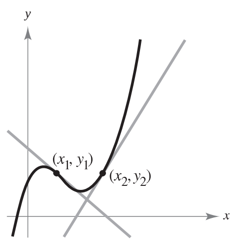
CHAPTER 2
Differentiation

C H A P T E R 2
Differentiation

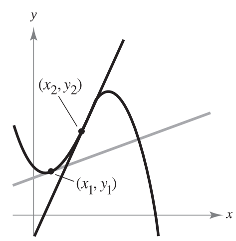
Section 2.1 The Derivative and the Slope of a Graph

Solutions to Even-Numbered Exercises

2. The tangent line at (x_1, y_1) has a negative slope.
The tangent line at (x_2, y_2) has a positive slope.



4. The tangent line at (x_1, y_1) has zero slope. The tangent line at (x_2, y_2) has a positive slope.



6. The slope is $m = \frac{4}{3}$.

8. The slope is $m = \frac{1}{4}$.

10. The slope is $m = -3$.

12. For 1998, $t = 8$ and $m \approx 500$.

For 2001, $t = 11$ and $m \approx -90$.

For 2003, $t = 13$ and $m \approx 500$.

14. (a) At t_1, $f'(t_1) > g'(t_1)$, so the runner given by f is running faster.

(b) At t_2, $g'(t_2) > f'(t_2)$, so the runner given by g is running faster. The runner given by f has traveled farther.

(c) At t_3, the runners are at the same location, but the runner given by g is running faster.

(d) The runner given by g will finish first, because that runner finishes the distance at a lesser value of t.

16. $f(x + \Delta x) = -4$

$f(x + \Delta x) - f(x) = -4 - (-4) = 0$

$$\frac{f(x + \Delta x) - f(x)}{\Delta x} = \frac{0}{\Delta x} = 0$$

$$\lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} = 0$$

18. $f(x + \Delta x) = \frac{1}{2}(x + \Delta x) + 5 = \frac{1}{2}x + \frac{1}{2}\Delta x + 5$

$f(x + \Delta x) - f(x) = \frac{1}{2}\Delta x$

$$\frac{f(x + \Delta x) - f(x)}{\Delta x} = \frac{1}{2}$$

$$\lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} = \frac{1}{2}$$

20. $f(x + \Delta x) = 1 - (x + \Delta x)^2 = 1 - x^2 - 2x\Delta x - (\Delta x)^2$

$f(x + \Delta x) - f(x) = -2x\Delta x - (\Delta x)^2$

$$\frac{f(x + \Delta x) - f(x)}{\Delta x} = -2x - \Delta x$$

$$\lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} = -2x$$

22. $f(x + \Delta x) = \sqrt{x + \Delta x + 2}$

$$f(x + \Delta x) - f(x) = \sqrt{x + \Delta x + 2} - \sqrt{x + 2}$$

$$\frac{f(x + \Delta x) - f(x)}{\Delta x} = \frac{\sqrt{x + \Delta x + 2} - \sqrt{x + 2}}{\Delta x} \cdot \frac{\sqrt{x + \Delta x + 2} + \sqrt{x + 2}}{\sqrt{x + \Delta x + 2} + \sqrt{x + 2}}$$

$$= \frac{(x + x\Delta + 2) - (x + 2)}{x\Delta\left[\sqrt{x + x\Delta + 2} + \sqrt{x + 2}\right]}$$

$$= \frac{1}{\sqrt{x + \Delta x + 2} + \sqrt{x + 2}}$$

$$\lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} = \frac{1}{2\sqrt{x + 2}}$$

24. $f(t + \Delta t) = (t + \Delta t)^3 + (t + \Delta t)^2 = t^3 + 3t^2\Delta t + 3t(\Delta t)^2 + (\Delta t)^3 + t^2 + 2t(\Delta t) + (\Delta t)^2$

$$f(t + \Delta t) - f(t) = 3t^2\Delta t + 3t(\Delta t)^2 + (\Delta t)^3 + 2t(\Delta t) + (\Delta t)^2$$

$$\frac{f(t + \Delta t) - f(t)}{\Delta t} = 3t^2 + 3t(\Delta t) + (\Delta t)^2 + 2t + \Delta t$$

$$\lim_{\Delta t \to 0} \frac{f(t + \Delta t) - f(t)}{\Delta t} = 3t^2 + 2t$$

26. $g(s + \Delta s) = \dfrac{1}{s + \Delta s - 1}$

$$g(s + \Delta s) - g(s) = \frac{1}{s + \Delta s - 1} - \frac{1}{s - 1} = \frac{(s - 1) - (s + \Delta x - 1)}{(s + \Delta s - 1)(s - 1)} = -\frac{\Delta s}{(s + \Delta s - 1)(s - 1)}$$

$$\frac{g(s + \Delta s) - g(s)}{\Delta s} = -\frac{1}{(s + \Delta s - 1)(s - 1)}$$

$$\lim_{\Delta s \to 0} \frac{g(s + \Delta s) - g(s)}{\Delta s} = -\frac{1}{(s - 1)^2}$$

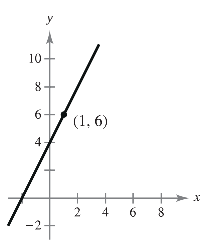
28. $f(x + \Delta x) = 2(x + \Delta x) + 4$

$$f(x + \Delta x) - f(x) = 2x + 2\Delta x + 4 - (2x + 4) = 2\Delta x$$

$$\frac{f(x + \Delta x) - f(x)}{\Delta x} = 2$$

$$\lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} = 2$$

At $(1, 6)$, the slope of the tangent line is $m = 2$.



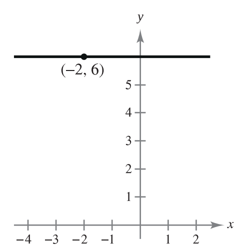
30. $f(x + \Delta x) = 6$

$$f(x + \Delta x) - f(x) = 6 - 6 = 0$$

$$\frac{f(x + \Delta x) - f(x)}{\Delta x} = \frac{0}{\Delta x} = 0$$

$$\lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} = 0$$

At $(-2, 6)$, the slope of the tangent line is $m = 0$.



32. $f(x + \Delta x) = (x + \Delta x)^2 + 2(x + \Delta x) + 1$

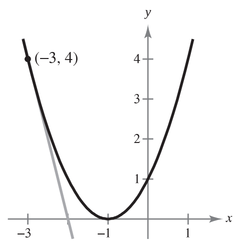
$$= x^2 + 2x\Delta x + (\Delta x)^2 + 2x + 2\Delta x + 1$$

$$f(x + \Delta x) - f(x) = 2x\Delta x + (\Delta x)^2 + 2\Delta x$$

$$\frac{f(x + \Delta x) - f(x)}{\Delta x} = 2x + \Delta x + 2$$

$$\lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} = 2x + 2$$

At $(-3, 4)$, the slope of the tangent line is $m = 2(-3) + 2 = -4$.



34. $f(x + \Delta x) = (x + \Delta x)^3 + 2(x + \Delta x)$

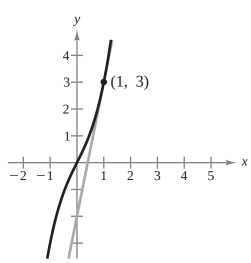
$$= x^3 + 3x^2\Delta x + 3x(\Delta x)^2 + (\Delta x)^3 + 2x + 2\Delta x$$

$$f(x + \Delta x) - f(x) = 3x^2\Delta x + 3x(\Delta x)^2 + (\Delta x)^3 + 2\Delta x$$

$$\frac{f(x + \Delta x) - f(x)}{\Delta x} = 3x^2 + 3x\,\Delta x + (\Delta x)^2 + 2$$

$$\lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} = 3x^2 + 2 = f'(x)$$

At $(1, 3)$, the slope of the tangent line is $m = 3(1)^2 + 2 = 5$. The figure shows the graph of f and the tangent line.



36. $f(x + \Delta x) = \sqrt{2(x + \Delta x) - 2}$

$$f(x + \Delta x) - f(x) = \sqrt{2x + 2\Delta x - 2} - \sqrt{2x - 2}$$

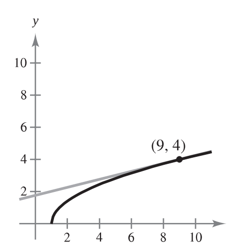
$$= \frac{\sqrt{2x + 2\Delta x - 2} - \sqrt{2x - 2}}{1} \cdot \frac{\sqrt{2x + 2\Delta x - 2} + \sqrt{2x - 2}}{\sqrt{2x + 2\Delta x - 2} + \sqrt{2x - 2}}$$

$$= \frac{2\Delta x}{\sqrt{2x + 2\Delta x - 2} + \sqrt{2x - 2}}$$

$$\frac{f(x + \Delta x) - f(x)}{\Delta x} = \frac{2}{\sqrt{2x + 2\Delta x - 2} + \sqrt{2x - 2}}$$

$$\lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} = \frac{2}{2\sqrt{2x - 2}} = \frac{1}{\sqrt{2x - 2}}$$

At $(9, 4)$, the slope of the tangent line is $m = \dfrac{1}{\sqrt{2(9) - 2}} = \dfrac{1}{4}$.



38. $f(x + \Delta x) = -(x + \Delta x)^2 = -x^2 - 2x\Delta x - (\Delta x)^2$

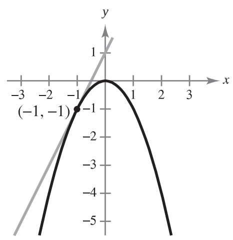
$$f(x + \Delta x) - f(x) = -2x\Delta x - (\Delta x)^2$$

$$\frac{f(x + \Delta x) - f(x)}{\Delta x} = -2x - \Delta x$$

$$\lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} = -2x$$

At the point $(-1, -1)$, the slope of the tangent line is $m = -2(-1) = 2$. The equation of the tangent line is

$$y - (-1) = 2[x - (-1)]$$
$$y = 2x + 1.$$



40. $f(x + \Delta x) = 2(x + \Delta x)^2 - 1$

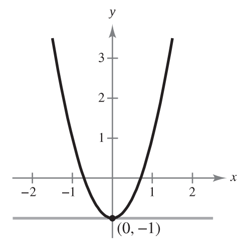
$$= 2x^2 + 4x\Delta x + 2(\Delta x)^2 - 1$$

$$f(x + \Delta x) - f(x) = 4x\Delta x + 2(\Delta x)^2$$

$$\frac{f(x + \Delta x) - f(x)}{\Delta x} = 4x + 2\Delta x$$

$$\lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} = 4x$$

At the point $(0, -1)$, the slope of the tangent line is $m = 4(0) = 0$. The equation of the horizontal tangent line is $y = -1$.

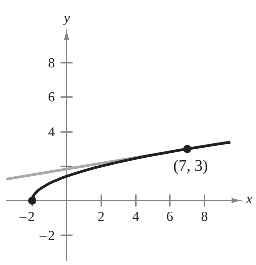


42. From Exercise 22, $\displaystyle\lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} = \frac{1}{2\sqrt{x + 2}}$.

At the point $(7, 3)$, the slope of the tangent line is $m = \dfrac{1}{6}$.

$$y - 3 = \frac{1}{6}(x - 7)$$

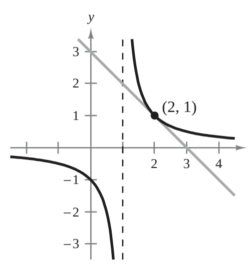
$$y = \frac{1}{6}x + \frac{11}{6}$$



44. From Exercise 26, $\displaystyle\lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} = \frac{-1}{(x - 1)^2}$.

At the point $(2, 1)$, the slope is -1.

$$y - 1 = -1(x - 2)$$
$$y = -x + 3$$



46. $f(x + \Delta x) = (x + \Delta x)^2 + 1 = x^2 + 2x\Delta x + (\Delta x)^2 + 1$

$$f(x + \Delta x) - f(x) = 2x\Delta x + (\Delta x)^2$$

$$\frac{f(x + \Delta x) - f(x)}{\Delta x} = 2x + \Delta x$$

$$\lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} = 2x \quad \text{(Slope of tangent line)}$$

Since the slope of the given line is -2, we have $2x = -2$ so $x = -1$ and $f(-1) = 2$. Therefore, at the point $(-1, 2)$, the tangent line parallel to $2x + y = 0$ is

$$y - 2 = -2[x - (-1)]$$
$$y = -2x.$$

48. $f(x + \Delta x) = (x + \Delta x)^2 - (x + \Delta x) = x^2 + 2x\Delta x + (\Delta x)^2 - x - \Delta x$

$f(x + \Delta x) - f(x) = 2x\Delta x + (\Delta x)^2 - \Delta x$

$$\frac{f(x + \Delta x) - f(x)}{\Delta x} = 2x + \Delta x - 1$$

$$\lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} = 2x - 1$$

Slope of line $x + 2y - 6 = 0$ is $m = -\frac{1}{2}$.

Equating slopes: $2x - 1 = -\frac{1}{2}$

$$2x = \frac{1}{2}$$

$$x = \frac{1}{4} \text{ and } f\left(\frac{1}{4}\right) = -\frac{3}{16}$$

$$y + \frac{3}{16} = -\frac{1}{2}\left(x - \frac{1}{4}\right)$$

$$y = -\frac{1}{2}x - \frac{1}{16} \quad \text{Tangent line.}$$

50. y is differentiable everywhere except the two cusps at $x = \pm 3$.

52. y is not differentiable at $x = 0$. At $(0, 0)$ the graph has a cusp.

54. y is differentiable everywhere except $x = \pm 2$, where f is not defined.

56. Since $x = 1$ is a nonremovable discontinuity, y is differentiable everywhere except at $x = 1$.

58.

x	-2	$-\frac{3}{2}$	-1	$-\frac{1}{2}$	0	$\frac{1}{2}$	1	$\frac{3}{2}$	2
$f(x)$	2	1.125	0.5	0.125	0	0.125	0.5	1.125	2
$f'(x)$	-2	-1.5	-1	-0.5	0	0.5	1	1.5	2

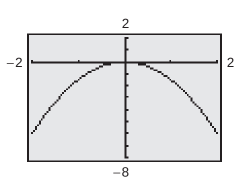
Analytically, the slope of $f(x) = \frac{1}{2}x^2$ is

$$m = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} = \lim_{\Delta x \to 0} \frac{\frac{1}{2}(x + \Delta x)^2 - \frac{1}{2}x^2}{\Delta x} = \lim_{\Delta x \to 0} \frac{x\Delta x + \frac{1}{2}(\Delta x)^2}{\Delta x} = \lim_{\Delta x \to 0} \left(x + \frac{1}{2}\Delta x\right) = x.$$

60.

x	-2	$-\frac{3}{2}$	-1	$-\frac{1}{2}$	0	$\frac{1}{2}$	1	$\frac{3}{2}$	2
$f(x)$	-6	-3.375	-1.5	-0.375	0	-0.375	-1.5	-3.375	-6
$f'(x)$	6	4.5	3	1.5	0	-1.5	-3	-4.5	-6

$f(x) = -\frac{3}{2}x^2 \qquad f'(x) = -3x$

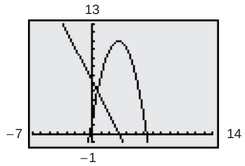


62. $f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} = \lim_{\Delta x \to 0} \frac{2 + 6(x + \Delta x) - (x + \Delta x)^2 - (2 + 6x - x^2)}{\Delta x}$

$= \lim_{\Delta x \to 0} \frac{6\Delta x - 2x\Delta x - (\Delta x)^2}{\Delta x} = \lim_{\Delta x \to 0} (6 - 2x - \Delta x) = 6 - 2x$

Graphs of $f(x) = 2 + 6x - x^2$ and $f'(x) = 6 - 2x$ (see figure)

The x-intercept of the derivative indicates a point of horizontal tangency for f.



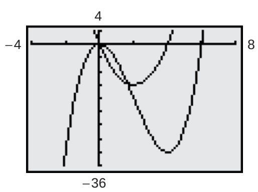
64. $f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$

$= \lim_{\Delta x \to 0} \frac{(x + \Delta x)^3 - 6(x + \Delta x)^2 - (x^3 - 6x^2)}{\Delta x}$

$= \lim_{\Delta x \to 0} \frac{x^3 - 3x^2\Delta x + 3x(\Delta x)^2 + (\Delta x)^3 - 6(x^2 + 2x\Delta x + (\Delta x)^2) - x^3 + 6x^2}{\Delta x}$

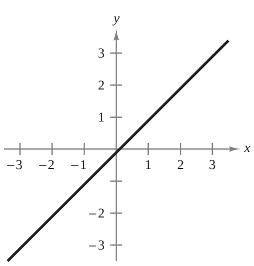
$= \lim_{\Delta x \to 0} (3x^2 + 3x\Delta x + (\Delta x)^2 - 12x - 6\Delta x)$

$= 3x^2 - 12x$



The x-intercepts of the derivative, $x = 0, 4$, indicate points of horizontal tangency for f.

66. One possible answer is $y = x$.



68. True

70. True (See page 89.)

Section 2.2 Some Rules for Differentiation

2. (a) $y = x^{3/2}$

$y' = \frac{3}{2}x^{1/2} = \frac{3\sqrt{x}}{2}$

At (1, 1), $y' = \frac{3}{2}$.

(b) $y = x^3$

$y' = 3x^2$

At (1, 1), $y' = 3$.

4. (a) $y = x^{-1/2}$

$y' = -\frac{1}{2}x^{-3/2} = -\frac{1}{2x^{3/2}}$

At (1, 1), $y' = -\frac{1}{2}$.

(b) $y = x^{-2}$

$y' = -2x^{-3} = -\frac{2}{x^3}$

At (1, 1), $y' = -2$.

6. $f'(x) = 0$

8. $g'(x) = 3$

10. $y' = 2t + 2$

12. $y' = 3x^2 - 18x$

14. $y' = 6x^2 - 2x + 3$

16. $h'(x) = \frac{5}{2}x^{3/2}$

18. $g(x) = 4x^{1/3} + 2$

$g'(x) = 4\left(\frac{1}{3}x^{-2/3}\right) = \frac{4}{3x^{2/3}}$

20. $s'(t) = 4(-1t^{-2}) = -\frac{4}{t^2}$

	Function	*Rewrite*	*Differentiate*	*Simplify*
22.	$y = \frac{2}{3x^2}$	$y = \frac{2}{3}x^{-2}$	$y' = -\frac{4}{3}x^{-3}$	$y' = -\frac{4}{3x^3}$
24.	$y = \frac{\pi}{(3x)^2}$	$y = \frac{\pi}{9}x^{-2}$	$y' = -\frac{2\pi}{9}x^{-3}$	$y' = -\frac{2\pi}{9x^3}$
26.	$y = \frac{4x}{x^{-3}}$	$y = 4x^4$	$y' = 16x^3$	$y' = 16x^3$

28. $f(t) = 4 - \frac{4}{3t} = 4 - \frac{4}{3}t^{-1}$

$f'(t) = \frac{4}{3}t^{-2} = \frac{4}{3t^2}$

$f'\left(\frac{1}{2}\right) = \frac{16}{3}$

30. $y = 3x\left(x^2 - \frac{2}{x}\right) = 3x^3 - 6$

$y' = 9x^2$

At (2, 18), $y' = 9(2)^2 = 36$.

32. $f(x) = 3(5 - x)^2 = 75 - 30x + 3x^2$

$f'(x) = -30 + 6x = -6(x - 5)$

At (5, 0), $f'(5) = 0$.

34. $f(x) = x^2 - 3x - 3x^{-2} + 5x^{-3}$

$f'(x) = 2x - 3 + 6x^{-3} - 15x^{-4}$

$= 2x - 3 + \frac{6}{x^3} - \frac{15}{x^4}$

36. $f(x) = x^2 + 4x + \frac{1}{x} = x^2 + 4x + x^{-1}$

$f'(x) = 2x + 4 - x^{-2} = 2x + 4 - \frac{1}{x^2}$

38. $f(x) = (x^2 + 2x)(x + 1) = x^3 + 3x^2 + 2x$

$f'(x) = 3x^2 + 6x + 2$

40. $f(x) = (3x^2 - 5x)(x^2 + 2) = 3x^4 - 5x^3 + 6x^2 - 10x$

$f'(x) = 12x^3 - 15x^2 + 12x - 10$

42. $f(x) = \dfrac{2x^2 - 3x + 1}{x} = 2x - 3 + x^{-1}$

$f'(x) = 2 - x^{-2} = 2 - \dfrac{1}{x^2} = \dfrac{2x^2 - 1}{x^2}$

44. $f(x) = \dfrac{-6x^3 + 3x^2 - 2x + 1}{x} = -6x^2 + 3x - 2 + x^{-1}$

$f'(x) = -12x + 3 - x^{-2} = -12x + 3 - \dfrac{1}{x^2}$

46. $f(x) = x^{1/3} - 1$

$f'(x) = \dfrac{1}{3}x^{-2/3} = \dfrac{1}{3x^{2/3}}$

48. $y' = 3x^2 + 1$

At $(-1, -2)$, the slope is $m = 4$.

The equation of the tangent line is

$$y - (-2) = 4[x - (-1)]$$
$$y + 2 = 4x + 4$$
$$y = 4x + 2.$$

50. $f(x) = x^{-2/3} - x$

$f'(x) = -\frac{2}{3}x^{-5/3} - 1$

$f'(-1) = -\frac{2}{3}(-1) - 1 = -\frac{1}{3}$

Tangent line: $y - 2 = -\frac{1}{3}(x + 1)$

$y = -\frac{1}{3}x + \frac{5}{3}$

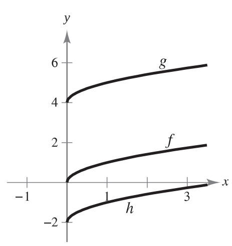
52. $y' = 3x^2 + 6x = 3x(x + 2) = 0$ when $x = 0, -2$.

The function has horizontal tangent lines at the points $(0, 0)$ and $(-2, 4)$.

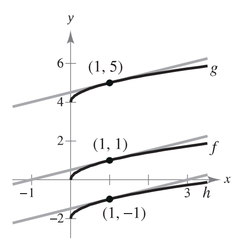
54. $y' = 2x + 2 = 0$ when $x = -1$.

The function has a horizontal tangent line at the point $(-1, -1)$.

56. (a)



(c)



(b) $f'(x) = g'(x) = h'(x) = \dfrac{1}{2}x^{-1/2} = \dfrac{1}{2\sqrt{x}}$

$f'(1) = g'(1) = h'(1) = \dfrac{1}{2}$

58. $R = -1.17879t^4 + 38.3641t^3 - 469.994t^2 + 2820.22t - 5577.7$

($t = 6$ corresponds to 1996; $6 \geq t \geq 12$.)

(a) $R'(t) = -4.71516t^3 + 115.0923t^2 - 939.988t + 2820.22$

1997: $R'(7) \approx 262.5$

2000: $R'(10) \approx 214.4$

2002: $R'(12) \approx -34.1$

(b) These results are close to the estimates.

(c) The units of R' are millions of dollars per year.

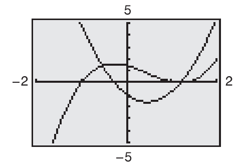
60. $C = 7.75x + 500$

$C' = 7.75$, which equals the variable cost.

62. (a) More men and women seem to suffer from migraines between 30 and 40 years old. More males than females suffer from migraines. Fewer people whose income is greater than or equal to \$30,000 suffer from migraines than people whose income is less than \$10,000.

(b) The derivatives are positive up to approximately 37 years old and negative after about 37 years of age. The percent of adults suffering from migraines increases up to about 37 years old, then decreases. The units of the derivative are percent of adults suffering from migraines per year.

64.



f has horizontal tangents $(y' = 0)$ at $(1.2, 0)$ and $(-0.267, 1.577)$.

66. True. c is a constant.

Section 2.3 Rates of Change: Velocity and Marginals

2. (a) $\dfrac{500 - 210}{10} = \$29$ billion per year

(b) $\dfrac{390 - 200}{10} = \$19$ billion per year

(c) $\dfrac{1210 - 500}{10} \approx \$71$ billion per year

(d) $\dfrac{780 - 390}{10} \approx \$39$ billion per year

(e) $\dfrac{1150 - 210}{22} \approx \$43$ billion per year

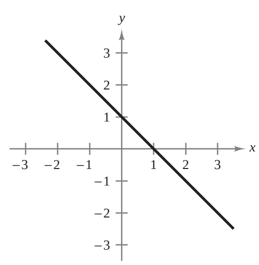
(f) $\dfrac{700 - 200}{22} \approx \$23$ billion per year

Answers will vary.

4. $h(x) = 1 - x$, $[0, 1]$, $h'(x) = -1$

Average rate of change: $\dfrac{h(1) - h(0)}{1 - 0} = \dfrac{0 - 1}{1} = -1$

Instantaneous rates of change: $h'(0) = h'(1) = -1$

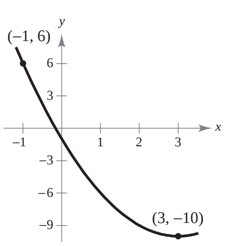


6. $f'(x) = 2x - 6$

Average rate of change:

$$\frac{\Delta y}{\Delta x} = \frac{f(3) - f(-1)}{3 - (-1)} = \frac{-10 - 6}{4} = -4$$

Instantaneous rates of change: $f'(-1) = -8$, $f'(3) = 0$

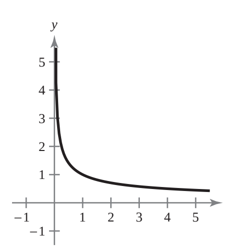


8. $f(x) = x^{-1/2}$ $\quad f'(x) = -\frac{1}{2}x^{-3/2} = \dfrac{-1}{2x^{3/2}}$

Average rate of change:

$$\frac{\Delta y}{\Delta x} = \frac{f(4) - f(1)}{4 - 1} = \frac{1/2 - 1}{3} = -\frac{1}{6}$$

Instantaneous rates of change: $f'(4) = -\dfrac{1}{16}$, $f'(1) = -\dfrac{1}{2}$

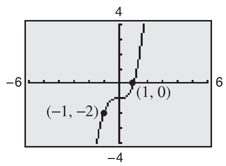


10. $g'(x) = 3x^2$

Average rate of change:

$$\frac{\Delta y}{\Delta x} = \frac{g(1) - g(-1)}{1 - (-1)} = \frac{0 - (-2)}{2} = 1$$

Instantaneous rates of change: $g'(-1) = 3,\ g'(1) = 3$



12. (a) From $t = 1$ to $t = 2$: $[1, 2]$: The average rate of change is increasing at the greatest rate. From $t = 5$ to $t = 6$: $[5, 6]$: the average rate of change is decreasing at the greatest rate.

(b) At $t = 4$, $m' \approx 0$. Over $[2, 5]$, the average rate of change is also 0. [Other answers possible.]

14. (a) $H'(v) = 33\left[10\left(\frac{1}{2}v^{-1/2}\right) - 1\right] = 33\left[\frac{5}{\sqrt{v}} - 1\right]$ Rate of change of heat loss.

(b) $H'(2) = 33\left[\frac{5}{\sqrt{2}} - 1\right] \approx 83.673 \div 3600 \approx 0.023$ kilocalories/meters^3

$H'(5) = 33\left[\frac{5}{\sqrt{5}} - 1\right] \approx 40.790 \div 3600 \approx 0.11$ kilocalories/meters^3

16. First leg: 0.75 km in 20 seconds

Second leg: 0.75 km in 25 seconds

(a) $\frac{0.75}{20} = 0.0375$ km/sec $= 37.5$ m/sec

(b) $\frac{1.50}{20 + 25} = 0.3\overline{3}$ km/sec $= 33.3\overline{3}$ m/sec

18. $\frac{dC}{dx} = 7200$

20. $\frac{dC}{dx} = 100\left[0 + 3\left(\frac{1}{2}x^{-1/2}\right)\right] = \frac{150}{\sqrt{x}}$

22. $\frac{dR}{dx} = 30 - 2x$

24. $\frac{dR}{dx} = 50\left[20 - \frac{3}{2}x^{1/2}\right] = 1000 - 75\sqrt{x}$

26. $\frac{dP}{dx} = -0.5x + 2000$

28. $\frac{dP}{dx} = -1.5x^2 + 60x - 164.25$

30. (a) $R(15) - R(14) = 2(15)[900 + 32(15) - 15^2] - 2(14)[900 + 32(14) - 14^2]$

$= 34{,}650 - 32{,}256 = 2394$ dollars

(b) $R = 1800x + 64x^2 - 2x^3$

$R'(x) = 1800 + 128x - 6x^2$

$R'(14) = 2416$ dollars

(c) The answers are nearly the same.

32. $P = 22t^2 + 52t + 10{,}000$

(a) $P(0) = 10{,}000$ people

$P(10) = 12{,}720$ people

$P(15) = 15{,}730$ people

$P(20) = 19{,}840$ people

The population is growing quadratically.

(b) $\dfrac{dP}{dt} = 44t + 52$

(c) $P'(0) = 52$ people per year

$P'(10) = 492$ people per year

$P'(15) = 712$ people per year

$P'(20) = 932$ people per year

The rate of growth is increasing.

34. $\dfrac{dP}{dx} = 2048\left(\dfrac{1}{2}x^{-1/2}\right) - \dfrac{1}{8}(-2x^{-3})$

$= \dfrac{1024}{\sqrt{x}} + \dfrac{1}{4x^3}$

(a) When $x = 150$, $\dfrac{dP}{dx} \approx \$83.61$.

(b) When $x = 175$, $\dfrac{dP}{dx} \approx \$77.41$.

(c) When $x = 200$, $\dfrac{dP}{dx} \approx \$72.41$.

(d) When $x = 225$, $\dfrac{dP}{dx} \approx \$68.27$.

(e) When $x = 250$, $\dfrac{dP}{dx} \approx \$64.76$.

(f) When $x = 275$, $\dfrac{dP}{dx} \approx \$61.75$.

36. (a) $TR = -10Q^2 + 160Q$

(b) $(TR)' = MR = -20Q + 160$

(c)

Q	0	2	4	6	8	10
Model	160	120	80	40	0	−40
Table	–	130	90	50	10	−30

38. (36,000, 6), (33,000, 7)

$\text{Slope} = \dfrac{7 - 6}{33{,}000 - 36{,}000} = \dfrac{-1}{3000}$

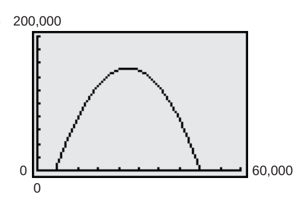
$p - 6 = -\dfrac{1}{3000}(x - 36{,}000)$

$p = -\dfrac{1}{3000}x + 18$ (demand function)

(a) $P = R - C = xp - C$

$= x\left(-\dfrac{1}{3000}x + 18\right) - (0.2x - 85{,}000)$

$= \dfrac{x^2}{3000} + 17.8x - 85{,}000$

(b) 

$P'(18{,}000) > 0 \implies$ positive slope

$P'(36{,}000) < 0 \implies$ negative slope

(c) $P'(x) = \dfrac{-x}{1500} + 17.8$

$P'(18{,}000) = 5.8$ dollars per ticket

$P'(36{,}000) = -6.2$ dollars per ticket

40. $C = v(x) + k$

Marginal cost: $C' = v'(x) + 0 = v'(x)$

Thus, the marginal cost is independent of the fixed cost.

42. $\dfrac{dC}{dQ} = -\dfrac{1{,}008{,}000}{Q^2} + \dfrac{63}{10}$

$C(351) - C(350) \approx -\$1.91$

$\dfrac{dC}{dQ} = -\$1.93$ per unit when $Q = 350$.

44. $C(x) = \dfrac{15{,}000 \text{ miles/year}}{x \text{ miles/gallon}}(1.30 \text{ dollars/gallon})$

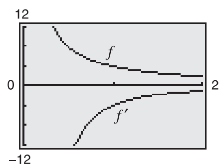
$= \dfrac{19{,}500}{x}$ dollars/year

$C'(x) = \dfrac{-19{,}500}{x^2}$

x	10	15	20	25	30	35	40
$C(x)$	1950	1300	975	780	650	557.14	487.50
$C'(x)$	-195	-86.67	-48.75	-31.2	-21.67	-15.92	-12.19

The car that gets 15 miles per gallon will benefit more.

46. (a)



(b) $f(x) = \dfrac{4}{x} = 4x^{-1},\ f'(x) = -4x^{-2} = \dfrac{-4}{x^2}$

x	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{2}$	1	2	3	4	5
$f(x)$	32	16	8	4	2	1.33	1	0.8
$f'(x)$	-256	-64	-16	-4	-1	-0.44	-0.25	-0.16

(c) $\left[\frac{1}{8}, \frac{1}{4}\right]$: average rate of change $= \dfrac{16 - 32}{(1/4) - (1/8)} = -128$

$\left[\frac{1}{4}, \frac{1}{2}\right]$: average rate of change $= -32$

$\left[\frac{1}{2}, 1\right]$: average rate of change $= -8$

$[1, 2]$: average rate of change $= -2$

$[2, 3]$: average rate of change $= -0.67$

$[3, 4]$: average rate of change $= -0.33$

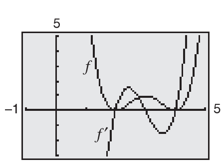
$[4, 5]$: average rate of change $= -0.2$

48. $f(x) = x^4 - 12x^3 + 52x^2 - 96x + 64$

$f'(x) = 4x^3 - 36x^2 + 104x - 96$

$f'(x) = 0$ when $x = 2, 3, 4$.

Therefore, f has horizontal tangents at $(2, 0)$, $(3, 1)$, and $(4, 0)$.



Section 2.4 The Product and Quotient Rules

2. $f'(x) = (x^2+1)(2) + (2x+5)(2x)$

$= 6x^2 + 10x + 2$

$f'(-1) = -2$

4. $f'(x) = \frac{1}{7}(-12x) = -\frac{12x}{7}$

$f'(1) = -\frac{12}{7}$

6. $g'(x) = (x^2-2x+1)(3x^2) + (x^3-1)(2x-2)$

$= 3x^4 - 6x^3 + 3x^2 + 2x^4 - 2x^3 - 2x + 2$

$= 5x^4 - 8x^3 + 3x^2 - 2x + 2$

$g'(1) = 0$

8. $h'(x) = \frac{(x+3)(2x) - (x^2)(1)}{(x+3)^2}$

$= \frac{2x^2 + 6x - x^2}{(x+3)^2} = \frac{x^2+6x}{(x+3)^2}$

$h'(-1) = -\frac{5}{4}$

10. $f(x) = \frac{3x}{x^2+4}$

$f'(x) = \frac{(x^2+4)(3) - 3x(2x)}{(x^2+4)^2} = \frac{-3x^2+12}{(x^2+4)^2}$

$f'(-1) = \frac{9}{25}$

12. $f'(x) = \frac{(x-1)(1) - (x+1)(1)}{(x-1)^2} = -\frac{2}{(x-1)^2}$

$f'(2) = -2$

14. $g(x) = \frac{4x-5}{x^2-1}$

$g'(x) = \frac{(x^2-1)(4) - (4x-5)(2x)}{(x^2-1)^2} = \frac{-4x^2+10x-4}{(x^2-1)^2}$

$g'(0) = -4$

	Function	*Rewrite*	*Differentiate*	*Simplify*
16.	$y = \frac{4x^{3/2}}{x}$	$y = 4x^{1/2}$	$y' = 2x^{-1/2}$	$y' = \frac{2}{\sqrt{x}}$
18.	$y = \frac{4}{5x^2}$	$y = \frac{4}{5}x^{-2}$	$y' = -\frac{8}{5}x^{-3}$	$y' = -\frac{8}{5x^3}$
20.	$y = \frac{3x^2-4x}{6x}$	$y = \frac{1}{2}x - \frac{2}{3}, x \neq 0$	$y' = \frac{1}{2}, x \neq 0$	$y' = \frac{1}{2}, x \neq 0$
22.	$y = \frac{x^2-4}{x+2}$	$y = x - 2, x \neq -2$	$y' = 1, x \neq -2$	$y' = 1, x \neq -2$

24. $h(t) = (t^5-1)(4t^2-7t-3)$

$h'(t) = (t^5-1)(8t-7) + (5t^4)(4t^2-7t-3)$

$= 8t^6 - 7t^5 - 8t + 7 + 20t^6 - 35t^5 - 15t^4$

$= 28t^6 - 42t^5 - 15t^4 - 8t + 7$

26. $h(p) = (p^3-2)^2$

$h(p) = p^6 - 4p^3 + 4$

$h'(p) = 6p^5 - 12p^2 = 6p^2(p^3-2)$

28. $f(x) = x^{1/3}(x + 1) = x^{4/3} + x^{1/3}$

$$f'(x) = \frac{4}{3}x^{1/3} + \frac{1}{3}x^{-2/3}$$
$$= \frac{4x^{1/3}}{3} + \frac{1}{3x^{2/3}} = \frac{4x + 1}{3x^{2/3}}$$

30. $f(x) = \dfrac{x^3 + 3x + 2}{x^2 - 1}$

$$f'(x) = \frac{(x^2 - 1)(3x^2 + 3) - (x^3 + 3x + 2)(2x)}{(x^2 - 1)^2}$$
$$= \frac{3x^4 - 3 - 2x^4 - 6x^2 - 4x}{(x^2 - 1)^2}$$
$$= \frac{x^4 - 6x^2 - 4x - 3}{(x^2 - 1)^2}$$

32. $f(x) = (x^5 - 3x)(1/x^2)$

$$f(x) = (x^5 - 3x)\left(\frac{1}{x^2}\right) = x^3 - \frac{3}{x}$$
$$f'(x) = 3x^2 + \frac{3}{x^2} = \frac{3x^4 + 3}{x^2} = \frac{3(x^4 + 1)}{x^2}$$

34. $h(t) = \dfrac{t + 2}{t^2 + 5t + 6}$

$$h'(t) = \frac{(t^2 + 5t + 6)(1) - (t + 2)(2t + 5)}{(t^2 + 5t + 6)^2}$$
$$= \frac{-t^2 - 4t - 4}{(t + 2)^2(t + 3)^2}$$
$$= -\frac{(t + 2)^2}{(t + 2)^2(t + 3)^2}$$
$$= \frac{-1}{(t + 3)^2},\ t \neq -2$$

Equivalently, note that

$$h(t) = \frac{t + 2}{(t + 2)(t + 3)} = \frac{1}{t + 3},\ t \neq -2.$$

36. $f(x) = \dfrac{x + 1}{\sqrt{x}}$

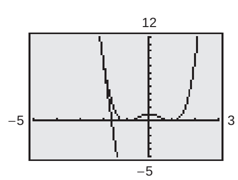
$$f(x) = \frac{x + 1}{\sqrt{x}} = x^{1/2} + x^{-1/2}$$
$$f'(x) = \frac{1}{2}x^{-1/2} - \frac{1}{2}x^{-3/2}$$
$$= \frac{1}{2}\left(\frac{1}{x^{1/2}} - \frac{1}{x^{3/2}}\right)$$
$$= \frac{1}{2}\left(\frac{x - 1}{x^{3/2}}\right)$$
$$= \frac{x - 1}{2x^{3/2}}$$

38. $f(x) = (3x^3 + 4x)(x - 5)(x + 1)$

$$f(x) = (3x^3 + 4x)(x - 5)(x + 1) = (3x^3 + 4x)(x^2 - 4x - 5)$$
$$f'(x) = (3x^3 + 4x)(2x - 4) + (x^2 - 4x - 5)(9x^2 + 4)$$
$$= (6x^4 - 12x^3 + 8x^2 - 16x) + (9x^4 - 36x^3 - 41x^2 - 16x - 20)$$
$$= 15x^4 - 48x^3 - 33x^2 - 32x - 20$$

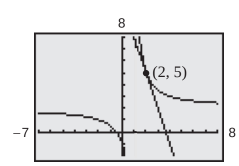
40. $h(x) = (x^2 - 1)^2, \quad (-2, 9)$

$$h'(x) = 2(x^2 - 1)(2x) = 4x(x^2 - 1)$$
$$h'(-2) = 4(-2)(4 - 1) = -24$$
$$y - 9 = -24(x + 2)$$
$$y = -24x - 39$$



42. $f(x) = \dfrac{2x + 1}{x - 1}, \quad (2, 5)$

$$f'(x) = \frac{(x - 1)2 - (2x + 1)}{(x - 1)^2} = \frac{-3}{(x - 1)^2}$$
$$f'(2) = -3$$
$$y - 5 = -3(x - 2)$$
$$y = -3x + 11$$



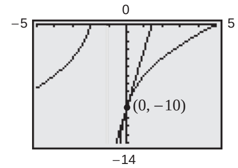
44. $g(x) = (x+2)\left(\dfrac{x-5}{x+1}\right) = \dfrac{x^2-3x-10}{x+1}, \quad (0, -10)$

$$g'(x) = \frac{(x+1)(2x-3)-(x^2-3x-10)}{(x+1)^2} = \frac{x^2+2x+7}{(x+1)^2}$$

$g'(0) = 7$

$y + 10 = 7(x - 0)$

$y = 7x - 10$



46. $f'(x) = \dfrac{(x^2+1)(2x)-(x^2)(2x)}{(x^2+1)^2} = \dfrac{2x}{(x^2+1)^2}$

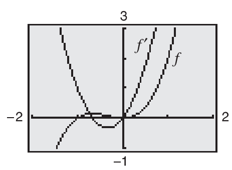
$f'(x) = 0$ when $2x = 0$, which implies that $x = 0$. Thus, the horizontal tangent line occurs at $(0, 0)$.

48. $f'(x) = \dfrac{(x^2+1)(4x^3)-(x^4+3)(2x)}{(x^2+1)^2} = \dfrac{2x(x^2+3)(x^2-1)}{(x^2+1)^2}$

$f'(x) = 0$ when $x = 0$ and $x = \pm 1$. Thus, the horizontal tangent line occurs at the points $(0, 3)$, $(1, 2)$, and $(-1, 2)$.

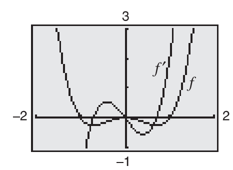
50. $f(x) = x^2(x+1) = x^3 + x^2$

$f'(x) = 3x^2 + 2x = x(3x+2)$



52. $f(x) = x^2(x+1)(x-1) = x^4 - x^2$

$f'(x) = 4x^3 - 2x = 2x(2x^2 - 1)$



54. $$\frac{dx}{dp} = 0 - 1 - \frac{(p+1)(2)-(2p)(1)}{(p+1)^2}$$

$$= -1 - \frac{2}{(p+1)^2}$$

$$= \frac{-(p+1)^2 - 2}{(p+1)^2}$$

$$= \frac{-p^2-2p-3}{(p+1)^2}$$

When $p = 3$, $\dfrac{dx}{dp} = \dfrac{-9-6-3}{16} = -\dfrac{9}{8}$.

56. The initial temperature is $T = 10\left(\dfrac{75}{10}\right) = 75$ deg.

$$\frac{dT}{dt} = 10\left[\frac{(t^2+4t+10)(8t+16)-(4t^2+16t+75)(2t+4)}{(t^2+4t+10)^2}\right] = 10\left[\frac{-70t-140}{(t^2+4t+10)^2}\right] = \frac{-700(t+2)}{(t^2+4t+10)^2}$$

(a) When $t = 1$, $\dfrac{dT}{dt} = \dfrac{-700(3)}{(15)^2} = -\dfrac{28}{3} \approx -9.33$ deg/hr.

(b) When $t = 3$, $\dfrac{dT}{dt} = \dfrac{-700(5)}{(31)^2} = -\dfrac{3500}{961} \approx -3.64$ deg/hr.

(c) When $t = 5$, $\dfrac{dT}{dt} = \dfrac{-700(7)}{(55)^2} = -\dfrac{196}{121} \approx -1.62$ deg/hr.

(d) When $t = 10$, $\dfrac{dT}{dt} = \dfrac{-700(12)}{(150)^2} = -\dfrac{28}{75} \approx -0.37$ deg/hr.

58. $\frac{dP}{dt} = \frac{50(t+2)(1) - (t+1750)(50)}{[50(t+2)]^2} = \frac{50[(t+2) - (t+1750)]}{2500(t+2)^2} = \frac{-1748}{50(t+2)^2} = \frac{-874}{25(t+2)^2}$

(a) When $t = 1$, $\frac{dP}{dt} = \frac{-874}{225} \approx -3.88$ percent/day.

(b) When $t = 10$, $\frac{dP}{dt} = \frac{-874}{3600} = -\frac{437}{1800} \approx -0.24$ percent/day.

60. (a) $P = ax^2 + bx + c$

When $x = 10$, $P = 50$: $50 = 100a + 10b + c$.

When $x = 12$, $P = 60$: $60 = 144a + 12b + c$.

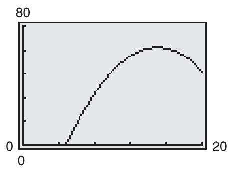
When $x = 14$, $P = 65$: $65 = 196a + 14b + c$.

Solving this system, we have

$a = -\frac{5}{8}$, $b = \frac{75}{4}$, and $c = -75$.

Thus, $P = -\frac{5}{8}x^2 + \frac{75}{4}x - 75$.

(b)



(c) Marginal profit: $P' = -\frac{5}{4}x + \frac{75}{4} = 0 \Rightarrow x = 15$

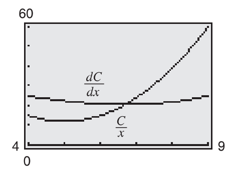
This is the maximum point on the graph of P.

62. $C = x^3 - 15x^2 + 87x - 73$, $4 \le x \le 9$

Marginal cost: $\frac{dC}{dx} = 3x^2 - 30x + 87$

Average cost: $\frac{C}{x} = x^2 - 15x + 87 - \frac{73}{x}$

(a)



(b) Point of intersection:

$$3x^2 - 30x + 87 = x^2 - 15x + 87 - \frac{73}{x}$$

$$2x^2 - 15x + \frac{73}{x} = 0$$

$$2x^3 - 15x^2 + 73 = 0$$

$$x \approx 6.683$$

When $x = 6.683$, $\frac{C}{x} = \frac{dC}{dx} \approx 20.50$.

Thus, the point of intersection is (6.683, 20.50). At this point average cost is at a minimum.

64. $M(t) = \frac{300t}{t^2 + 1} + 8$

(a) $M'(t) = \frac{(t^2+1)(300) - 300t(2t)}{(t^2+1)^2} = \frac{300(1 - t^2)}{(t^2+1)^2}$

(b) $M(3) = 98$

$M'(3) = -24$

(c) $M(24) \approx 20.48$

$M'(24) = -0.52$

66. Answers will vary.

Section 2.5 The Chain Rule

	$y = f(g(x))$	$u = g(x)$	$y = f(u)$
2.	$y = (x^2 - 2x + 3)^3$	$u = x^2 - 2x + 3$	$y = u^3$
4.	$y = (x^2 + 1)^{4/3}$	$u = x^2 + 1$	$y = u^{4/3}$
6.	$y = \sqrt{9 - x^2}$	$u = 9 - x^2$	$y = u^{1/2}$
8.	$y = (x + 1)^{-1/2}$	$u = x + 1$	$y = u^{-1/2}$

10. $f(x) = \dfrac{2x}{1 - x^3}$ Quotient Rule (d)

12. $f(x) = \sqrt[3]{x^2} = x^{2/3}$ Simple Power Rule (a)

14. (a) First rewrite $f(x) = x^{7/2} - 2x^{1/2} + x^{-1/2}$, then use the Simple Power Rule.

16. (c) Rewrite as $f(x) = 5(x^2 + 2)^{-1}$ and use the General Power Rule.

18. $y' = 4(3x^2 + 1)^3(6x) = 24x(3x^2 + 1)^3$

20. $h'(t) = 4(1 - t^2)^3(-2t) = -8t(1 - t^2)^3$

22. $f'(x) = 3(4x - x^2)^2(4 - 2x) = 6(2 - x)(4x - x^2)^2$

24. $f'(t) = \dfrac{2}{3}(9t + 2)^{-1/3}(9) = 6(9t + 2)^{-1/3} = \dfrac{6}{\sqrt[3]{9t + 2}}$

26. $g(x) = \sqrt{2x + 3} = (2x + 3)^{1/2}$

$$g'(x) = \frac{1}{2}(2x + 3)^{-1/2}(2) = \frac{1}{\sqrt{2x + 3}}$$

28. $y = \sqrt[3]{3x^3 + 4x} = (3x^3 + 4x)^{1/3}$

$$y' = \frac{1}{3}(3x^3 + 4x)^{-2/3}(9x^2 + 4) = \frac{9x^2 + 4}{3(3x^3 + 4x)^{2/3}}$$

30. $y = 2(4 - x^2)^{1/2}$

$$y' = 2\left(\frac{1}{2}\right)(4 - x^2)^{-1/2}(-2x) = \frac{-2x}{\sqrt{4 - x^2}}$$

32. $f(x) = (25 + x^2)^{-1/2}$

$$f'(x) = -\frac{1}{2}(25 + x^2)^{-3/2}(2x) = \frac{-x}{(25 + x^2)^{3/2}}$$

34. $f'(x) = -\dfrac{5}{2}(4 - 3x)^{-7/2}(-3) = \dfrac{15}{2(4 - 3x)^{7/2}}$

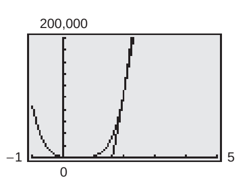
36. $g(x) = 3(9x - 4)^4$, $(2, 115{,}248)$

$$g'(x) = 12(9x - 4)^3(9) = 108(9x - 4)^3$$

$$g'(2) = 12(14)^3(9) = 296{,}352$$

$$y - 115{,}248 = 296{,}352(x - 2)$$

$$y = 296{,}352x - 477{,}456$$



38. $f(x) = x\sqrt{x^2 + 5} = x(x^2 + 5)^{1/2}$

Point: $(2, f(2)) = (2, 6)$

$$f'(x) = x\left[\frac{1}{2}(x^2 + 5)^{-1/2}(2x)\right] + (x^2 + 5)^{1/2}(1)$$

$$= x^2(x^2 + 5)^{-1/2} + (x^2 + 5)^{1/2}$$

$$= (x^2 + 5)^{-1/2}[x^2 + (x^2 + 5)]$$

$$= \frac{2x^2 + 5}{\sqrt{x^2 + 5}}$$

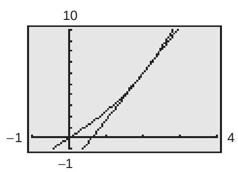
When $x = 2$, the slope is $f'(2) = \frac{13}{3}$ and the equation of the tangent line is

$$y - 6 = \frac{13}{3}(x - 2)$$

$$3y - 18 = 13x - 26$$

$$0 = 13x - 3y - 8$$

$$y = \frac{13}{3}x - \frac{8}{3}.$$



40. $g(x) = (4 - 3x^2)^{-2/3}$

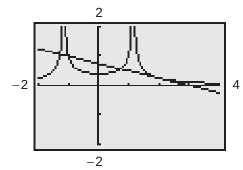
$$g(2) = (-8)^{-2/3} = \frac{1}{4}$$

$$g'(x) = -\frac{2}{3}(4 - 3x^2)^{-5/3}(-6x) = \frac{4x}{(4 - 3x^2)^{5/3}}$$

$$g'(2) = \frac{4(2)}{(-8)^{5/3}} = \frac{8}{-32} = -\frac{1}{4}$$

$$y - \frac{1}{4} = -\frac{1}{4}(x - 2)$$

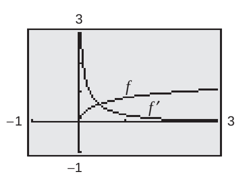
$$y = -\frac{1}{4}x + \frac{3}{4}$$



42. $f(x) = \sqrt{\dfrac{2x}{x + 2}}$

$$f'(x) = \frac{\sqrt{2}}{2\sqrt{x}(x + 1)^{3/2}}$$

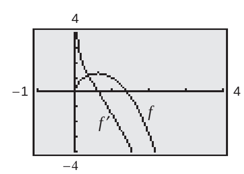
f' is never 0.



44. $f(x) = \sqrt{x}(2 - x^2)$

$$f'(x) = \frac{2 - 5x^2}{2\sqrt{x}}$$

f has a horizontal tangent when $f' = 0$.



46. $s(t) = \dfrac{1}{t^2 + 3t - 1} = (t^2 + 3t - 1)^{-1}$

$$s'(t) = -1(t^2 + 3t - 1)^{-2}(2t + 3)$$

$$= -\frac{2t + 3}{(t^2 + 3t - 1)^2}$$

48. $f(x) = \dfrac{3}{(x^3 - 4)^2} = 3(x^3 - 4)^{-2}$

$$f'(x) = 3(-2)(x^3 - 4)^{-3}(3x^2) = \frac{-18x^2}{(x^3 - 4)^3}$$

50. $y = \dfrac{1}{\sqrt{x + 2}} = (x + 2)^{-1/2}$

$$y' = -\frac{1}{2}(x + 2)^{-3/2} = \frac{-1}{2(x + 2)^{3/2}}$$

52. $g(x) = \dfrac{3}{\sqrt[3]{x^3 - 1}} = 3(x^3 - 1)^{-1/3}$

$$g'(x) = 3\left(-\frac{1}{3}\right)(x^3 - 1)^{-4/3}(3x^2) = \frac{-3x^2}{(x^3 - 1)^{4/3}}$$

54. $f(x) = x^3(x-4)^2$

$= x^3(x^2 - 8x + 16)$

$= x^5 - 8x^4 + 16x^3$

$f'(x) = 5x^4 - 32x^3 + 48x^2$

$= x^2(5x^2 - 32x + 48)$

$= x^2(5x - 12)(x - 4)$

56. $y = t\sqrt{t+1} = t(t+1)^{1/2}$

$y' = t\left[\frac{1}{2}(t+1)^{-1/2}(1)\right] + (t+1)^{1/2}(1)$

$= \frac{1}{2}t(t+1)^{-1/2} + (t+1)^{1/2}$

$= \frac{1}{2}(t+1)^{-1/2}[t + 2(t+1)]$

$= \frac{1}{2}(t+1)^{-1/2}(3t+2)$

$= \dfrac{3t+2}{2\sqrt{t+1}}$

58. $y = \sqrt{x}(x-2)^2 = x^{1/2}(x-2)^2$

$y' = x^{1/2}[2(x-2)^1(1)] + (x-2)^2\left(\frac{1}{2}x^{-1/2}\right)$

$= 2\sqrt{x}(x-2) + \dfrac{(x-2)^2}{2\sqrt{x}}$

$= \dfrac{4x(x-2) + (x-2)^2}{2\sqrt{x}}$

$= \dfrac{(x-2)[4x + (x-2)]}{2\sqrt{x}}$

$= \dfrac{(x-2)(5x-2)}{2\sqrt{x}}$

60. $g(t) = \dfrac{3t^2}{\sqrt{t^2+2t-1}}$

$g'(t) = \dfrac{(t^2+2t-1)^{1/2}(6t) - 3t^2\left(\frac{1}{2}\right)(t^2+2t-1)^{-1/2}(2t+2)}{t^2+2t-1}$

$= \dfrac{(t^2+2t-1)^{-1/2}[(t^2+2t-1)(6t) - 3t^2(t+1)]}{(t^2+2t-1)}$

$= \dfrac{6t^3 + 12t^2 - 6t - 3t^3 - 3t^2}{(t^2+2t-1)^{3/2}}$

$= \dfrac{3t^3 + 9t^2 - 6t}{(t^2+2t-1)^{3/2}}$

$= \dfrac{3t(t^2+3t-2)}{(t^2+2t-1)^{3/2}}$

62. $g(x) = \sqrt{x-1} + \sqrt{x+1} = (x-1)^{1/2} + (x+1)^{1/2}$

$g'(x) = \frac{1}{2}(x-1)^{-1/2}(1) + \frac{1}{2}(x+1)^{-1/2}(1)$

$= \dfrac{1}{2\sqrt{x-1}} + \dfrac{1}{2\sqrt{x+1}}$

64. $y = \left(\dfrac{4x^2}{3-x}\right)^3 = 64\left[\dfrac{x^6}{(3-x)^3}\right]$

$y' = 64\left[\dfrac{(3-x)^3 6x^5 - x^6(3)(3-x)^2(-1)}{(3-x)^6}\right]$

$= 64\left\{\dfrac{x^5[6(3-x) + 3x]}{(3-x)^4}\right\} = 192\left[\dfrac{x^5(6-x)}{(3-x)^4}\right]$

66. $s(x) = (x^2 - 3x + 4)^{-1/2}, \quad \left(3, \frac{1}{2}\right)$

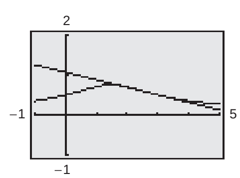
$s'(x) = -\frac{1}{2}(x^2 - 3x + 4)^{-3/2}(2x - 3)$

$= \dfrac{3 - 2x}{2(x^2 - 3x + 4)^{3/2}}$

$s'(3) = \dfrac{3-6}{2(4)^{3/2}} = -\dfrac{3}{16}$

$y - \frac{1}{2} = -\frac{3}{16}(x - 3)$

$y = -\frac{3}{16}x + \frac{17}{16}$



68. $y = \dfrac{2x}{\sqrt{x+1}}$

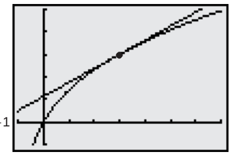
$y' = \dfrac{\sqrt{x+1}(2) - 2x\frac{1}{2}(x+1)^{-1/2}}{x+1}$

$= \dfrac{2(x+1) - x}{(x+1)^{3/2}} = \dfrac{x+2}{(x+1)^{3/2}}$

$y'(3) = \dfrac{5}{8}$

$y - 3 = \dfrac{5}{8}(x-3)$

$y = \dfrac{5}{8}x + \dfrac{9}{8}$



70. $y = \dfrac{x}{\sqrt{25+x^2}} = x(25+x^2)^{-1/2}$

$y' = x\left[-\dfrac{1}{2}(25+x^2)^{-3/2}(2x)\right] + (25+x^2)^{-1/2}(1)$

$= -x^2(25+x^2)^{-3/2} + (25+x^2)^{-1/2}$

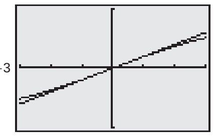
$= (25+x^2)^{-3/2}[-x^2 + (25+x^2)]$

$= \dfrac{25}{(25+x^2)^{3/2}}$

$y'(0) = \dfrac{1}{5}$

$y - 0 = \dfrac{1}{5}(x-0)$

$y = \dfrac{1}{5}x$



72. $P = 0.25(0.5n^2 + 5n + 25)^{1/2}$

$P'(n) = \dfrac{1}{4}\left(\dfrac{1}{2}\right)(0.5n^2 + 5n + 25)^{-1/2}(n+5)$

$P'(12{,}000) = \dfrac{12{,}000 + 5}{8[0.5(12{,}000)^2 + 5(12{,}000) + 25]^{1/2}}$

≈ 0.177

74. (a) $V = \dfrac{k}{\sqrt{t+1}}$

When $t = 0$, $V = 10{,}000$.

$10{,}000 = \dfrac{k}{\sqrt{0+1}} \Longrightarrow k = 10{,}000$

$V = \dfrac{10{,}000}{\sqrt{t+1}}$

(b) $V = 10{,}000(t+1)^{-1/2}$

$\dfrac{dV}{dt} = -5000(t+1)^{-3/2}(1) = -\dfrac{5000}{(t+1)^{3/2}}$

When $t = 1$,

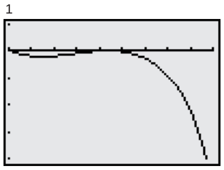
$\dfrac{dV}{dt} = -\dfrac{5000}{(2)^{3/2}} = -\dfrac{2500}{\sqrt{2}} \approx -\$1767.77$ per year.

(c) When $t = 3$, $\dfrac{dV}{dt} = -\dfrac{5000}{(4)^{3/2}} = -\$625.00$ per year.

76. (a) Using the General Power Rule,

$r'(t) = \frac{1}{2}(-0.14239t^4 + 3.939t^3 - 39.0835t^2 + 161.0373t + 22.13)^{-1/2}(-0.56956t^3 + 11.817t^2 - 78.167t + 161.0373)$

(b)



(c) $|r(t)|$ is changing most rapidly near $t = 12$.

(d) $|r(t)|$ is changing the least when $r'(t) = 0$, near $t = 4$ and $t = 9$.

78. True.

Section 2.6 Higher-Order Derivatives

2. $f'(x) = 3$

$f''(x) = 0$

4. $f'(x) = 6x + 4$

$f''(x) = 6$

6. $f(x) = 4(x^2 - 1)^2 = 4(x^4 - 2x^2 + 1)$

$f'(x) = 4(4x^3 - 4x)$

$f''(x) = 4(12x^2 - 4) = 16(3x^2 - 1)$

8. $g(t) = t^{-1/3}$

$g'(t) = -\frac{1}{3}t^{-4/3}$

$g''(t) = \frac{4}{9}t^{-7/3} = \frac{4}{9t^{7/3}}$

10. $f(x) = x\sqrt[3]{x} = x^{4/3}$

$f'(x) = \frac{4}{3}x^{1/3}$

$f''(x) = \frac{4}{9}x^{-2/3} = \frac{4}{9x^{2/3}}$

12. $g(t) = -\frac{4}{(t + 2)^2} = -4(t + 2)^{-2}$

$g'(t) = 8(t + 2)^{-3}$

$g''(t) = -24(t + 2)^{-4} = -\frac{24}{(t + 2)^4}$

14. $h(s) = s^3(s^2 - 2s + 1) = s^5 - 2s^4 + s^3$

$h'(s) = 5s^4 - 8s^3 + 3s^2$

$h''(s) = 20s^3 - 24s^2 + 6s = 2s(10s^2 - 12s + 3)$

16. $f(x) = x^4 - 2x^3$

$f'(x) = 4x^3 - 6x^2$

$f''(x) = 12x^2 - 12x$

$f'''(x) = 24x - 12 = 12(2x - 1)$

18. $f(x) = (x - 1)^2$

$f'(x) = 2(x - 1)$

$f''(x) = 2$

$f'''(x) = 0$

20. $f(x) = \frac{1}{x} = x^{-1}$

$f'(x) = -x^{-2}$

$f''(x) = 2x^{-3}$

$f'''(x) = -6x^{-4} = -\frac{6}{x^4}$

22. $f(x) = 9 - x^2$

$f'(x) = -2x$

$f''(x) = -2$

$f''(-\sqrt{5}) = -2$

24. $f(t) = \sqrt{2t + 3} = (2t + 3)^{1/2}$

$f'(t) = \frac{1}{2}(2t + 3)^{-1/2}(2) = (2t + 3)^{-1/2}$

$f''(t) = -\frac{1}{2}(2t + 3)^{-3/2}(2) = -(2t + 3)^{-3/2}$

$f'''(t) = \frac{3}{2}(2t + 3)^{-5/2}(2) = \frac{3}{(2t + 3)^{5/2}}$

$f'''\left(\frac{1}{2}\right) = \frac{3}{32}$

26. $g(x) = 2x^5 - 10x^4 + 8x^3$

$g'(x) = 10x^4 - 40x^3 + 24x^2$

$g''(x) = 40x^3 - 120x^2 + 48x$

$g'''(x) = 120x^2 - 240x + 48$

$g'''(0) = 48$

28. $f''(x) = 20x^3 - 36x^2$

$f'''(x) = 60x^2 - 72x = 12x(5x - 6)$

30. $f'''(x) = 2\sqrt{x-1} = 2(x-1)^{1/2}$

$f^{(4)}(x) = 2\left(\frac{1}{2}\right)(x-1)^{-1/2}(1) = \frac{1}{\sqrt{x-1}}$

32. $f(x) = x^3 - 2x$

$f'(x) = 3x^2 - 2$

$f''(x) = 6x$

34. $f(x) = 3x^3 - 9x + 1$

$f'(x) = 9x^2 - 9$

$f''(x) = 18x = 0$

$f''(x) = 0$ when $x = 0$.

36. $f(x) = (x+2)(x-2)(x+3)(x-3)$

$= (x^2-4)(x^2-9)$

$= x^4 - 13x^2 + 36$

$f'(x) = 4x^3 - 26x$

$f''(x) = 12x^2 - 26$

$f''(x) = 0 \implies 12x^2 = 26$

$x = \pm\sqrt{\frac{13}{6}} = \pm\frac{\sqrt{78}}{6}$

38. $f(x) = x\sqrt{4-x^2} = x(4-x^2)^{1/2}$

$f'(x) = x\left[\frac{1}{2}(4-x^2)^{-1/2}(-2x)\right] + (4-x^2)^{1/2}(1)$

$= -x^2(4-x^2)^{-1/2} + (4-x^2)^{1/2}$

$= (4-x^2)^{-1/2}[-x^2 + (4-x^2)]$

$= (4-2x^2)(4-x^2)^{-1/2}$

$f''(x) = (4-2x^2)\left[-\frac{1}{2}(4-x^2)^{-3/2}(-2x)\right] + (4-x^2)^{-1/2}(-4x)$

$= 2x(2-x^2)(4-x^2)^{-3/2} - 4x(4-x^2)^{-1/2}$

$= 2x(4-x^2)^{-3/2}[(2-x^2) - 2(4-x^2)]$

$= 2x(4-x^2)^{-3/2}(x^2-6) = \frac{2x(x^2-6)}{(4-x^2)^{3/2}} = 0$

$f''(x) = 0$ when $x = 0$.

[**Note**: $x = \pm\sqrt{6}$ are not in the domain of f.]

40. $f(x) = \frac{x}{x^2+1}$

$f'(x) = \frac{(x^2+1)(1) - (x)(2x)}{(x^2+1)^2}$

$= \frac{1-x^2}{(x^2+1)^2}$

$= (1-x^2)(x^2+1)^{-2}$

$f''(x) = (1-x^2)[-2(x^2+1)^{-3}(2x)] + (x^2+1)^{-2}(-2x)$

$= -2x(x^2+1)^{-3}[2(1-x^2) + (x^2+1)]$

$= -2x(x^2+1)^{-3}(3-x^2)$

$= \frac{2x(x^2-3)}{(x^2+1)^3} = 0$

$f''(x) = 0$ when $x = 0, \pm\sqrt{3}$.

42. (a) $s(t) = -16t^2 + 1250$

(b) $v(t) = s'(t) = -32t$

$a(t) = v'(t) = -32$

(c) $s(t) = 0$ when $16t^2 = 1250$, or $t = \sqrt{78.125} \approx 8.8$ sec.

(d) $v(8.8) \approx -282.8$ ft/sec

44. $s(t) = -8.25t^2 + 66t$

$v(t) = s'(t) = -16.50t + 66$

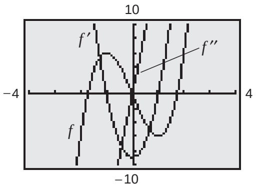
$a(t) = s''(t) = -16.50$

t	0	1	2	3	4	5
$s(t)$	0	57.75	99	123.75	132	123.75
$v(t)$	66	49.50	33	16.50	0	−16.50
$a(t)$	−16.50	−16.50	−16.50	−16.50	−16.50	−16.50

46. $f(x) = 3x^3 - 9x$

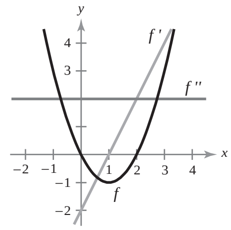
$f'(x) = 9x^2 - 9$

$f''(x) = 18x$



The degrees of the successive derivatives decrease by 1.

48.



The degrees of the successive derivatives decrease by 1.

50. (a) $s(t) = -16t^2 + 48t + 64$

(b) $v(t) = s'(t) = -32t + 48$

$a(t) = v'(t) = -32$

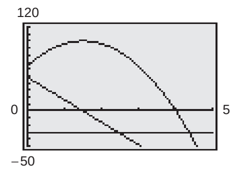
(c) $s(t) = -16t^2 + 48t + 64 = 0 \Rightarrow -16(t^2 - 3t + 4)$

$= 0 \Rightarrow (t - 4)(t + 1) = 0 \Rightarrow t = 4$ sec

(d) $v(t) = -32t + 48 = 0$ when $32t = 48$ or $t = \frac{3}{2}$ sec.

(e) $s\left(\frac{3}{2}\right) = -16\left(\frac{3}{2}\right)^2 + 48\left(\frac{3}{2}\right) + 64 = 100$ feet high

(f)



The position function is a quadratic function, the velocity function is a linear function, and the acceleration function is a constant function.

52. True. The fifth derivative of a fourth degree polynomial is 0.

54. True. The $(n + 1)^{\text{st}}$ derivative of an n^{th} degree polynomial is 0.

56. True. $\dfrac{dC}{dt} = 0$ where $v(t) = C$ is constant.

Section 2.7 Implicit Differentiation

2. $\frac{1}{2}x^2 - y = 6x$

$x - y' = 6$

$y' = x - 6$

4. $4x^2y - 3y^{-1} = 0$

$4x^2y' + 8xy + 3y^{-2}y' = 0$

$\left(4x^2 + \dfrac{3}{y^2}\right)y' = -8xy$

$\dfrac{4x^2y^2 + 3}{y^2}y' = -8xy$

$y' = \dfrac{-8xy \cdot y^2}{4x^2y^2 + 3} = \dfrac{-8xy^3}{4x^2y^2 + 3}$

6. $xy^2 + 4xy = 10$

$y^2 + 2xyy' + 4y + 4xy' = 0$

$(2xy + 4x)y' = -y^2 - 4y$

$y' = -\dfrac{y^2 + 4y}{2xy + 4x}$

8. $2xy^3 - x^2y = 2$

$2y^3 + 6xy^2y' - 2xy - x^2y' = 0$

$(6xy^2 - x^2)y' = 2xy - 2y^3$

$y' = \dfrac{2xy - 2y^3}{6xy^2 - x^2}$

10. $\dfrac{xy - y^2}{y - x} = 1$

$xy - y^2 = y - x$

$y(x - y) = -(x - y)$

$y = -1$

$y' = 0$

12. $\dfrac{2x+y}{x-5y} = 1$

$$2x + y = x - 5y$$
$$6y = -x$$
$$y = -\frac{1}{6}x$$
$$y' = -\frac{1}{6}$$

14. $x^2 - y^2 = 16$

$$2x - 2yy' = 0$$
$$y' = \frac{x}{y}$$

At $(4, 0)$, y' is undefined.

16. $x^2 - y^3 = 3$

$$2x - 3y^2y' = 0$$
$$y' = \frac{2x}{3y^2}$$

At $(2, 1)$, $y' = \dfrac{2(2)}{3(1)} = \dfrac{4}{3}$.

18. $x^2y + y^2x = -2$

$$x^2y' + 2xy + y^2 + 2yy'x = 0$$
$$y'(x^2 + 2xy) = -2xy - y^2$$
$$y' = -\frac{y(2x+y)}{x(x+2y)}$$

At $(2, -1)$, y' is undefined.

20. $x^3 + y^3 = 2xy$

$$3x^2 + 3y^2y' = 2xy' + 2y$$
$$y'(3y^2 - 2x) = 2y - 3x^2$$
$$y' = \frac{2y - 3x^2}{3y^2 - 2x}$$

At $(1, 1)$, $y' = -1$.

22. $\sqrt{xy} = x - 2y$

$$\sqrt{x}\sqrt{y} = x - 2y$$
$$\sqrt{x}\left(\frac{1}{2}y^{-1/2}y'\right) + \sqrt{y}\left(\frac{1}{2}x^{-1/2}\right) = 1 - 2y'$$
$$\frac{\sqrt{x}}{2\sqrt{y}}y' + 2y' = 1 - \frac{\sqrt{y}}{2\sqrt{x}}$$
$$y' = \frac{1 - \dfrac{\sqrt{y}}{2\sqrt{x}}}{\dfrac{\sqrt{x}}{2\sqrt{y}} + 2} \cdot \frac{2\sqrt{x}\sqrt{y}}{2\sqrt{x}\sqrt{y}}$$
$$= \frac{2\sqrt{xy} - y}{x + 4\sqrt{xy}}$$
$$= \frac{2(x - 2y) - y}{x + 4(x - 2y)}$$
$$= \frac{2x - 5y}{5x - 8y}$$

At $(4, 1)$, $y' = \frac{1}{4}$.

24. $(x + y)^3 = x^3 + y^3$

$$3(x + y)^2(1 + y') = 3x^2 + 3y^2y'$$
$$3(x + y)^2 + 3(x + y)^2y' = 3x^2 + 3y^2y'$$
$$(x + y)^2y' - y^2y' = x^2 - (x + y)^2$$
$$y'[(x + y)^2 - y^2] = x^2 - (x^2 + 2xy + y^2)$$
$$y' = \frac{-(2xy + y^2)}{x^2 + 2xy} = -\frac{y(2x + y)}{x(x + 2y)}$$

At $(-1, 1)$, $y' = -1$.

26. $4x^2 + 2y - 1 = 0$

$$8x + 2y' = 0$$
$$y' = -\frac{8x}{2} = -4x$$
$$y'(-1) = -4(-1) = 4$$

28. $4x^2 + y^2 = 4$

$$8x + 2yy' = 0$$
$$yy' = -4x$$
$$y' = -4\frac{x}{y}$$

At $(0, -2)$, $y' = 0$.

30. $x^2 - y^3 = 0$

$$2x - 3y^2y' = 0$$
$$y' = \frac{2x}{3y^2}$$

At $(-1, 1)$, $y' = -\frac{2}{3}$.

32. Implicitly: $18x + 32y\dfrac{dy}{dx} = 0$

$$\frac{dy}{dx} = -\frac{9x}{16y}$$

Explicitly: $y = \pm\left(\frac{1}{4}\right)\sqrt{144 - 9x^2}$

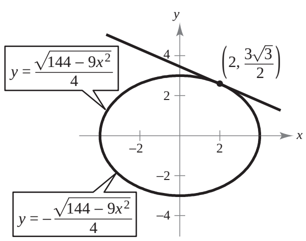
$$\frac{dy}{dx} = \pm\left(\frac{1}{8}\right)(144 - 9x^2)^{-1/2}(-18x)$$

$$= \pm\frac{-9x}{4\sqrt{144 - 9x^2}}$$

$$= -\frac{9x}{16\left[\pm(1/4)\sqrt{144 - 9x^2}\right]}$$

$$= -\frac{9x}{16y}$$

At $\left(2, \frac{3\sqrt{3}}{2}\right)$, $\dfrac{dy}{dx} = -\dfrac{\sqrt{3}}{4}$.



34. Implicitly: $8yy' - 2x = 0$

$$y' = \frac{x}{4y}$$

Explicitly: $y = \pm\frac{1}{2}\sqrt{x^2 + 7}$

$$= \pm\frac{1}{2}(x^2 + 7)^{1/2}$$

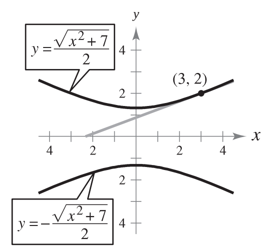
$$y' = \pm\frac{1}{4}(x^2 + 7)^{-1/2}(2x)$$

$$= \pm\frac{x}{2\sqrt{x^2 + 7}}$$

$$= \frac{x}{4\left(\pm\frac{1}{2}\sqrt{x^2 + 7}\right)}$$

$$= \frac{x}{4y}$$

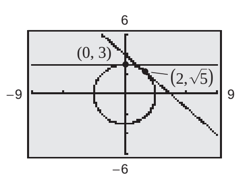
At $(3, 2)$, $y' = \frac{3}{8}$.



36. $x^2 + y^2 = 9$

$2x + 2yy' = 0$

$$y' = -\frac{x}{y}$$



At $(0, 3)$:

$$m = 0$$

$$y - 3 = 0(x - 0)$$

$$y - 3 = 0$$

At $\left(2, \sqrt{5}\right)$:

$$m = -\frac{2}{\sqrt{5}}$$

$$y - \sqrt{5} = -\frac{2}{\sqrt{5}}(x - 2)$$

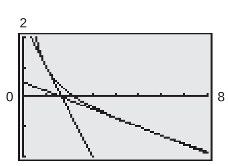
$$\sqrt{5}y - 5 = -2x + 4$$

$$2x + \sqrt{5}y - 9 = 0$$

38. $4xy + x^2 = 5$

$4xy' + 4y + 2x = 0$

$$y' = -\frac{4y + 2x}{4x}$$



At $(1, 1)$:

$$y' = -\frac{6}{4} = -\frac{3}{2}$$

$$y - 1 = -\frac{3}{2}(x - 1)$$

$$y = -\frac{3}{2}x + \frac{5}{2}$$

At $(5, -1)$:

$$y' = -\frac{-4 + 10}{20} = -\frac{3}{10}$$

$$y + 1 = -\frac{3}{10}(x - 5)$$

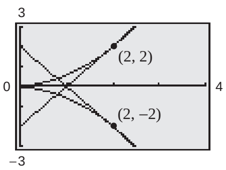
$$y = -\frac{3}{10}x + \frac{1}{2}$$

40. $y^2 = \dfrac{x^3}{4 - x}$

$2yy' = \dfrac{(4 - x)(3x^2) - (x^3)(-1)}{(4 - x)^2}$

$2yy' = \dfrac{2x^2(6 - x)}{(4 - x)^2}$

$y' = \dfrac{x^2(6 - x)}{y(4 - x)^2}$



At (2, 2):

$m = 2$

$y - 2 = 2(x - 2)$

$y - 2 = 2x - 4$

$0 = 2x - y - 2$

At (2, −2):

$m = -2$

$y + 2 = -2(x - 2)$

$y + 2 = -2x + 4$

$2x + y - 2 = 0$

42. $p = 0.002x^4 + 0.01x^2 + 5, \quad x \geq 0$

$1 = 0.008x^3 \dfrac{dx}{dp} + 0.02x \dfrac{dx}{dp}$

$\dfrac{dx}{dp} = \dfrac{1}{0.008x^3 + 0.02x}$

44. $p = \sqrt{\dfrac{500 - x}{2x}}, \quad 0 < x \leq 500$

$p^2 = \dfrac{500 - x}{2x}$

$2xp^2 = 500 - x$

$2x(2p) + p^2\left(2\dfrac{dx}{dp}\right) = -\dfrac{dx}{dp}$

$2p^2\dfrac{dx}{dp} + \dfrac{dx}{dp} = -4xp$

$\dfrac{dx}{dp}(2p^2 + 1) = -4xp$

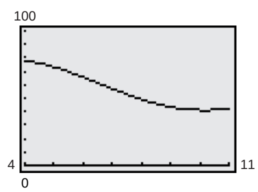
$\dfrac{dx}{dp} = -\dfrac{4xp}{2p^2 + 1}$

46. $y^2 + 4436 = -4.2460t^4 + 146.821t^3 - 1728.0t^2 + 7456.6t$

$y = [-4.246t^4 + 146.821t^3 - 1728.0t^2 + 7456.6t - 4436]^{1/2}$

$4 \leq t \leq 11$ ($t = 4$ corresponds to 1994.)

(a)



The number of cases decrease until around 2000 ($t = 10$).

(b) The graph seems to decrease most rapidly around $t \approx 6.7$, or during 1996.

(c)

t	4	5	6	7	8	9	10	11
y	77.8	73.1	65.6	57.0	49.0	43.4	41.4	41.9
y'	−2.6	−6.4	−8.3	−8.6	−7.1	−4.1	−0.5	1.6

Section 2.8 Related Rates

2. $y = x^2 - 3x,\ \dfrac{dy}{dt} = 2x\dfrac{dx}{dt} - 3\dfrac{dx}{dt},\ \dfrac{dy}{dt} = (2x-3)\dfrac{dx}{dt},\ \dfrac{dx}{dt} = \left(\dfrac{1}{2x-3}\right)\dfrac{dy}{dt}$

(a) When $x = 3$ and $\dfrac{dx}{dt} = 2,\ \dfrac{dy}{dt} = [2(3) - 3](2) = 6.$

(b) When $x = 1$ and $\dfrac{dy}{dt} = 5,\ \dfrac{dx}{dt} = \left(\dfrac{1}{2(1) - 3}\right)(5) = -5.$

4. $x^2 + y^2 = 25,\ 2x\dfrac{dx}{dt} + 2y\dfrac{dy}{dt} = 0,\ \dfrac{dy}{dt} = -\dfrac{x}{y}\dfrac{dx}{dt},\ \dfrac{dx}{dt} = -\dfrac{y}{x}\dfrac{dy}{dt}$

(a) When $x = 3$, $y = 4$, and $\dfrac{dx}{dt} = 8,\ \dfrac{dy}{dt} = -\dfrac{3}{4}(8) = -6.$

(b) When $x = 4$, $y = 3$, and $\dfrac{dy}{dt} = -2,\ \dfrac{dx}{dt} = -\dfrac{3}{4}(-2) = \dfrac{3}{2}.$

6. $V = \dfrac{4}{3}\pi r^3,\ \dfrac{dr}{dt} = 2,\ \dfrac{dV}{dt} = 4\pi r^2\dfrac{dr}{dt} = 8\pi r^2$

(a) When $r = 6,\ \dfrac{dV}{dt} = 8\pi(6)^2 = 288\pi$ in³/min.

(b) When $r = 24,\ \dfrac{dV}{dt} = 8\pi(24)^2 = 4608\pi$ in³/min.

8. $V = \dfrac{4}{3}\pi r^3,\ \dfrac{dV}{dt} = 4\pi r^2\dfrac{dr}{dt}$

If dr/dt is constant, dV/dt is *not* constant since it is proportional to the square of r.

10. $V = \dfrac{1}{3}\pi r^2 h = \dfrac{1}{3}\pi r^2(3r) = \pi r^3$ [since $h = 3r$]

$\dfrac{dV}{dt} = 3\pi r^2\dfrac{dr}{dt} = 6\pi r^2$ [since $dr/dt = 2$]

(a) When $r = 6,\ \dfrac{dV}{dt} = 6\pi(6)^2 = 216\pi$ in³/min.

(b) When $r = 24,\ \dfrac{dV}{dt} = 6\pi(24)^2 = 3456\pi$ in³/min.

12. $C = 75{,}000 + 1.05x,\ R = 500x - \dfrac{x^2}{25}$

(a) $\dfrac{dC}{dt} = 1.05\dfrac{dx}{dt} = 1.05(250) = 262.5$ dollars/week

(b) $\dfrac{dR}{dt} = \left(500 - \dfrac{2x}{25}\right)\dfrac{dx}{dt} = \left(500 - \dfrac{2(5000)}{25}\right)(250)$

$= 25{,}000$ dollars/week

(c) $P = R - C$

$\dfrac{dP}{dt} = \dfrac{dR}{dt} - \dfrac{dC}{dt} = 25{,}000 - 262.5$

$= 24{,}737.5$ dollars/week

14. $A = 6x^2,\ \dfrac{dx}{dt} = 3,\ \dfrac{dA}{dt} = 12x\dfrac{dx}{dt} = 36x$

(a) When $x = 1,\ \dfrac{dA}{dt} = 36(1) = 36$ cm²/sec.

(b) When $x = 10,\ \dfrac{dA}{dt} = 36(10) = 360$ cm²/sec.

16. $y = \dfrac{1}{1 + x^2},\ \dfrac{dx}{dt} = 2,\ \dfrac{dy}{dt} = -\dfrac{2x}{(1 + x^2)^2}\dfrac{dx}{dt} = -\dfrac{4x}{(1 + x^2)^2}$

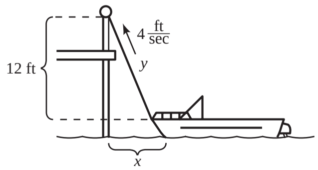
(a) When $x = -2,\ \dfrac{dx}{dt} = -\dfrac{4(-2)}{[1 + (-2)^2]^2} = \dfrac{8}{25}$ cm/min.

(b) When $x = 2,\ \dfrac{dx}{dt} = -\dfrac{4(2)}{[1 + (2)^2]^2} = -\dfrac{8}{25}$ cm/min.

(c) When $x = 0,\ \dfrac{dx}{dt} = -\dfrac{4(0)}{[1 + (0)^2]^2} = 0$ cm/min.

(d) When $x = 10,\ \dfrac{dx}{dt} = -\dfrac{4(10)}{[1 + (10)^2]^2} = -\dfrac{40}{10{,}201}$ cm/min.

18.



$12^2 + x^2 = y^2$

$$\frac{dy}{dt} = -4$$

$$2x\frac{dx}{dt} = 2y\frac{dy}{dt}$$

$$\frac{dx}{dt} = \frac{y}{x}\frac{dy}{dt}$$

When $y = 13$, $x = 5$ and $\frac{dx}{dt} = \frac{13}{5}(-4) = -10.4$ ft/sec.

As $x \to 0$, $\frac{dx}{dt}$ increases.

20. $x^2 + 6^2 = s^2$

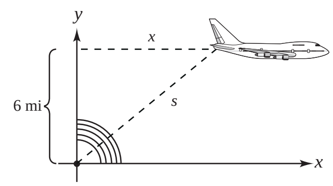
$$2x\frac{dx}{dt} = 2s\frac{ds}{dt}$$

$$\frac{dx}{dt} = \frac{s}{x}\frac{ds}{dt}$$

When $s = 10$, $x = 8$ and $\frac{ds}{dt} = -240$:

$$\frac{dx}{dt} = \frac{10}{8}(-240) = -300 \text{ mi/hr.}$$

The **speed** of the plane is 300 mi/hr.



22. $S = 2250 + 50x + 0.35x^2$

$$\frac{dS}{dt} = 50\frac{dx}{dt} + 0.70x\frac{dx}{dt}$$

$$\frac{dS}{dt} = 50(125) + 0.70(1500)(125) = \$137{,}500 \text{ per week}$$

24. $P = R - C$

$= xp - C$

$= x[50 - 0.01x] - (4000 + 40x - 0.02x^2)$

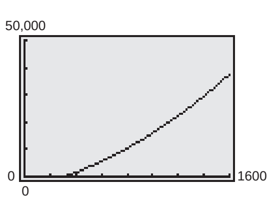
$= 50x - 0.01x^2 - 4000 - 40x + 0.02x^2$

$= 0.01x^2 + 10x - 4000$

$$\frac{dP}{dt} = 0.02x\frac{dx}{dt} + 10\frac{dx}{dt}$$

When $x = 800$ and $\frac{dx}{dt} = 25$,

$$\frac{dP}{dt} = 0.02(800)(25) + (10)(25) = \$650/\text{week.}$$



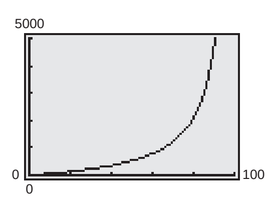
26. (a) $C = 528p(100 - p)^{-1}$ and $\frac{dp}{dt} = 5\%/\text{yr.}$

$$\frac{dC}{dt} = 528p(-1)(100 - p)^{-2}(-1)\frac{dp}{dt} + 528(100 - p)^{-1}\frac{dp}{dt}$$

$$= \frac{528p\frac{dp}{dt} + 528(100 - p)\frac{dp}{dt}}{(100 - p)^2}$$

$$\left.\frac{dC}{dt}\right|_{p=30} = \frac{528(30)(5) + 528(70)(5)}{70^2} = \$23.70/\text{yr}$$

(b) $\left.\frac{dC}{dt}\right|_{p=60} = \$165/\text{yr.}$



As $p \to 100^+$, C increases without bound.

Review Exercises for Chapter 2

2. Slope $\approx \frac{4}{2} = 2$

4. Slope $\approx \frac{-2}{4} = -\frac{1}{2}$

6. $t = 4$: slope $\approx$ 8000 thousand per year per year

$t = 8$: slope $\approx$ 13,500 thousand per year per year

$t = 8$: slope $\approx$ 27,500 thousand per year per year

8. (a) At t_1, $f'(t_1) > g'(t_1)$, so rafter f is traveling faster.

(b) At t_2, $g'(t_2) > f'(t_2)$, so rafter g is faster.

(c) At t_3, g is faster.

(d) g finishes first, because g completes 10 miles at a lesser value of t.

10.
$$\begin{aligned} f'(x) &= \lim_{\Delta x\to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x} \\ &= \lim_{\Delta x\to 0} \frac{7(x+\Delta x) + 3 - (7x+3)}{\Delta x} \\ &= \lim_{\Delta x\to 0} \frac{7\Delta x}{\Delta x} = 7 \end{aligned}$$
$$f'(-1) = 7$$

12.
$$\begin{aligned} f'(x) &= \lim_{\Delta x\to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x} \\ &= \lim_{\Delta x\to 0} \frac{(x+\Delta x)^2 + 10 - (x^2+10)}{\Delta x} \\ &= \lim_{\Delta x\to 0} \frac{2x\Delta x + (\Delta x)^2}{\Delta x} \\ &= \lim_{\Delta x\to 0} (2x + \Delta x) = 2x \end{aligned}$$
$$f'(-1) = -2$$

14.
$$\begin{aligned} f(x) &= \lim_{\Delta x\to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x} \\ &= \lim_{\Delta x\to 0} \frac{\sqrt{x+\Delta x-1} - \sqrt{x-1}}{\Delta x} \cdot \frac{\sqrt{x+\Delta x-1} + \sqrt{x-1}}{\sqrt{x+\Delta x-1} + \sqrt{x-1}} \\ &= \lim_{\Delta x\to 0} \frac{(x+\Delta x-1) - (x-1)}{\Delta x\left[\sqrt{x+\Delta x-1} + \sqrt{x-1}\right]} \\ &= \lim_{\Delta x\to 0} \frac{1}{\sqrt{x+\Delta x-1} + \sqrt{x-1}} = \frac{1}{2\sqrt{x-1}} \end{aligned}$$
$$f'(10) = \frac{1}{6}$$

16.
$$\begin{aligned} f'(x) &= \lim_{\Delta x\to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x} \\ &= \lim_{\Delta x\to 0} \frac{\dfrac{1}{x+\Delta x+4} - \dfrac{1}{x+4}}{\Delta x} \\ &= \lim_{\Delta x\to 0} \frac{(x+4) - (x+\Delta x+4)}{\Delta x[(x+\Delta x+4)(x+4)]} \\ &= \lim_{\Delta x\to 0} \frac{-1}{(x+\Delta x+4)(x+4)} \\ &= \frac{-1}{(x+4)^2} \end{aligned}$$
$$f'(-3) = -1$$

18. $f(x) = 2 - 3x$

$f'(x) = -3$

$f(1) = -3$

20. $f(x) = 4 - x^2$

$f'(x) = -2x$

$f'(-1) = 2$

22. $f(x) = 2x^{1/2} + 1$

$f'(x) = x^{-1/2} = \dfrac{1}{\sqrt{x}}$

$f'(4) = \dfrac{1}{2}$

24. $f(x) = 2x^{-1} - 1$

$f'(x) = -2x^{-2} = \dfrac{-2}{x^2}$

$f'\left(\dfrac{1}{2}\right) = \dfrac{-2}{(1/2)^2} = -8$

26. $y = -|x| + 3$ is not differentiable at $x = 0$.

28. y is not differentiable at $x = -1$.

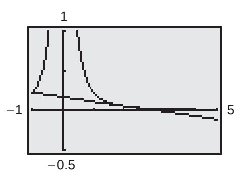
30. $h(x) = \dfrac{2}{(3x)^2} = \dfrac{2}{9}x^{-2}, \quad \left(2, \dfrac{1}{18}\right)$

$h'(x) = \dfrac{-4}{9}x^{-3} = \dfrac{-4}{9x^3}$

$h'(2) = -\dfrac{1}{18}$

$y - \dfrac{1}{18} = -\dfrac{1}{18}(x - 2)$

$y = -\dfrac{1}{18}x + \dfrac{1}{6}$



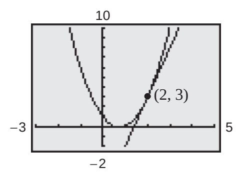
32. $f(x) = 2x^2 - 3x + 1, \quad (2, 3)$

$f'(x) = 4x - 3$

$f'(2) = 5$

$y - 3 = 5(x - 2)$

$y = 5x - 7$

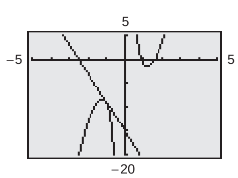


34. $y = x^3 - 5 + \dfrac{3}{x^3} = x^3 - 5 + 3x^{-3}, \quad y(-1) = -9$

$y' = 3x^2 + 3(-3x^{-4}) = 3x^2 - \dfrac{9}{x^4}, \quad y'(-1) = -6$

$y - (-9) = -6[x - (-1)] = -6x - 6$

$y = -6x - 15$



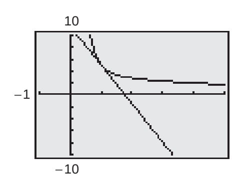
36. $f(x) = 2x^{-3} + 4 - \sqrt{x},$

$f'(x) = -6x^{-4} - \dfrac{1}{2\sqrt{x}}$

$f'(1) = -6 - \dfrac{1}{2} = -\dfrac{13}{2}$

$y - 5 = -\dfrac{13}{2}(x - 1)$

$y = -\dfrac{13}{2}x + \dfrac{23}{2}$



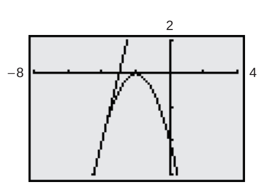
38. $f(x) = -x^2 - 4x - 4$

$f'(x) = -2x - 4$

$f'(-4) = 4$

$y + 4 = 4(x + 4)$

$y = 4x + 12$



40. $f(x) = x^3 + x, \quad [-2, 2]$

Average rate of change $= \dfrac{f(2) - f(-2)}{2 - (-2)}$

$= \dfrac{10 - (-10)}{4} = 5$

$f'(x) = 3x^2 + 1$

$f'(-2) = 13$

$f'(2) = 13$

42. (a) 1997 to 2002: $\dfrac{147{,}560 - 57{,}309}{12 - 7} \approx 18{,}050$ thousand per year per year

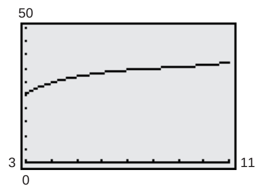
(b) 1997: $S'(7) \approx 11{,}867$ thousand per year per year

2002: $S'(7) \approx 27{,}456$ thousand per year per year

(c) Answers will vary.

44. $T = \sqrt{2.489t^3 - 62.062t^2 + 553.16t - 509.4}$

(a)



(b) $T'(t) = \frac{1}{2}(2.489t^3 - 62.062t^2 + 553.16t - 509.4)^{-1/2}(7.467t^2 - 124.124t + 553.16)$

1993: $T'(3) \approx 4.8$ million tons per year

1998: $T'(8) \approx 0.5$ million tons per year

2001: $T'(11) \approx 1.2$ million tons per year

(c) Yes, $T'(t) > 0$ for $t \geq 3$. The graph is increasing on $3 \leq t \leq 11$. The amount of recycled paper products is increasing.

46. $S = 2t^{3/2}, \quad 0 \leq t \leq 8$

$v = 3t^{1/2}$

t	0	2	4	6	8
v	0	4.24	6	7.35	8.49

48. $P = R - C = xp - C$

$= x(1.89 - 0.0083x) - (21 + 0.65x)$

$= -0.0083x^2 + 1.24x - 21$

50. $C = 225x + 4500$

$C'(x) = 225$

52. $\dfrac{dC}{dx} = 5.25\left(\dfrac{2}{3}x^{-1/3}\right) = \dfrac{3.5}{\sqrt[3]{x}}$

54. $R = 150x - \dfrac{3}{4}x^2$

$R'(x) = 150 - \dfrac{3}{2}x$

56. $R = x\left(5 + \dfrac{10}{\sqrt{x}}\right) = 5x + 10x^{1/2}$

$\dfrac{dR}{dx} = 5 + 5x^{-1/2} = 5 + \dfrac{5}{\sqrt{x}} = \dfrac{5\left(\sqrt{x} + 1\right)}{\sqrt{x}}$

58. $\dfrac{dP}{dx} = -\dfrac{1}{5}x^2 + 8000x - 120$

60. $y = (3x^2 + 7)(x^2 - 2x)$

$y' = (3x^2 + 7)(2x - 2) + (6x)(x^2 - 2x)$

$= 6x^3 + 14x - 6x^2 - 14 + 6x^3 - 12x^2$

$= 12x^3 - 18x^2 + 14x - 14$

62. $s = \left(4 - \dfrac{1}{t^2}\right)(t^2 - 3t) = 4t^2 - 12t - 1 + 3t^{-1}$

$s' = 8t - 12 - 3t^{-2} = 8t - 12 - \dfrac{3}{t^2}$

64. $f(x) = \dfrac{x^2 + x - 1}{x^2 - 1}$

$$f'(x) = \frac{(x^2 - 1)(2x + 1) - (x^2 + x - 1)(2x)}{(x^2 - 1)^2}$$

$$= \frac{2x^3 + x^2 - 2x - 1 - 2x^3 - 2x^2 + 2x}{(x^2 - 1)^2}$$

$$= \frac{-x^2 - 1}{(x^2 - 1)^2} = -\frac{x^2 + 1}{(x^2 - 1)^2}$$

66. $f(x) = \sqrt[3]{x^2 - 1} = (x^2 - 1)^{1/3}$

$$f'(x) = \frac{1}{3}(x^2 - 1)^{-2/3}(2x) = \frac{2x}{3(x^2 - 1)^{2/3}}$$

68. $g(x) = (x^6 - 12x^3 + 9)^{1/2}$

$$g'(x) = \frac{1}{2}(x^6 - 12x^3 + 9)^{-1/2}(6x^5 - 36x^2)$$

$$= \frac{3x^5 - 18x^2}{\sqrt{x^6 - 12x^3 + 9}}$$

70. $g(t) = \dfrac{t}{(1 - t)^3}$

$$g'(t) = \frac{(1 - t)^3(1) - t(3)(1 - t)^2(-1)}{(1 - t)^6}$$

$$= \frac{(1 - t)^3 + 3t(1 - t)^2}{(1 - t)^6}$$

$$= \frac{(1 - t) + 3t}{(1 - t)^4} = \frac{2t + 1}{(1 - t)^4}$$

72. $f(x) = \left(x^2 + \dfrac{1}{x}\right)^5 = (x^2 + x^{-1})^5$

$$f'(x) = 5(x^2 + x^{-1})^4(2x - x^{-2})$$

$$= 5\left(x^2 + \frac{1}{x}\right)^4\left(2x - \frac{1}{x^2}\right)$$

74. $f(x) = [(x - 2)(x + 4)]^2 = (x^2 + 2x - 8)^2$

$$f'(x) = 2(x^2 + 2x - 8)^1(2x + 2)$$

$$= 2(x - 2)(x + 4)(2)(x + 1)$$

$$= 4(x - 2)(x + 4)(x + 1)$$

76. $f(s) = s^3(s^2 - 1)^{5/2}$

$$f'(s) = s^3\left(\tfrac{5}{2}\right)(s^2 - 1)^{3/2}(2s) + 3s^2(s^2 - 1)^{5/2}$$

$$= s^2(s^2 - 1)^{3/2}[5s^2 + 3(s^2 - 1)]$$

$$= s^2(s^2 - 1)^{3/2}(8s^2 - 3)$$

78. $g(x) = \dfrac{(3x + 1)^2}{(x^2 + 1)^2}$

$$g'(x) = \frac{(x^2 + 1)^2 2(3x + 1)(3) - (3x + 1)^2 2(x^2 + 1)2x}{(x^2 + 1)^4}$$

$$= \frac{(3x + 1)[6(x^2 + 1) - (3x + 1)4x]}{(x^2 + 1)^3}$$

$$= \frac{(3x + 1)(-6x^2 - 4x + 6)}{(x^2 + 1)^3}$$

$$= \frac{-2(3x + 1)(3x^2 + 2x - 3)}{(x^2 + 1)^3}$$

80. When $L = 12$,

$$V = \frac{L}{16}(D - 4)^2 = \frac{12}{16}(D - 4)^2 = \frac{3}{4}(D - 4)^2$$

$$\frac{dV}{dD} = \frac{3}{2}(D - 4) = (1.5)(D - 4).$$

(a) When $D = 8$, $\dfrac{dV}{dD} = (1.5)(8 - 4) = 6$ board ft/in.

(b) When $D = 16$, $\dfrac{dV}{dD} = (1.5)(16 - 4) = 18$ board ft/in.

(c) When $D = 24$, $\dfrac{dV}{dD} = (1.5)(24 - 4) = 30$ board ft/in.

(d) When $D = 36$, $\dfrac{dV}{dD} = (1.5)(36 - 4) = 48$ board ft/in.

82. $f'(x) = 5x^4 - 6x^2 + 2x$

$$f''(x) = 20x^3 - 12x + 2 = 2(10x^3 - 6x + 1)$$

$$f'''(x) = 60x^2 - 12 = 12(5x^2 - 1)$$

84. $f(x) = \sqrt{x} = x^{1/2}$

$f'(x) = \frac{1}{2}x^{-1/2}$

$f''(x) = -\frac{1}{4}x^{-3/2}$

$f'''(x) = \frac{3}{8}x^{-5/2}$

$f^{(4)}(x) = \frac{-15}{16}x^{-7/2} = \frac{-15}{16x^{7/2}}$

86. $f(x) = x^2 + 3x^{-1}$

$f'(x) = 2x - 3x^{-2}$

$f''(x) = 2 + 6x^{-3} = 2 + \frac{6}{x^3}$

88. $f'''(x) = 20x^4 - 2x^{-3}$

$f^{(4)}(x) = 80x^3 + 6x^{-4}$

$f^{(5)}(x) = 240x^2 - 24x^{-5}$

$= 240x^2 - \frac{24}{x^5}$

90. $s(t) = \frac{1}{t^2 + 2t + 1} = (t + 1)^{-2}$

$v(t) = s'(t) = -2(t + 1)^{-3} = -\frac{2}{(t + 1)^3}$

$a(t) = s''(t) = 6(t + 1)^{-4} = \frac{6}{(t + 1)^4}$

92. $x^2 + 9xy + y^2 = 0$

$2x + 9y + 9xy' + 2yy' = 0$

$(9x + 2y)y' = -2x - 9y$

$y' = \frac{-2x - 9y}{9x + 2y}$

94. $y^2 + x^2 - 6y - 2x - 5 = 0$

$2yy' + 2x - 6y' - 2 = 0$

$y'(2y - 6) = 2 - 2x$

$y' = \frac{2 - 2x}{2y - 6} = \frac{1 - x}{y - 3}$

96. $2x^{1/3} + 3y^{1/2} = 10$

$\frac{2}{3}x^{-2/3} + \frac{3}{2}y^{-1/2}y' = 0$

$y' = -\frac{4y^{1/2}}{9x^{2/3}}$

At (8, 4),

$\frac{2}{3}\left(\frac{1}{4}\right) + \frac{3}{2}\left(\frac{1}{2}\right)y' = 0$

$\frac{3}{4}y' = -\frac{1}{6}$

$y' = -\frac{2}{9}.$

$y - 4 = -\frac{2}{9}(x - 8)$

$y = -\frac{2}{9}x + \frac{52}{9}$

98. $y^3 - 2x^2y + 3xy^2 = -1$

$3y^2y' - 2x^2y' - 4xy + 6xyy' + 3y^2 = 0$

$y'(3y^2 - 2x^2 + 6xy) = 4xy - 3y^2$

$y' = \frac{4xy - 3y^2}{3y^2 - 2x^2 + 6xy}$

At $(0, -1)$,

$y' = -1$

$y + 1 = -1(x - 0)$

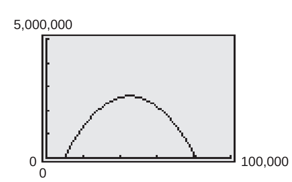
$y = -x - 1.$

100. (a) $P = R - C = xp - C = x(211 - 0.002x) - (30x + 1{,}500{,}000) = 181x - 0.002x^2 - 1{,}500{,}000$

(b) $P'(x) = 181 - 0.004x$

$P'(80{,}000) = \$-139$ per unit

(c)



The maximum profit occurs for $x = 45{,}250$, which means $p = 211 - 0.002(45{,}250) = \$120.50$.