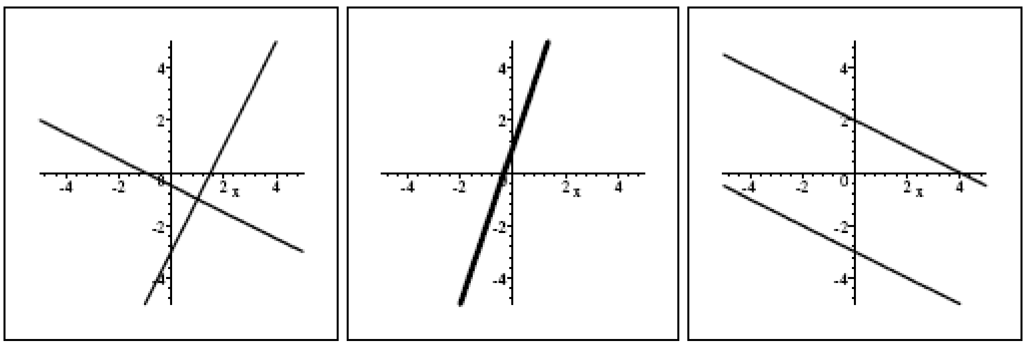
Summer MA 15200 Lesson 24, Section 5.1

A linear equation with two variables has infinite solutions (many ordered pairs, example $2x - 3y = 6$) However, if a second equation is also considered, and we want to know what ordered pairs they may have in common, how many solutions may there be? When two linear equations are considered together, it is known as a **system of linear equations or a linear system.** The **solution(s)** of the system is/are any **ordered pair(s)** they have in common.

Consider graphing two lines. How many points may they have in common?



$2x - y = 3,\ x + 2y = -1$ $\quad$ $y = 3x + 1,\ 6x - 2y = -2$ $\quad$ $y = -\frac{1}{2}x - 3,\ x + 2y = 4$

As demonstrated in the graphs above, there are 3 situations.

1. The graphs may intersect at one point. If the lines of the equations intersect, the solution is one ordered pair.
2. The graphs may intersect at an infinite number of points. If the lines of the equations form the same line, the solution is an infinite number of ordered pairs.
3. The graphs may not intersect. If the lines of the equations do not intersect, there is no solution.

<u>Ex 1:</u> Given the ordered pair, determine if it is a solution of the shown system.

$a)\ \begin{cases} 2x + 5y = 16 \\ 3x + y = -2 \end{cases} \quad (-2, 4)$

$b)\ \begin{cases} 5x + 2y = 1 \\ 10x + 5y = 1 \end{cases} \quad \left(\frac{3}{5}, 1\right)$

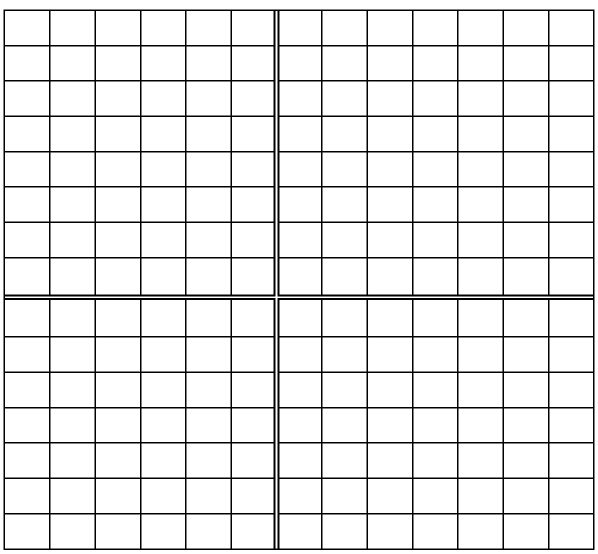
I Solving a system of Linear Equations

There are three methods that may be used to find the solution of a system of linear equations. Graphing the lines could be done. Two more algebraic methods are covered in this lesson.

1. **The Graphing Method**-Graph each line and identify any common point (ordered pair), or determine if the lines are the same or if the lines are parallel.
2. **The Substitution Method**
3. **The Addition Method**

For the next example we will use the graphing method. The graphing method is not really the best method. Graphing is not very accurate. And, if the solution has coordinates that are fractions or decimals, the coordinates of the point of intersection (if there is one) would be difficult to determine. There is a good study tip on using the graphing method on page 483 of the textbook.

Ex 1: Find the solution to the system $\begin{cases} 3x + 2y = 2 \\ 2x - 3y = -16 \end{cases}$ by graphing.



Substitution Method:

1. Solve either of the equations for one variable in terms of the other.
2. Substitute the expression found in step 1 for that variable in the *other equation.* This will result in an equation with only one variable.
3. Solve the equation for that variable.
4. Back-substitute the value found in step 3 into one of the original equations and solve for the remaining variable.
5. Check and write the solution as an ordered pair or identify x and y.

Ex 2: Solve each system of linear equations using the substitution method.

$a)\quad \begin{cases} 2x+3y=0 \\ y=3x-11 \end{cases}$

Note: It is easier to use the substitution method when a variable of one of the equations has a coefficient of 1 or -1. This avoids the use of fractions.

$b)\quad \begin{cases} 4x+5y=4 \\ 8x-15y=3 \end{cases}$

Examine these examples.

$A\quad \begin{cases} x+3y=4 \\ y=-\frac{1}{3}x+\frac{4}{3} \end{cases}$

$x+3\left(-\frac{1}{3}x+\frac{4}{3}\right)=4$

$2x+6\left(-\frac{1}{3}x+\frac{4}{3}\right)=8$

$2x-2x+8=8$

$8=8$

$B\quad \begin{cases} y=-x+3 \\ x+y=7 \end{cases}$

$x+(-x+3)=7$

$x-x+3=7$

$3=7$

You will notice that both variables **'dropped out'** in both example A and example B.

A In example A, a true statement was the result (8 = 8). This indicates that there are several solutions, because the statement is true. Graphically, these are the same line. There are an **infinite** number of solutions, every point on the line. The **general solution** may be written as a set this way: $\left\{(x, y) \mid y = -\frac{1}{3}x + \frac{4}{3}\right\}$.

Although there are an infinite number of solutions in such a system, this does not mean that any ordered pair is a solution. It must be an ordered pair that satisfies the equation of the line.

B In example B, the result was a false statement (3 = 7). Because the result was false, there is no ordered pair that is common to both equations. The system has **no solution**. Graphically, these lines would be parallel.

This system is called an **inconsistent system**.

Ex 3: Solve the system $\begin{cases} 3x - y = 12 \\ y = 3x - 12 \end{cases}$.

Addition Method: As with the substitution method, the goal of the addition method is to eliminate one of the variables.

1. Write both equations in general (standard) form ($Ax + By = C$).
2. If necessary, multiply one or both of the equations by appropriate nonzero numbers so that the sum of the *x*-coefficients or the sum of the *y*-coefficients is 0.
3. Add the equations in step 2. The sum will be an equation in one variable.
4. Solve the equation.
5. Back-substitute the value you found in step 4 into either of the given equations and solve for the remaining variable.
6. Check. Write the solution as an ordered pair or give values of *x* and *y*.

Ex 4: Solve each system.

$a)\quad \begin{cases} 2x+3y=8 \\ 5x-y=3 \end{cases}$

$b)\quad \begin{cases} 3x-3y=y-9 \\ 5(x+y)=-15 \end{cases}$

$c)\quad \begin{cases} 3x=4(2-y) \\ 3x-6+4y=0 \end{cases}$

II Applied Problems

Ex 5: Three times a first number decreased by 5 times a second number is -9. The first number is one less than the second number. Find the numbers.

Ex 6: For a linear function $f(x) = mx + b$, $f(2) = -1$ and $f(5) = 8$. Find the values of *m* and *b*.

Break-Even Analysis

The **Revenue** of a company is the money generated by sells of the product. The **Cost** of a company is the sum of any fixed costs and the cost of producing the product.

If a company produces and sells *x* number of a product, its **Revenue Function and Cost Function** are represented by

$$y = R(x) = (\text{price per unit})x$$

$$y = C(x) = \text{fixed cost} + (\text{cost per unit})x$$

These are both linear functions. In the revenue function, the price per unit is the slope. In the cost function, the cost per unit is the slope. When graphed, these lines will intersect. The **point of intersection** is called the **break-even point**. The *x*-coordinate will be the number of units of the product and the *y*-coordinate will be the amount of money coming in from sells and the amount of money going out for costs.

Ex 7: A manufacturer of CDs have determined that monthly fixed costs to make the CDs are \$337,500. It costs \$3.50 to make a CD. Each CD that is made can be sold for \$9.75. Find an equation to represent the costs of the manufacturer and an equation to represent to revenue where *x* represents the number of CDs made and sold during a month. What is the break-even point and what do the coordinates represent?

Ex 8: The weekly revenue of a company is represented by $y = R(x) = 12x$ and its weekly cost is represented by $y = C(x) = 5x + 12,000$. Find the value of $R(5,000) - C(5,000)$. What does this represent? Find the value of $R(800) - C(800)$. What does it represent?

The **Profit** of a company is found by subtracting costs from the revenue. If the profit is profit is positive, there truly is a profit or gain. If it is negative, there is a loss for the company.

The **Profit Function** is found by $y = P(x) = R(x) - C(x)$.

Ex 9: The quarterly Cost function for a business that sells umbrellas (based on *x* umbrellas) is $C(x) = 9x + 26,500$. The quarterly Revenue function for this business is $R(x) = 21x$.

a) Find the profit function, $P(x)$.

b) How much profit (or lose) will the business incur during a quarter of the year, if they make and sell 4000 umbrellas?

c) Find $P(1200)$ and describe what it means.

d) How many umbrellas must the business make and sell quarterly in order to break even? Round to the nearest umbrella, if necessary.

Ex 10: A company that manufactures tricycles has a fixed monthly cost of $25,000. It costs $40 to produce each tricycle. The selling price of a tricycle is $65.

a) Write a cost function, letting x represent the number of tricycles made and sold during the month.

b) Write a revenue function.

c) Write a profit function.

d) Determine the break-even point.

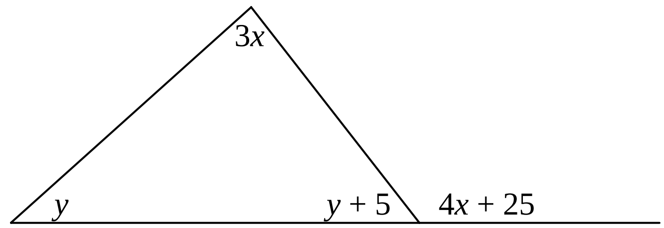
Ex 11: The percentage of Purdue students in group A (y) who like a certain TV show can be modeled by $x+3y=214$, where x is the number of weeks after the show premièred. The percentage of Purdue students in group B (y) who liked the TV show can be modeled by $2y-x=136$, where x is the number of weeks after the show premiered. In what week after the premier was the percentage in both group A and group B the same? What is that percentage?

Ex 12: At a local fast food restaurant, the cost of two cheeseburgers and three fries was \$12.25. The cost of 5 cheeseburgers and two fries was \$21. Find the cost of a single cheeseburger and a single fries.

Ex13: A 500 seat theater has tickets for sale at \$15 or \$25. How many tickets should be sold at each price to generate revenue of \$9200?

Ex 14: The rectangular backyard of the Jones family has a **perimeter** of 170 feet. The family decides to fence in the 2 shorter sides and one longer side of the backyard (the house itself forms the 2^{nd} longer side) at a cost of \$8 per foot for a total of \$960. Find the dimensions of the backyard.

Ex 15: The figure below represents the 3 angles inside of a triangle and an angle supplementary to one of the angles. Find the measures of each angle of the triangle.



Ex 16: When rowing with the current, a team can row 33 miles in 3 hours. When rowing against the current, the team rows 14 miles in 2 hours. Find the rowing speed of the team (as if in still water) and the speed of the current.

	Distance	Rate	Time
With current			
Against current			