Summer MA 15200 Lesson 22 Section 4.3

This lesson is on the **properties of logarithms.** Properties of logarithms model the properties of exponents.

I Product Rule

Product Rule of Exponents: $b^m b^n = b^{m+n}$

Notice: When the bases were the same, the **exponents were added** when multiplication was performed. Likewise **logarithms are added** when multiplication is performed in the argument.

Product Rule of Logarithms: $\log_b(MN) = \log_b M + \log_b N$

In words, the logarithm of a product is the sum of the logarithms.

Informal Proof:

$\log 100 = 2 \qquad \log 1000 = 3$

$\log(100 \cdot 1000)$

$= \log(100,000)$

$= 5$

$= 2 + 3$

$= \log 100 + \log 1000$

When a single logarithm is written using this product rule, we say we are **expanding the logarithmic expression.**

Ex 1: Assume all variables represent positive values.
Use the product rule to expand each expression **and simplify where possible.**

$a)$ $\log_2(7r) =$

$b)$ $\log_b\left(2x^2 y\right) =$

$c)$ $\log(100ab) =$

$d)$ $\ln(20e^5) =$

II Quotient Rule

Quotient Rule for Exponents: $\dfrac{b^m}{b^n} = b^{m-n}$

Notice: When the bases were the same, the **exponents were subtracted** when division was performed. Likewise, **logarithms are subtracted** when division is performed in the argument.

CAUTION:
$\log_b (M \pm N) \neq \log_b M \pm \log_b N$
$\log_b \frac{M}{N} \neq \frac{\log_b M}{\log_b N}$

Quotient Rule for Logarithms: $\log_b \left(\frac{M}{N} \right) = \log_b M - \log_b N$
In words, the logarithm of a quotient is the difference of the logarithms.

We can also **expand a logarithm** by using the quotient rule.

Ex 2: Assume all variables represent positive values.
Use the quotient rule to expand each logarithm and **simplify where possible.**

$a)$ $\log_3 \left(\frac{9}{y} \right)$

$b)$ $\log \left(\frac{x}{1000} \right)$

Note: Our text and online homework does not usually use parenthesis around the argument. However, it would be better to write as in the following.
$\ln \left(\frac{4x^3}{yz^5} \right)$

III Power Rule

Power Rule for Exponents: $(b^m)^n = b^{mn}$
Note: When a power is raised to another power, the **exponents are multiplied**.
Likewise, when a logarithm has an exponent in the argument, **the exponent is multiplied by the logarithm**.

Power Rule for Logarithms: $\log_b M^p = p \log_b M$
In words, the logarithm of a power is the product of the exponent and the logarithm.

We can also **expand a logarithm** by using the product rule.

Ex 3: Assume all variable represent positive values.
Use the power rule to expand each logarithm and **simplify where possible.**

$a)$ $\log x^8 =$

$b)$ $\log_5 \left(25^3 \right) =$

$c)$ $\ln \sqrt{y} =$

IV Here is a summary of all the properties of logarithms.

Assume all variables represent positive values and that all bases are positive number (not 1).

1. $\log_b(MN) = \log_b M + \log_b N$ Product Rule
2. $\log_b\left(\frac{M}{N}\right) = \log_b M - \log_b N$ Quotient Rule
3. $\log_b\left(M^p\right) = p\log_b M$ Power Rule

<u>Ex 4:</u> Use the properties to **expand** each logarithmic expression. Assume all variables represent positive values.

$a)$ $\log\frac{xy}{\sqrt{z}} =$

$b)$ $\ln\frac{4x^3}{yz^5} =$

$c)$ $\log_2 3\sqrt{xy} =$

$d)$ $\log_3\left(27x^2\sqrt[3]{y}\right)$

In opposite of expanding a logarithmic expression is **condensing a logarithmic expression**. This is writing a logarithmic expression as a single logarithm.

Ex 5: Condense each expression. In other words, write as a single logarithm. Assume all variables represent positive values.

$a)$ $\log 3 - \log x + 2\log y - \frac{1}{2}\log z =$

$b)$ $\frac{1}{3}\log(x-2) + 2\log x - 2\log 4 =$

$c)$ $\frac{1}{2}(\ln x + 3\ln y) - 3\ln(x+2)$

Ex 6: If $\log_b m = 2.3892, \log_b n = -1.2389,$ and $\log_b r = 0.8881$, use the properties of logs to find the following values.

$a)$ $\log_b(m^2 n) =$

$b)$ $\log_b\left(\frac{\sqrt{n}}{r}\right) =$

Ex 7: If $\log_b 8 = 1.8928, \log_b 11 = 2.1827,$ and $\log_b 2 = 0.6309$. Use these values and the properties of logs to find the following values.

There is more than 1 way to determine these values.

$a)$ $\log_b 4 =$

$b)$ $\log_b 88 =$

$c)$ $\log_b 121 =$

$d)$ $\log_b 44 =$

Ex 8: Let $\log_2 4 = A$ and $\log_2 5 = B$. Write each expression in terms of *A* and/or *B*.

$a)$ $\log_2(125) =$

$b)$ $\log_2\left(\frac{5}{4}\right) =$

V Change of Base Formula

Your scientific calculator will approximate or find common logarithms (base 10) or natural logarithms (base *e*). How can logarithms with other bases be approximated?

$$\log_b M = \frac{\log M}{\log b} \text{ or } \frac{\ln M}{\ln b}$$

Note: $\log\left(\frac{M}{N}\right) \neq \frac{\log M}{\log N}$

The formula above is known as the change of base formula.

Ex 9: Approximate each logarithm to 4 decimal places.

$a)$ $\log_3(22.8) =$

$b)$ $\log_{0.2}(285) =$