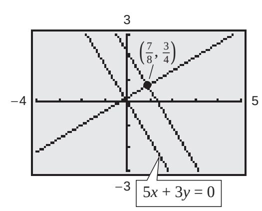
66. Given line: $y = -\frac{5}{3}x$

(a) Parallel: $m_1 = -\frac{5}{3}$

$$y - \frac{3}{4} = -\frac{5}{3}\left(x - \frac{7}{8}\right)$$
$$y - \frac{3}{4} = -\frac{5}{3}x + \frac{35}{24}$$
$$24y - 18 = -40x + 35$$
$$40x + 24y - 53 = 0$$

(b) Perpendicular: $m_2 = \frac{3}{5}$

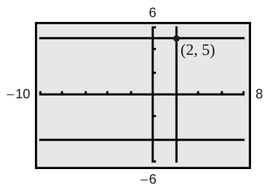
$$y - \frac{3}{4} = \frac{3}{5}\left(x - \frac{7}{8}\right)$$
$$y - \frac{3}{4} = \frac{3}{5}x - \frac{21}{40}$$
$$40y - 30 = 24x - 21$$
$$0 = 24x - 40y + 9$$



68. Given line: $y + 4 = 0$ is horizontal

(a) Parallel: $m = 0, y = 5$

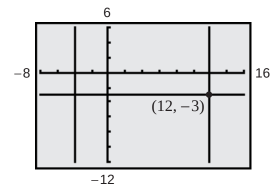
(b) Perpendicular m is undefined, $x = 2$



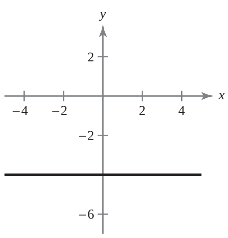
70. Given line: $x + 4 = 0$ is vertical

(a) Parallel: slope is undefined, $x = 12$

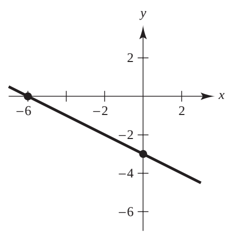
(b) Perpendicular: $m = 0, y = -3$



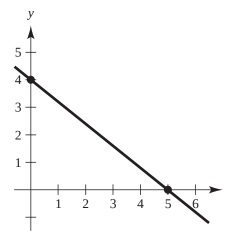
72. $y = -4$ is a horizontal line with y-intercept $(0, -4)$.



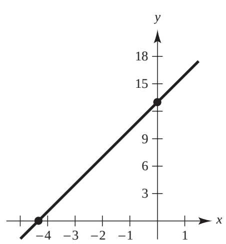
74. $x + 2y + 6 = 0$ has intercepts at $(-6, 0)$ and $(0, -3)$.



76. $4x + 5y - 20 = 0$ has intercepts at $(5, 0)$ and $(0, 4)$.



78. $y = 3x + 13$ has intercepts at $(0, 13)$ and $\left(-\frac{13}{3}, 0\right)$.



80. (a) Slope: $m = \dfrac{29{,}700 - 26{,}300}{2004 - 2002} = \dfrac{3400}{2} = 1700$

$$y - 26{,}300 = 1700(t - 2)$$

$$y = 1700t + 22{,}900$$

(b) For 2008, $t = 8$ and $y = 36{,}500$

82. Use $F = \frac{9}{5}C + 32$ and $C = \frac{5}{9}(F - 32)$.

(a) If $F = 102.5$, $C = \frac{5}{9}(102.5 - 32) \approx 39.2°$

(b) If $F = 74$, the $C = \frac{5}{9}(74 - 32) \approx 23.3°$

84. (a) $W = 0.80x + 9.25$ (union plan)

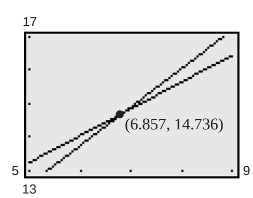
$W = 1.15x + 6.85$ (corporation plan)

(b) $0.8x + 9.25 = 1.15x + 6.85$

$$2.4 = .35x$$

$$x = \frac{240}{35} \approx 6.857$$

$$W \approx 14.736$$



(c) The point of intersection indicates the number of units (6.857) a worker needs to produce for the two plans to be equivalent.

86. Use the points (0, 825,000) and (25, 75,000).

$$y - 825{,}000 = \frac{825{,}000 - 75{,}000}{0 - 25}(t - 0)$$

$$y - 825{,}000 = -30{,}000t$$

$$y = -30{,}000t + 825{,}000, \quad 0 \le t \le 25$$

88. Let $t = 0$ represent 2002.

$$\text{Slope} = m = \frac{2702 - 2546}{2 - 0} = 78$$

$$y = 78t + 2546$$

For 2008, $t = 6$ and $y = 3014$ students.

90. (a) $C = (5.25 + 9.50)t + 26{,}500 = 14.75t + 26{,}500$

(b) $R = 25t$

(c) $P = R - C = 25t - (14.75t + 26{,}500) = 10.25t - 26{,}500$

(d) $R = C$

$$25t = 14.75t + 26{,}500$$

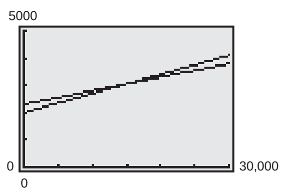
$$10.25t = 26{,}500$$

$$t \approx 2585.4 \text{ hours}$$

92. (a) $W = 2000 + .07S$

(b) $W = 2300 + .05S$

(c)



(d) No. You will make more money (if sales are \$20,000) at your current job ($w = \$3400$) than in the offered job ($w = \$3300$).

The lines intersect at (15,000, 3050). If you sell \$15,000, then both jobs would yield wages of \$3050.

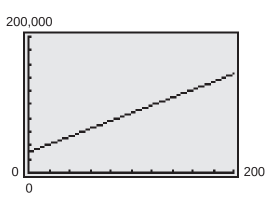
94. $C = 30{,}000 + 575x$ where $C \le 100{,}000$.

Thus, $30{,}000 + 575x \le 100{,}000$

$$575x \le 70{,}000$$

$$x \le 121.739 \approx 122 \text{ units.}$$

Therefore, $x \le 121$ units.

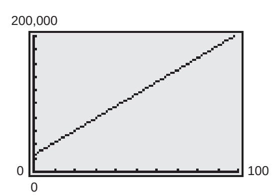


96. $C = 24{,}900 + 1785x \le 100{,}000$

$$1785x \le 75{,}100$$

$$x \le 42.07$$

$$x \le 42 \text{ units}$$

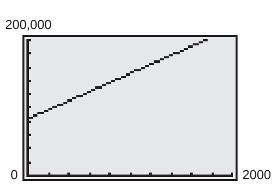


98. $C = 83{,}620 + 67x \le 100{,}000$

$$67x \le 16{,}380$$

$$x \le 244.48$$

$$x \le 244 \text{ units}$$

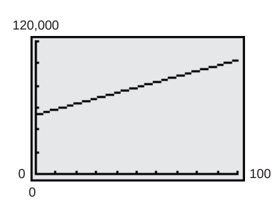


100. $C = 53{,}500 + 495x \le 100{,}000$

$$495x \le 46{,}500$$

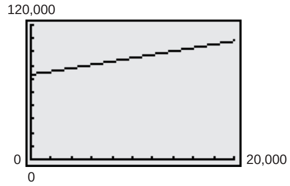
$$x \le 93.94$$

$$x \le 93 \text{ units}$$



102. $C = 75{,}500 + 1.50x \le 100{,}000$

$$1.50x \le 24{,}500$$
$$x \le 16{,}333.3$$
$$x \le 16{,}333 \text{ units}$$



Section 1.4 Functions

2. $y = \pm\sqrt{4 - x}$

y is *not* a function of x since there are two values of y for some x.

4. $y = \dfrac{3x + 5}{2}$

y *is* a function of x since there is only one value of y for each x.

6. $(x^2 - 2x + 1) + (y^2 - 4y + 4) = -1 + 1 + 4$

$$(x - 1)^2 + (y - 2)^2 = 4$$

The graph is a circle; therefore, by the vertical line test, y is *not* a function of x.

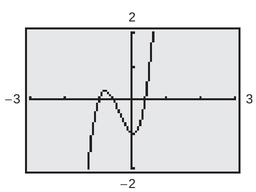
8. $y(x^2 + 4) = x^2$

$$y = \frac{x^2}{x^2 + 4}$$

y *is* a function of x since there is only one value of y for each x. [**Note:** It is not a one-to-one function.]

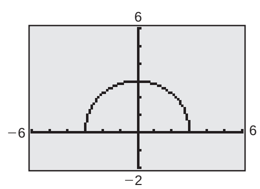
10. Domain: $(-\infty, \infty)$

Range: $(-\infty, \infty)$



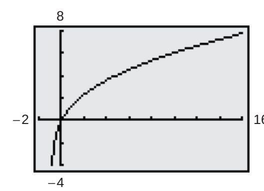
12. Domain: $[-3, 3]$

Range: $[0, 3]$



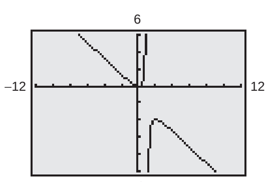
14. Domain: $(-1, \infty)$

Range: $(-\infty, \infty)$



16. Domain: $(-\infty, 1) \cup (1, \infty)$

Range: $(-\infty, -4] \cup [0, \infty)$



18. Domain: $\left[\frac{3}{2}, \infty\right)$

Range: $[0, \infty)$

20. Domain: $(-\infty, \infty)$

Range: $[0, \infty)$

22. $f(x) = x^2 - 2x + 2$

(a) $f\left(\frac{1}{2}\right) = \left(\frac{1}{2}\right)^2 - 2\left(\frac{1}{2}\right) + 2 = \frac{5}{4}$

(b) $f(-1) = (-1)^2 - 2(-1) + 2 = 5$

(c) $f(c + 2) = (c + 2)^2 - 2(c + 2) + 2$

$$= c^2 + 4c + 4 - 2c - 4 + 2$$
$$= c^2 + 2c + 2$$

(d) $f(x + \Delta x) = (x + \Delta x)^2 - 2(x + \Delta x) + 2$

$$= x^2 + 2x\Delta x + (\Delta x)^2 - 2x - 2\Delta x + 2$$

24. $f(x) = |x| + 4$

(a) $f(2) = |2| + 4 = 6$

(b) $f(-2) = |-2| + 4 = 6$

(c) $f(x + 2) = |x + 2| + 4$

(d) $f(x + \Delta x) - f(x) = |x + \Delta x| + 4 - (|x| + 4)$

$$= |x + \Delta x| - |x|$$

26. $$\frac{h(2+\Delta x)-h(2)}{\Delta x} = \frac{(2+\Delta x)^2-(2+\Delta x)+1-(4-2+1)}{\Delta x}$$
$$= \frac{4+4\Delta x+(\Delta x)^2-2-\Delta x+1-3}{\Delta x}$$
$$= \frac{\Delta x(3+\Delta x)}{\Delta x}$$
$$= 3+\Delta x,\ \Delta x \neq 0$$

28. $$\frac{f(x)-f(2)}{x-2} = \frac{\left(1/\sqrt{x-1}\right)-1}{x-2}$$
$$= \frac{1-\sqrt{x-1}}{(x-2)\sqrt{x-1}} \cdot \frac{1+\sqrt{x-1}}{1+\sqrt{x-1}}$$
$$= \frac{1-(x-1)}{(x-2)\sqrt{x-1}\left(1+\sqrt{x-1}\right)}$$
$$= \frac{2-x}{(x-2)\sqrt{x-1}\left(1+\sqrt{x-1}\right)}$$
$$= \frac{-1}{\sqrt{x-1}\left(1+\sqrt{x-1}\right)},\ x \neq 2$$

30. $$\frac{f(x+\Delta x)-f(x)}{\Delta x} = \frac{\dfrac{1}{x+\Delta x+4}-\dfrac{1}{x+4}}{\Delta x}$$
$$= \frac{(x+4)-(x+\Delta x+4)}{\Delta x\,[x+\Delta x+4][x+4]}$$
$$= \frac{-1}{(x+\Delta x+4)(x+4)},\quad \Delta x \neq 0$$

32. y *is* a function of x.

34. y is *not* a function of x.

36. (a) $f(x)+g(x) = (2x-5)+(2-x) = x-3$

(b) $f(x)\,g(x) = (2x-5)(2-x) = -2x^2+9x-10$

(c) $f(x)/g(x) = \dfrac{2x-5}{2-x}$

(d) $f(g(x)) = f(2-x) = 2(2-x)-5 = -2x-1$

(e) $g(f(x)) = g(2x-5) = 2-(2x-5) = -2x+7$

38. (a) $f(x)+g(x) = x^2+5+\sqrt{1-x},\quad x \le 1$

(b) $f(x)\cdot g(x) = (x^2+5)\sqrt{1-x},\quad x \le 1$

(c) $\dfrac{f(x)}{g(x)} = \dfrac{x^2+5}{\sqrt{1-x}},\quad x < 1$

(d) $$f(g(x)) = f\left(\sqrt{1-x}\right)$$
$$= \left(\sqrt{1-x}\right)^2+5$$
$$= 6-x,\quad x \le 1$$

(e) $g(f(x))$ is not defined since the domain of g is $(-\infty, 1]$ and the range of f is $[5, \infty)$. The range of f is not in the domain of g.

40. (a) $f(x)+g(x) = \dfrac{x}{x+1}+x^3 = \dfrac{x^4+x^3+x}{x+1}$

(b) $f(x)\cdot g(x) = \left(\dfrac{x}{x+1}\right)(x^3) = \dfrac{x^4}{x+1}$

(c) $\dfrac{f(x)}{g(x)} = \dfrac{\left(\dfrac{x}{x+1}\right)}{x^3} = \dfrac{1}{x^2(x+1)}$

(d) $f(g(x)) = f(x^3) = \dfrac{x^3}{x^3+1}$

(e) $g(f(x)) = g\left(\dfrac{x}{x+1}\right) = \left(\dfrac{x}{x+1}\right)^3 = \dfrac{x^3}{(x+1)^3}$

42. $f(x) = \dfrac{1}{x},\ g(x) = x^2-1$

(a) $f(g(2)) = f(2^2-1) = f(3) = \dfrac{1}{3}$

(b) $g(f(2)) = g\left(\dfrac{1}{2}\right) = \left(\dfrac{1}{2}\right)^2-1 = -\dfrac{3}{4}$

(c) $f\left(g\left(\dfrac{1}{\sqrt{2}}\right)\right) = f\left(\dfrac{1}{2}-1\right) = f\left(-\dfrac{1}{2}\right) = -2$

(d) $g\left(f\left(\dfrac{1}{\sqrt{2}}\right)\right) = g\left(\sqrt{2}\right) = 2-1 = 1$

(e) $f(g(x)) = f(x^2-1) = \dfrac{1}{(x^2-1)}$

(f) $g(f(x)) = g\left(\dfrac{1}{x}\right) = \left(\dfrac{1}{x}\right)^2-1$

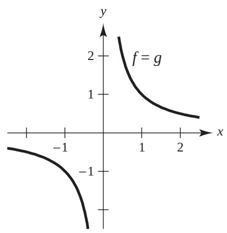
44. The data fits the function (a) $f(x) = \frac{1}{4}x$, with $c = \frac{1}{4}$.

46. The data fits the function (c) $h(x) = 3\sqrt{|x|}$, with $c = 3$.

48. $f(g(x)) = f\left(\dfrac{1}{x}\right) = \dfrac{1}{1/x} = x,\ x \neq 0$

$g(f(x)) = g\left(\dfrac{1}{x}\right) = \dfrac{1}{1/x} = x,\ x \neq 0$

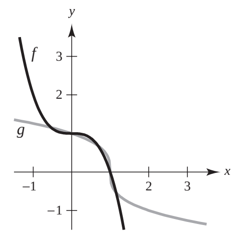
See accompanying graph.



50. $f(g(x)) = f\left(\sqrt[3]{1-x}\right) = 1 - \left(\sqrt[3]{1-x}\right)^3 = x$

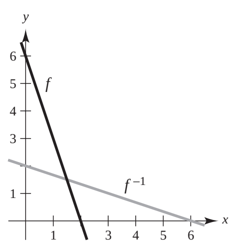
$g(f(x)) = g(1 - x^3) = \sqrt[3]{1 - (1 - x^3)} = x$

See accompanying graph.



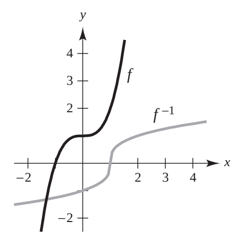
52.

$$f(x) = 6 - 3x = y$$
$$x = 6 - 3y$$
$$y = \frac{6 - x}{3}$$
$$f^{-1}(x) = \frac{6 - x}{3}$$



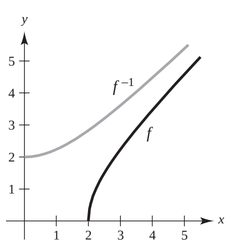
54.

$$f(x) = x^3 + 1 = y$$
$$x = y^3 + 1$$
$$\sqrt[3]{x - 1} = y$$
$$f^{-1}(x) = \sqrt[3]{x - 1}$$



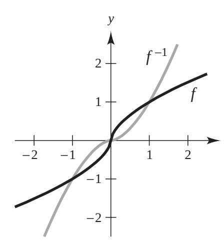
56.

$$f(x) = \sqrt{x^2 - 4} = y,\ x \geq 2$$
$$x = \sqrt{y^2 - 4}$$
$$x^2 + 4 = y^2$$
$$y = \sqrt{x^2 + 4},\ x \geq 0$$
$$f^{-1}(x) = \sqrt{x^2 + 4},\ x \geq 0$$



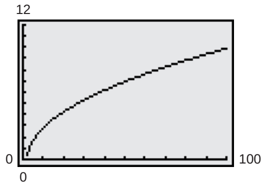
58.

$$f(x) = x^{3/5} = y$$
$$x = y^{3/5}$$
$$x^{5/3} = y$$
$$f^{-1}(x) = x^{5/3}$$



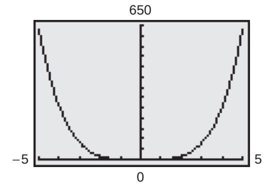
60. $f(x) = \sqrt{x - 2}$ *is* one-to-one for $x \geq 2$.

$f^{-1}(x) = x^2 + 2$ where $x \geq 0$.



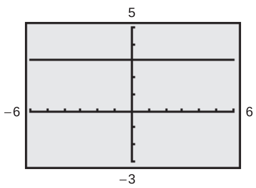
$f(x)$ is one-to-one. $f^{-1}(x) = x^2 + 2, \quad x \geq 0$

62. $f(x) = x^4$ is *not* one-to-one since $f(2) = 16 = f(-2)$.



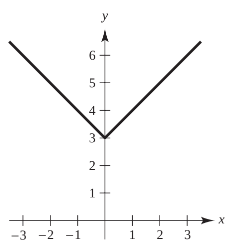
$f(x)$ is not one-to-one.

64. $f(x) = 3$ is *not* one-to-one since $f(0) = 3 = f(1)$.

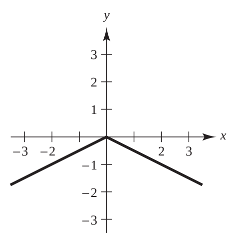


$f(x)$ is not one-to-one.

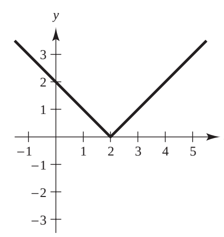
66. (a)



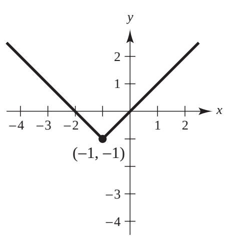
(b)



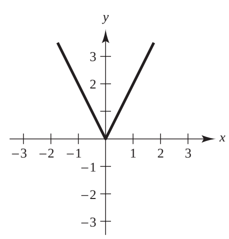
(c)



(d)



(e)



68. Value $= V = 2500x + 750{,}000$

70. (a) $C = 0.95x + 6000$

(b) $\overline{C} = \dfrac{C}{x} = \dfrac{0.95x + 6000}{x} = 0.95 + \dfrac{6000}{x}$

(c) $0.95 + \dfrac{6000}{x} < 1.69$

$$\frac{6000}{x} < 0.74$$

$$\frac{6000}{0.74} < x \text{ since } x > 0.$$

$$8108.108 < x$$

Must sell 8109 units before the average cost per unit falls below the selling price.

72. Cost = Cost on land + Cost underwater

$$= 10(5280)(3 - x) + 15(5280)\sqrt{x^2 + \tfrac{1}{4}}$$

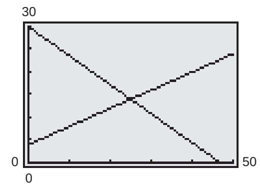
$$= 5(5280)\left[2(3 - x) + 3\sqrt{x^2 + \tfrac{1}{4}}\right]$$

74. (a) Graphing utility graph

(b) $(25, 14)$ is equilibrium point.

(c) Demand exceeds supply for $x < 25$.

(d) Supply exceeds demand for $x > 25$.

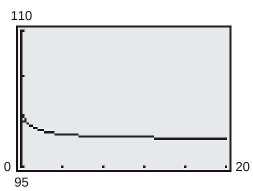


76. (a) Cost $= C = 98{,}000 + 12.30x$

(b) Revenue $= R = 17.98x$

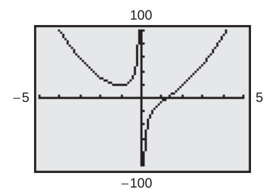
(c) Profit $= R - C = 17.98x - (12.30x + 98{,}000) = 5.68x - 98{,}000$

78. $F(t) = 98 + \dfrac{3}{t + 1}$



The function is valid for all $t \geq 0$ because the patient's temperature can only be affected by the drug from the time that it is administered.

80.

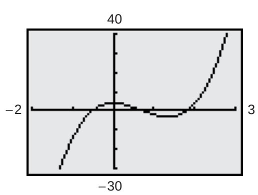


$f(x) = 2\left(3x^2 - \dfrac{6}{x}\right)$

Zero: $x = 1.2599$

The function is not one-to-one.

82.

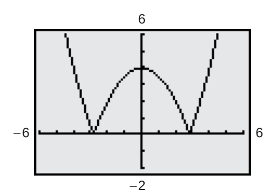


$h(x) = 6x^3 - 12x^2 + 4$

Zeros: $x \approx -0.5419, 0.7224, 1.7925$

The function is *not* one-to-one.

84.

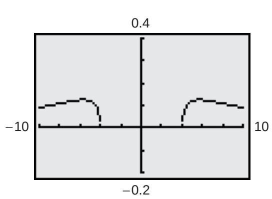


$f(x) = \left|\dfrac{1}{2}x^2 - 4\right|$

Zeros: $x = \pm 2\sqrt{2}$

The function is not one-to-one.

86.



$f(x) = \dfrac{\sqrt{x^2 - 16}}{x^2}$

Domain: $|x| \geq 4$

Zeros: $x = \pm 4$

Section 1.5 Limits

2.

x	1.9	1.99	1.999	2	2.001	2.01	2.1
$f(x)$	−1.09	−1.0099	−1.000999	−1	−0.998999	−0.9899	−0.89

$\lim_{x\to 2} (x^2 - 3x + 1) = -1$

4.

x	1.9	1.99	1.999	2	2.001	2.01	2.1
$f(x)$	72.39	79.20	79.92	undefined	80.08	80.80	88.41

$\lim_{x\to 2} \dfrac{x^5 - 32}{x - 2} = 80$

6.

x	−0.1	−0.01	−0.001	0	0.001	0.01	0.1
$f(x)$	0.3581	0.3540	0.3536	undefined	0.3535	0.3531	0.3492

$\lim_{x\to 0} \dfrac{\sqrt{x+2} - \sqrt{2}}{x} = \dfrac{1}{2\sqrt{2}} \approx 0.354$

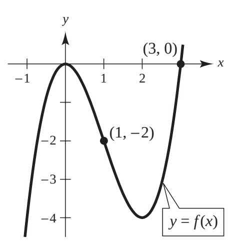
8.

x	0.5	0.1	0.01	0.001	0
$f(x)$	−0.1	−0.1190	−0.1244	−0.1249	undefined

$\lim_{x\to 0^+} \dfrac{[1/(2+x)] - (1/2)}{2x} = -\dfrac{1}{8} = -0.125$

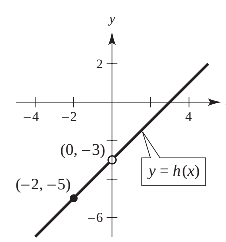
10. (a) $\lim_{x\to 1} f(x) = -2$

(b) $\lim_{x\to 3} f(x) = 0$



12. (a) $\lim_{x\to -2} h(x) = -5$

(b) $\lim_{x\to 0} h(x) = -3$



14. (a) $\lim_{x\to c} [f(x) + g(x)] = \lim_{x\to c} f(x) + \lim_{x\to c} g(x) = \dfrac{3}{2} + \dfrac{1}{2} = 2$

(b) $\lim_{x\to c} [f(x) \cdot g(x)] = \left[\lim_{x\to c} f(x)\right]\left[\lim_{x\to c} g(x)\right] = \left(\dfrac{3}{2}\right)\left(\dfrac{1}{2}\right) = \dfrac{3}{4}$

(c) $\lim_{x\to c} \dfrac{f(x)}{g(x)} = \dfrac{\lim_{x\to c} f(x)}{\lim_{x\to c} g(x)} = \dfrac{3/2}{1/2} = 3$

16. (a) $\lim_{x\to c} \sqrt{f(x)} = \sqrt{9} = 3$

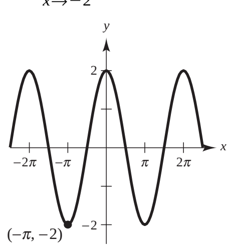
(b) $\lim_{x\to c} (3f(x)) = 3(9) = 27$

(c) $\lim_{x\to c} [f(x)]^2 = 9^2 = 81$

18. (a) $\lim_{x\to -2^+} f(x) = -2$

(b) $\lim_{x\to -2^-} f(x) = -2$

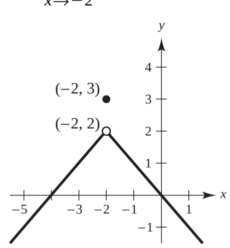
(c) $\lim_{x\to -2} f(x) = -2$



20. (a) $\lim_{x\to -2^+} f(x) = 2$

(b) $\lim_{x\to -2^-} f(x) = 2$

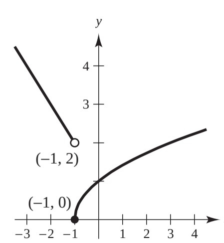
(c) $\lim_{x\to -2} f(x) = 2$



22. (a) $\lim_{x\to -1^+} f(x) = 0$

(b) $\lim_{x\to -1^-} f(x) = 2$

(c) $\lim_{x\to -1} f(x)$ does not exist.



24. $\lim_{x\to -2} x^3 = (-2)^3 = -8$

26. $\lim_{x\to 0} (2x - 3) = 2(0) - 3 = -3$

28. $\lim_{x\to 2} (-x^2 + x - 2) = -4 + 2 - 2 = -4$

30. $\lim_{x\to 4} \sqrt[3]{x + 4} = \sqrt[3]{4 + 4} = 2$

32. $\lim_{x\to -2} \dfrac{3x + 1}{2 - x} = \dfrac{3(-2) + 1}{2 - (-2)} = \dfrac{-5}{4}$

34. $\lim_{x\to -1} \dfrac{4x - 5}{3 - x} = \dfrac{4(-1) - 5}{3 - (-1)} = \dfrac{-9}{4} = -\dfrac{9}{4}$

36. $\lim_{x\to 3} \dfrac{\sqrt{x + 1}}{x - 4} = \dfrac{\sqrt{3 + 1}}{3 - 4} = \dfrac{2}{-1} = -2$

38. $\lim_{x\to 5} \dfrac{\sqrt{x + 4} - 2}{x} = \dfrac{3 - 2}{5} = \dfrac{1}{5}$

40. $\lim_{x\to 2} \dfrac{\dfrac{1}{x + 2} - \dfrac{1}{2}}{2} = \dfrac{\dfrac{1}{4} - \dfrac{1}{2}}{2} = -\dfrac{1}{8}$

42. $\lim_{x\to -1} \dfrac{2x^2 - x - 3}{x + 1} = \lim_{x\to -1} \dfrac{(x + 1)(2x - 3)}{x + 1}$

$= \lim_{x\to -1} (2x - 3) = -5$

44. $\lim_{x\to 2} \dfrac{2 - x}{x^2 - 4} = \lim_{x\to 2} \dfrac{-(x - 2)}{(x + 2)(x - 2)}$

$= \lim_{x\to 2} \dfrac{-1}{x + 2} = -\dfrac{1}{4}$

46. $\lim_{t\to 1} \dfrac{t^2 + t - 2}{t^2 - 1} = \lim_{t\to 1} \dfrac{(t - 1)(t + 2)}{(t + 1)(t - 1)}$

$= \lim_{t\to 1} \dfrac{t + 2}{t + 1} = \dfrac{3}{2}$

48. $\lim_{x\to 1} \dfrac{x^3 - 1}{x - 1} = \lim_{x\to 1} \dfrac{(x - 1)(x^2 + x + 1)}{x - 1}$

$= \lim_{x\to 1} (x^2 + x + 1) = 3$

50. $\lim_{x\to 2^-} \dfrac{|x - 2|}{x - 2} = -1$

$\lim_{x\to 2^+} \dfrac{|x - 2|}{x - 2} = 1$

Therefore, $\lim_{x\to 2} \dfrac{|x - 2|}{x - 2}$ does not exist.

52. $\lim_{s\to 1^-} f(s) = \lim_{s\to 1^-} s = 1$

$\lim_{s\to 1^+} f(s) = \lim_{s\to 1^+} (1 - s) = 0$

Therefore, $\lim_{s\to 1} f(s)$ does not exist.

54. $\lim_{\Delta x\to 0} \dfrac{4(x + \Delta x) - 5 - (4x - 5)}{\Delta x} = \lim_{\Delta x\to 0} \dfrac{4\Delta x}{\Delta x} = 4$

56. $\lim_{\Delta x\to 0} \dfrac{\sqrt{x + \Delta x} - \sqrt{x}}{\Delta x} = \lim_{\Delta x\to 0} \dfrac{\sqrt{x + \Delta x} - \sqrt{x}}{\Delta x} \cdot \dfrac{\sqrt{x + \Delta x} + \sqrt{x}}{\sqrt{x + \Delta x} + \sqrt{x}}$

$= \lim_{\Delta x\to 0} \dfrac{(x + \Delta x) - x}{\Delta x\left[\sqrt{x + \Delta x} + \sqrt{x}\right]}$

$= \lim_{\Delta x\to 0} \dfrac{1}{\sqrt{x + \Delta x} + \sqrt{x}}$

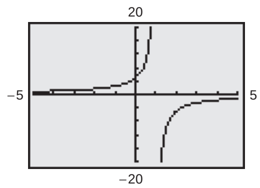
$= \dfrac{1}{2\sqrt{x}}$

58. $\lim_{\Delta t\to 0} \dfrac{(t + \Delta t)^2 - 4(t + \Delta t) + 2 - (t^2 - 4t + 2)}{\Delta t}$

$= \lim_{\Delta t\to 0} \dfrac{t^2 + 2t\,\Delta t + (\Delta t)^2 - 4t - 4\Delta t - t^2 + 4t}{\Delta t}$

$= \lim_{\Delta t\to 0} \dfrac{2t\,\Delta t + (\Delta t)^2 - 4\Delta t}{\Delta t}$

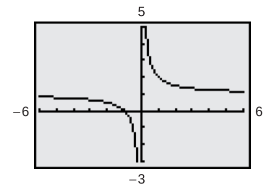
$= \lim_{\Delta t\to 0} (2t + \Delta t - 4) = 2t - 4$

60. 

$\lim_{x\to 1^+} \frac{5}{1-x} = -\infty$

x	2	1.5	1.1	1.01
$f(x)$	−5	−10	−50	−500

x	1.001	1.0001	1
$f(x)$	−5000	−50,000	Undefined

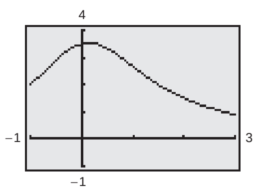
62. 

$\lim_{x\to 0^-} \frac{x+1}{x} = -\infty$

x	−1	−0.5	−0.1	−0.01
$f(x)$	0	−1	−9	−99

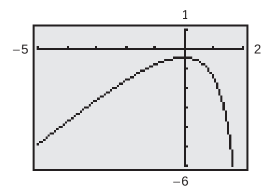
x	−0.001	−0.0001	0
$f(x)$	−999	−9999	Undefined

64. $\lim_{x\to 1} \frac{x^2+6x-7}{x^3-x^2+2x-2} = \lim_{x\to 1} \frac{(x-1)(x+7)}{(x-1)(x^2+2)}$



$\frac{8}{3} \approx 2.667$

66. $\lim_{x\to -2} \frac{4x^3+7x^2+x+6}{3x^2-x-14} = \lim_{x\to -2} \frac{(x+2)(4x^2-x+3)}{(x+2)(3x-7)}$



$\frac{21}{-13} \approx -1.615$

68. Because $4 - x^2 \le f(x) \le 4 + x^2$,

$$\lim_{x\to 0}(4-x^2) \le \lim_{x\to 0} f(x) \le \lim_{x\to 0}(4+x^2)$$
$$4 \le \lim_{x\to 0} f(x) \le 4.$$

Hence, $\lim_{x\to 0} f(x) = 4$.

70. $\lim_{r\to 0.06} A = \lim_{r\to 0.06} 1000\left(1 + \frac{r}{4}\right)^{40} = 1814.02$ Yes, the limit exists.

Section 1.6 Continuity

2. The polynomial $f(x) = (x^2 - 1)^3$ is continuous on the entire real line.

4. $f(x) = \frac{1}{9-x^2} = \frac{1}{(3-x)(3+x)}$ is continuous on $(-\infty, -3)$, $(-3, 3)$ and $(3, \infty)$.

6. $f(x) = \frac{3x}{x^2+1}$ is continuous on $(-\infty, \infty)$.

8. f is not continuous on the entire real line. f is not defined at $x = 1, 5$.

10. g is not continuous, on the entire real line. g is not defined at $x = \pm 4$.

12. $f(x) = \dfrac{1}{x^2 - 4}$ is continuous on $(-\infty, -2)$, $(-2, 2)$ and $(2, \infty)$.

14. $f(x)$ is continuous on $(-\infty, 2)$ and $(2, \infty)$.

16. $f(x)$ is continuous on $(-\infty, \infty)$.

18. $f(x) = \dfrac{x - 3}{x^2 - 9}$ is continuous on $(-\infty, -3)$, $(-3, 3)$ and $(3, \infty)$.

20. $f(x) = \dfrac{1}{x^2 + 1}$ is continuous on $(-\infty, \infty)$.

22. $f(x) = \dfrac{x - 1}{x^2 + x - 2} = \dfrac{x - 1}{(x - 1)(x + 2)}$ is continuous on $(-\infty, -2)$, $(-2, 1)$ and $(1, \infty)$.

24. $f(x) = \dfrac{[\![x]\!]}{2} + x$ is continuous on all intervals of the form $(c, c + 1)$ where c is an integer.

26. $\lim\limits_{x \to 2^-} f(x) = \lim\limits_{x \to 2^-} (3 + x) = 5$

$\lim\limits_{x \to 2^+} f(x) = \lim\limits_{x \to 2^+} (x^2 + 1) = 5$

Since $f(2) = 5$, f is continuous on the entire real line.

28. $\lim\limits_{x \to 0^-} f(x) = -4$

$\lim\limits_{x \to 0^+} f(x) = 1$

f is not continuous at $x = 0$. f is continuous on $(-\infty, 0)$ and $(0, \infty)$.

30. $\lim\limits_{x \to 4^-} \dfrac{|4 - x|}{4 - x} = 1$

$\lim\limits_{x \to 4^+} \dfrac{|4 - x|}{4 - x} = -1$

$\lim\limits_{x \to 4} \dfrac{|4 - x|}{4 - x}$ does not exist.

f is continuous on $(-\infty, 4)$ and $(4, \infty)$.

32. $\lim\limits_{x \to c^-} (x - [\![x]\!]) = c - (c - 1) = 1$, c is any integer

$\lim\limits_{x \to c^+} (x - [\![x]\!]) = c - c = 0$, c is any integer

f is continuous on all intervals $(c, c + 1)$.

34. $h(x) = f(g(x)) = f(x^2 + 5)$

$$= \frac{1}{(x^2 + 5) - 1} = \frac{1}{x^2 + 4}$$

Thus, h is continuous on the entire real line.

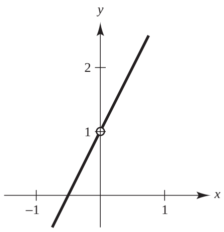
36. $f(x) = \dfrac{5}{x^2 + 1}$ is continuous on $[-2, 2]$.

[**Note:** f is continuous on the entire real line.]

38. $f(x) = \dfrac{x}{(x - 1)(x - 3)}$ has nonremovable discontinuities at $x = 1$ and $x = 3$.

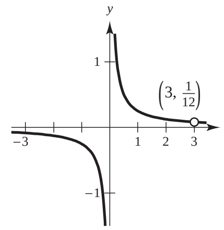
40. $f(x) = \dfrac{2x^2 + x}{x} = \dfrac{x(2x+1)}{x}$

f has a removable discontinuity at $x = 0$; continuous on $(-\infty, 0)$ and $(0, \infty)$.

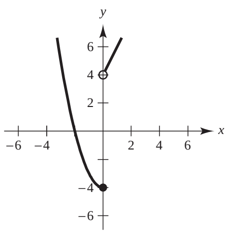


42. $f(x) = \dfrac{x-3}{4x^2 - 12x} = \dfrac{x-3}{4x(x-3)} = \dfrac{1}{4x}, \quad x \neq 3$

f has a removable discontinuity at $x = 3$, and a nonremovable discontinuity at $x = 0$. f is continuous on $(-\infty, 0)$, $(0, 3)$, $(3, \infty)$.



44. f is continuous on $(-\infty, 0)$ and $(0, \infty)$.



46. $\lim\limits_{x\to -1^-} f(x) = 2$

$\lim\limits_{x\to -1^+} f(x) = -a + b$

$\lim\limits_{x\to 3^-} f(x) = 3a + b$

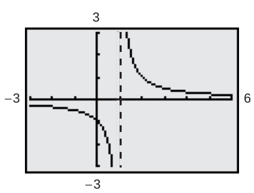
$\lim\limits_{x\to 3^+} f(x) = -2$

Thus:

$$\begin{aligned} -a + b &= 2 \\ 3a + b &= -2 \\ \hline -4a &= 4 \\ a &= -1 \\ b &= 1 \end{aligned}$$

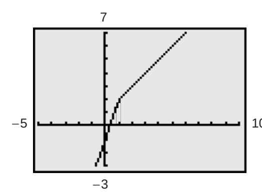
48. $f(x) = \dfrac{x-4}{x^2 - 5x + 4} = \dfrac{x-4}{(x-4)(x-1)} = \dfrac{1}{x-1}, \quad x \neq 4$

is not continuous at $x = 1$ and $x = 4$.

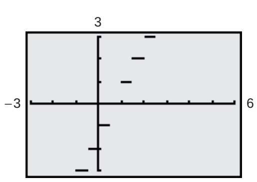


There is a hole at $\left(4, \frac{1}{3}\right)$.

50. f is continuous on $(-\infty, \infty)$.



52.



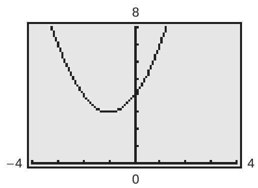
f is not continuous at all $\frac{1}{2}c$, where c is an integer.

54. $f(x) = x\sqrt{x+3}$ is continuous on $[-3, \infty)$.

56. $f(x) = \dfrac{x+1}{\sqrt{x}}$ is continuous on $(0, \infty)$.

58. $f(x) = \dfrac{x^3 - 8}{x - 2} = \dfrac{(x - 2)(x^2 + 2x + 4)}{(x - 2)}$

appears to be continuous on $[-4, 4]$. But, it is not continuous at $x = 2$ (removable discontinuity).



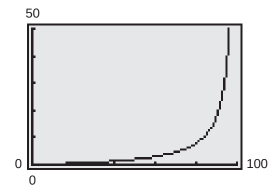
60. $C = \dfrac{2x}{100 - x}$

(a) $[0, 100)$; Negative x values do not make sense in this context nor do values greater than 100. Also, $C(100)$ is undefined.

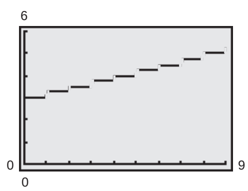
(b) C is continuous on its domain because all rational functions are continuous on their domains.

(c) For $x = 75$,

$$C = \frac{2(75)}{100 - 75} = \frac{150}{25} = 6 \text{ million dollars.}$$



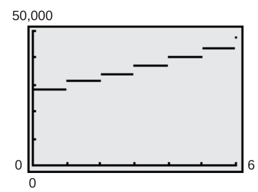
62. $C = \begin{cases} 3, & n = 0 \\ 3 + 0.25[\![n]\!], & n > 0, n \text{ is not an integer.} \\ 3 + 0.25(n - 1), & n > 0, n \text{ is an integer.} \end{cases}$



C is continuous at all intervals $(n, n + 1)$, n is a nonnegative integer.

[**Note:** $C = 3 - 0.25[\![1 - n]\!], \quad n > 0$]

64. (a) Nonremovable discontinuities at $t = 1, 2, 3, 4, 5$



(b) For $t = 5$, $S = \$43{,}850.78$.

66. Yes, a linear model is a continuous function. No, actual revenue would probably not be continuous because the actual revenue would probably not follow the model exactly, which may introduce some discontinuities.

Review Exercises for Chapter 1

2. Matches (c)

4. Matches (d)

6. Distance $= \sqrt{(1 - 4)^2 + (2 - 3)^2} = \sqrt{9 + 1} = \sqrt{10}$.

8. Distance $= \sqrt{[6 - (-3)]^2 + (8 - 7)^2} = \sqrt{81 + 1} = \sqrt{82}$

10. Midpoint $= \left(\dfrac{0 - 4}{2}, \dfrac{0 + 8}{2}\right) = (-2, 4)$

12. Midpoint $= \left(\dfrac{7 - 3}{2}, \dfrac{-9 + 5}{2}\right) = (2, -2)$

14. 1999: $R \approx \$120$ thousand
$C \approx \$70$ thousand
$P \approx \$50$ thousand

2000: $R \approx \$170$ thousand
$C \approx \$92$ thousand
$P \approx \$78$ thousand

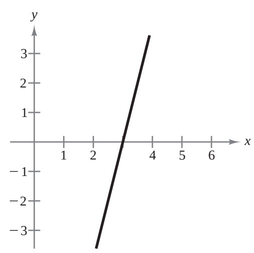
2001: $R \approx \$70$ thousand
$C \approx \$33$ thousand
$P \approx \$37$ thousand

2002: $R \approx \$200$ thousand
$C \approx \$110$ thousand
$P \approx \$90$ thousand

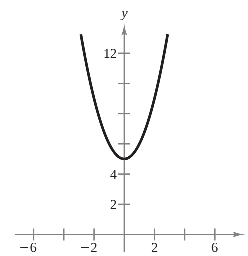
2003: $R \approx \$260$ thousand
$C \approx \$135$ thousand
$P \approx \$125$ thousand

16. $(-2, 1) \to (2, 0)$
$(-1, 2) \to (3, 1)$
$(1, 0) \to (5, -1)$
$(0, -1) \to (4, -2)$

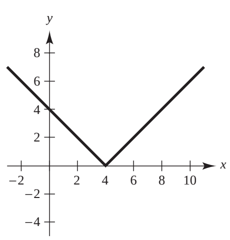
18.



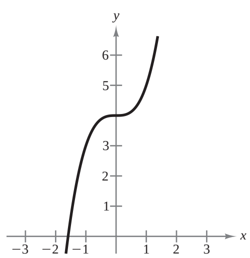
20.



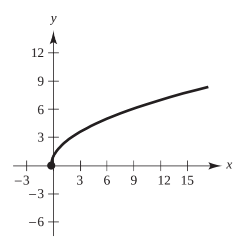
22. $y = |4 - x|$



24.



26. $y = \sqrt{4x + 1}$



28. y-intercept: $x = 0 \Longrightarrow y = -3 \Longrightarrow (0, -3)$

x-intercept: $y = 0 \Longrightarrow x = -\frac{3}{4} \Longrightarrow \left(-\frac{3}{4}, 0\right)$

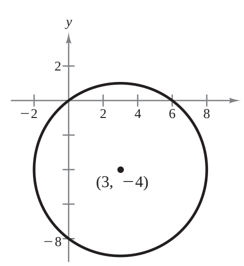
30.
$$(x - 0)^2 + (y - 0)^2 = r^2$$
$$x^2 + y^2 = r^2$$
$$2^2 + \left(\sqrt{5}\right)^2 = r^2$$
$$9 = r^2$$
$$3 = r$$

Equation: $x^2 + y^2 = 3^2$

32.
$$x^2 + y^2 - 6x + 8y = 0$$
$$(x^2 - 6x + 9) + (y^2 + 8y + 16) = 9 + 16$$
$$(x - 3)^2 + (y + 4)^2 = 25$$

Center: $(3, -4)$

Radius: 5



34. $x + y = 2 \Longrightarrow y = 2 - x$

$2x - y = 1 \Longrightarrow y = 2x - 1$

$$2 - x = 2x - 1$$
$$3 = 3x$$
$$1 = x$$

Point of intersection: $(1, 1)$

36. $y = x^3$

$y = x$

$$x^3 = x$$
$$x^3 - x = 0$$
$$x(x - 1)(x + 1) = 0$$

Points of intersection: $(0, 0), (1, 1), (-1, -1)$

38. (a) $C = 200 + 2x + 8x = 200 + 10x$

$R = 14x$

(b) $C = R$

$$200 + 10x = 14x$$
$$200 = 4x$$
$$x = 50 \text{ shirts}$$

$(x, R) = (x, C) = (50, 700)$.

40. $p = 91.4 - 0.009x = 6.4 + 0.008x$

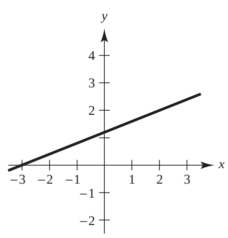
$$85 = 0.017x$$
$$x = 5000 \text{ units}$$
$$p = \$46.40$$

42. $-\frac{1}{3}x + \frac{5}{6}y = 1$

$$\frac{5}{6}y = \frac{1}{3}x + 1$$
$$y = \frac{2}{5}x + \frac{6}{5}$$

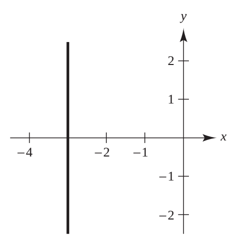
Slope: $m = \frac{2}{5}$

y-intercept: $\left(0, \frac{6}{5}\right)$



44. $x = -3$

Slope: undefined (vertical line)
No y-intercept

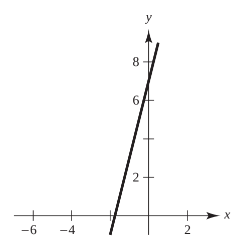


46. $3.2x - 0.8y + 5.6 = 0$

$$8y = 32x + 56$$
$$y = 4x + 7$$

Slope: 4

y-intercept: $(0, 7)$

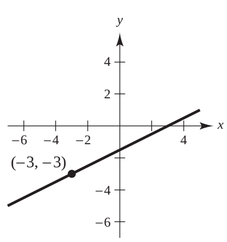


48. Slope $= \dfrac{7 - 5}{-5 - (-1)} = \dfrac{2}{-4} = -\dfrac{1}{2}$

50. Slope $= \dfrac{-3 - (-3)}{-1 - (-11)} = \dfrac{0}{10} = 0$ (horizontal line)

52. $y - (-3) = \frac{1}{2}[x - (-3)]$

$$y = \frac{1}{2}x - \frac{3}{2}$$



54. (a) Slope $= 0 \rightarrow y = -3$

(b) Slope undefined $\rightarrow x = 1$

(c) $-4x + 5y = -3$

$$y = \frac{4}{5}x - \frac{3}{5}$$
$$y - (-3) = \frac{4}{5}(x - 1)$$
$$y = \frac{4}{5}x - \frac{19}{5}$$

(d) $5x - 2y = 3$

$$y = \frac{5}{2}x - \frac{3}{2}$$

Slope of perpendicular is $-\frac{2}{5}$.

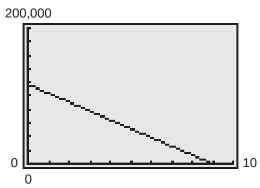
$$y - (-3) = -\frac{2}{5}(x - 1)$$
$$y = -\frac{2}{5}x - \frac{13}{5}$$

56. $(0, 117{,}000), (9, 0)$

$$m = \frac{117{,}000}{-9} = -13{,}000$$

(a) $v = -13{,}000(t - 9) = -13{,}000t + 117{,}000$

(b) Graphing utility



(c) $v(4) = \$65{,}000$

(d) $v = 84{,}000$ when $t \approx 2.54$ years

58. $x^2 + y^2 = 4$

No

60. $y = |x + 4|$

Yes

62. $f(x) = x^2 + 4x + 3$

(a) $f(0) = 0^2 + 4(0) + 3 = 3$

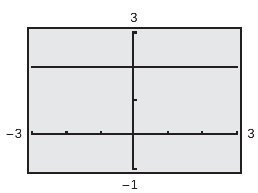
(b) $f(x - 1) = (x - 1)^2 + 4(x - 1) + 3 = x^2 + 2x$

(c) $f(x + \Delta x) - f(x) = (x + \Delta x)^2 + 4(x + \Delta x) + 3 - (x^2 + 4x + 3)$

$= 2x\Delta x + (\Delta x)^2 + 4\Delta x$

64. $f(x) = 2$

Domain: $(-\infty, \infty)$
Range: $\{2\}$

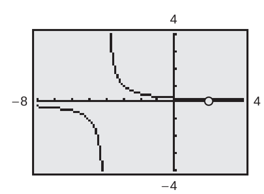


66. $f(x) = \dfrac{x - 3}{x^2 + x - 12} = \dfrac{(x - 3)}{(x - 3)(x + 4)}$

$= \dfrac{1}{x + 4}, \ x \neq 3$

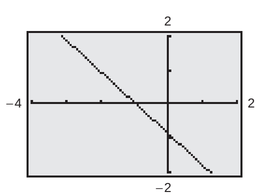
Domain: $(-\infty, -4) \cup (-4, 3) \cup (3, \infty)$

Range: $(-\infty, 0) \cup \left(0, \dfrac{1}{7}\right) \cup \left(\dfrac{1}{7}, \infty\right)$



68. $f(x) = \dfrac{-12}{13}x - \dfrac{7}{8}$

Domain: $(-\infty, \infty)$
Range: $(-\infty, \infty)$



70. (a) $f(x) + g(x) = 2x - 3 + \sqrt{x + 1}$

(b) $f(x) - g(x) = 2x - 3 - \sqrt{x + 1}$

(c) $f(x)g(x) = (2x - 3)\sqrt{x + 1}$

(d) $\dfrac{f(x)}{g(x)} = \dfrac{2x - 3}{\sqrt{x + 1}}$

(e) $f(g(x)) = f\left(\sqrt{x + 1}\right) = 2\sqrt{x + 1} - 3$

(f) $g(f(x)) = g(2x - 3) = \sqrt{(2x - 3) + 1} = \sqrt{2x - 2}$

72. $f(x) = |x + 1|$ does not have an inverse by the horizontal line test.

74. $f(x) = x^3 - 1$ has an inverse by the horizontal line test.

$$y = x^3 - 1$$
$$x = y^3 - 1$$
$$x + 1 = y^3$$
$$y = \sqrt[3]{x + 1}$$
$$f^{-1}(x) = \sqrt[3]{x + 1}$$

76. $\lim_{x\to 2} (2x + 9) = 2(2) + 9 = 13$

78. $\lim_{x\to 2} \frac{5x - 3}{2x + 9} = \frac{5(2) - 3}{2(2) + 9} = \frac{7}{13}$

80. $\lim_{t\to 0^-} \frac{t^2 + 1}{t} = -\infty$

$\lim_{t\to 0^+} \frac{t^2 + 1}{t} = \infty$

$\lim_{t\to 0} \frac{t^2 + 1}{t}$ does not exist.

82. $\lim_{t\to 2^-} \frac{t + 1}{t - 2} = -\infty$

$\lim_{t\to 2^+} \frac{t + 1}{t - 2} = \infty$

$\lim_{t\to 2} \frac{t + 1}{t - 2}$ does not exist.

84. $\lim_{x\to 3^-} \frac{x^2 - 9}{x - 3} = \lim_{x\to 3^-} (x + 3) = 6$

86. $\lim_{x\to 1/2} \frac{2x - 1}{6x - 3} = \lim_{x\to 1/2} \frac{1}{3} = \frac{1}{3}$

88. $\lim_{x\to 0} \frac{\frac{1}{x - 4} - \frac{1}{4}}{x} = \lim_{x\to 0} \frac{4 - (x - 4)}{x(x - 4)4} = \lim_{x\to 0} \frac{8 - x}{4x(x - 4)}$; $\lim_{x\to 0^+} \frac{8 - x}{4x(x - 4)} = -\infty$; $\lim_{x\to 0^-} \frac{8 - x}{4x(x - 4)} = \infty$;

$\lim_{x\to 0} \frac{\frac{1}{x - 4} - \frac{1}{4}}{x}$ does not exist.

90.
$$\lim_{s\to 0} \frac{\left(1/\sqrt{1 + s}\right) - 1}{s} = \lim_{s\to 0} \frac{1 - \sqrt{1 + s}}{s\sqrt{1 + s}} \cdot \frac{1 + \sqrt{1 + s}}{1 + \sqrt{1 + s}}$$
$$= \lim_{s\to 0} \frac{1 - (1 + s)}{s\sqrt{1 + s}\left(1 + \sqrt{1 + s}\right)} = \lim_{s\to 0} \frac{-1}{\sqrt{1 + s}\left(1 + \sqrt{1 + s}\right)} = -\frac{1}{2}$$

92.
$$\lim_{\Delta x\to 0} \frac{\left[1 - (x + \Delta x)^2\right] - (1 - x^2)}{\Delta x} = \lim_{\Delta x\to 0} \frac{1 - x^2 - 2x\Delta x - (\Delta x)^2 - 1 + x^2}{\Delta x}$$
$$= \lim_{\Delta x\to 0} \frac{\Delta x(-2x - \Delta x)}{\Delta x} = \lim_{\Delta x\to 0} (-2x - \Delta x) = -2x$$

94.

x	1.1	1.01	1.001	1.0001
$f(x)$	-0.3228	-0.3322	-0.3332	-0.3333

$\lim_{x\to 1^+} \frac{1 - \sqrt[3]{x}}{x - 1} = -\frac{1}{3}$

96. The statement $\lim_{x\to 0} x^3 = 0$ is true.

98. The statement $\lim_{x\to 0} \sqrt[3]{x} = 0$ is true.

100. The statement $\lim_{x\to 3} f(x) = 1$ is true since

$\lim_{x\to 3^-} f(x) = \lim_{x\to 3^-} (x - 2) = 1$

$\lim_{x\to 3^+} f(x) = \lim_{x\to 3^+} (-x^2 + 8x - 14) = 1.$

102. $f(x) = \dfrac{x + 2}{x}$ is continuous on the intervals $(-\infty, 0)$ and $(0, \infty)$.

104. $f(x) = \dfrac{x + 1}{2x + 2}$ is continuous on the intervals $(-\infty, -1)$ and $(-1, \infty)$.

106. $f(x) = [\![x]\!] - 2$ is continuous on all intervals of the form $(c, c + 1)$, where c is an integer.

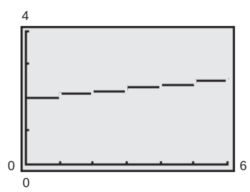
108. $f(x)$ is continuous on $(-\infty, \infty)$.

110. $\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} (x + 1) = 2$

$\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} (2x + a) = 2 + a$

Thus, $2 = 2 + a$ and $a = 0$.

112. $$C = \begin{cases} 2 & t < 1 \\ 2 + 0.1[\![t]\!], & t > 1,\ t \text{ is not an integer.} \\ 2 + 0.1(t - 1), & t \geq 1,\ t \text{ is an integer.} \end{cases}$$



114. Nonremovable discontinuities at $t = 1, 2, 3, \ldots$

Yellow sweet maize:

Intercepts: $(0, 45), (5, 0)$

Line: $y - 45 = \dfrac{45 - 0}{0 - 5}(x - 0)$

$y = -9x + 45$

White flint maize:

Intercepts: $(0, 30), (5.5, 0)$

Line: $y - 30 = \dfrac{30 - 0}{0 - 5.5}(x - 0)$

$y = -5.45x + 30$