

Summer Lesson 25 MA 152, Appendix G, Section 2.8

Many application problems can be solved by using a system of equations. Usually the addition method or the substitution method is used to solve the problem.

STEPS:

1. Define two variables for the problem.
2. Write two equations representing statements from the problem.
3. Solve using either the substitution method or the addition method.
4. Answer the question posed in the problem.

Ex 1: A rectangular soccer field has a perimeter of 360 yards. Its length is 20 yards more than its width. What are the dimensions of the field?

w = width

L = length

Ex 2: In 2002-2003, the average total SAT score for high school students was 1026, with the average math score exceeding the verbal score by 12 points. What were the average math score and average verbal score on the SAT for that year?

*Source: College Entrance Examination Board

Ex 3: In his motorboat, Nigel travels upstream at top speed to his favorite spot, a distance of 36 miles, in two hours. Returning downstream, still at top speed, the trip only takes $\frac{3}{2}$ hours. Find the top speed of the boat (as if it was in still water) and the speed of the current.

	Rate	Time	Distance
upstream			
downstream			

Ex 4: At a local diner, 2 cheeseburgers and 4 french fries cost \$13.20. Three cheeseburgers and 3 fries cost \$14.10. Find the cost of a single cheeseburger and a single fries.

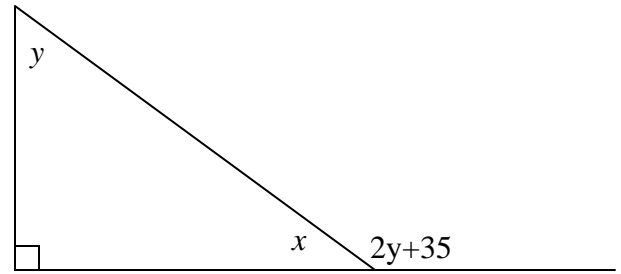
Ex 5: Gail must mix some pure antifreeze with a solution of 20% antifreeze to obtain 27 Liters of 56% antifreeze. Find how much of the pure antifreeze and how much of the 20% antifreeze she must use to get the desired result. Round to the nearest whole liter for each.

Ex 6: Diana's furniture outlet regularly sells two popular types of dinette sets; one is in oak and the other in walnut. In July, the outlet sold 5 of the oak sets and 4 of the walnut and collected \$18400 from the sales of these dinette sets. In August, they sold 2 of the oak and 6 of the walnut and collected \$18030. What is the price of each type of dinette set?

Ex 7: Terry wants to mix some peanuts that sell for \$6.50 a pound with some cashews that sell for \$8.50 a pound to make 10 pounds of a nut mix that sells for \$7.30 a pound. How many pounds of peanuts and how many pounds of cashews should she use?

Ex 8: A train travels 75 miles in the same time a small plane travels 1260 miles. If the speed of the plane is 20 miles per hour more than 16 times the speed of the train, find both speeds.

Ex 9: Given the figure at the right, find the measurements of angles x and y .



I Circles

An earlier lesson has the distance and midpoint formulas. You will need to remember those formulas for this lesson.

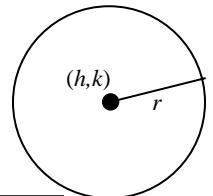
$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$M : \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

A **circle** is a set of all points in a plane that are a fixed distance, called the **radius**, and from a point called its **center**.

Equation of a circle: The distance formula can be used to find the equation of a circle. Let the center of the circle be represented by the ordered pair (h, k) and the radius of the circle by r . Every point (x, y) is r units from (h, k) .

$$r = \sqrt{(x - h)^2 + (y - k)^2} \quad \text{This is the standard equation for a circle.}$$
$$r^2 = (x - h)^2 + (y - k)^2$$



Standard Equation of a Circle:

If the center of a circle is (h, k) and the radius of the circle is r units, the an equation for the circle is $(x - h)^2 + (y - k)^2 = r^2$.

If the center of the circle is the origin, then the standard equation of the circle is $x^2 + y^2 = r^2$.

The **general equation for a circle** is an equation where the parentheses are cleared from the standard form (binomials are squared)

General Equation of a Circle:

$x^2 + y^2 + cx + dy + e = 0$, where c , d , and e represent real numbers.

Ex 10: Find the standard equation in (a) and the general form in (b) for each circle described.

a) center: $(2, -3)$, radius: 4 units

b) center: $(-3, 6)$, a point of the circle: $(12, 2)$

Ex 11: Identify the center and length of radius for each circle.

a) $(x - 4)^2 + y^2 = 49$

b) $(x + 9)^2 + (y - \frac{1}{2})^2 = 21$

To change an equation of a circle from general form to standard form, a completing the square process must be used. Examine this example.

$$2x^2 + 2y^2 - 8x + 12y - 24 = 0$$

$$x^2 + y^2 - 4x + 6y - 12 = 0$$

$$(x^2 - 4x \quad) + (y^2 + 6y \quad) = 12 \quad \text{center: } (2, -3), \text{ radius: } 5$$

$$(x^2 - 4x + 4) + (y^2 + 6y + 9) = 12 + 4 + 9$$

$$(x - 2)^2 + (y + 3)^2 = 25$$

The following steps were used to convert above from general to standard.

1. If necessary, divide so each squared term has coefficients of 1.
2. Arrange the x terms together and the y terms together and move the constant to the other side.
3. Complete the square for the x's and for the y's. Balance the equation by adding the numbers found to the other side as well.
4. Write with binomial squared terms and combine the numbers.

Ex 12: Write each circle equation in standard form.

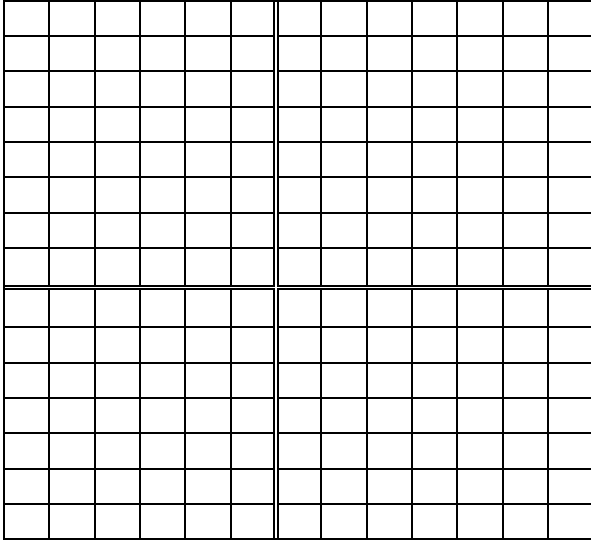
a) $x^2 + y^2 - 10x - 20y - 2 = 0$

b) $4x^2 + 4y^2 - 4x + 8y + 1 = 0$

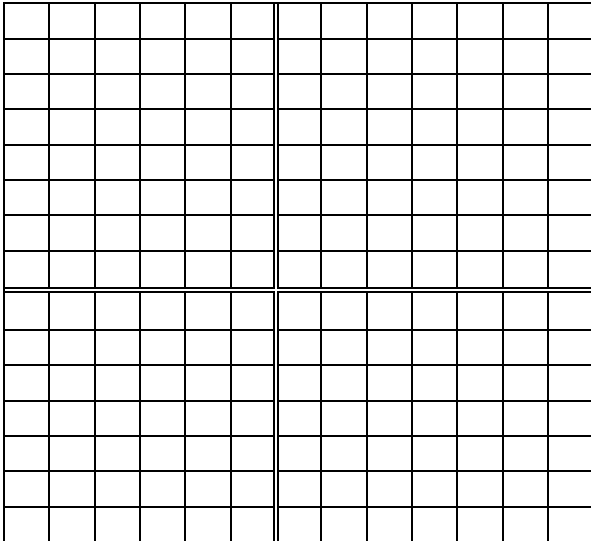
Graphing a circle: To graph circle, locate the center. Find points r units up, down, left, and right from the center. (Sometimes it may be valuable to locate other points in a table.) Draw a smooth circle. Identify the domain and range.

Ex 13: Graph each circle:

a) $(x-1)^2 + (y+3)^2 = 25$



b) $x^2 + y^2 + 2x - 4y - 4 = 0$



Ex 14: Find the equation (in standard form) for a circle with endpoints of a diameter of the circle at (3, -2) and (5, 8).

Ex 15: Find the equation of a circle (in general form) if the circle has a radius of 8 units and the center is at the intersection of lines $x + 2y = 8$ and $2x - 3y = -5$.

Ex16: The picture below represents two gears, a larger gear on the left that has a circle equation $x^2 + y^2 = 16$. The smaller gear has a center at (7, 0) and only touches the larger gear where they meet. Find an equation for the smaller gear.

