

- Find the angle between the vectors  $\mathbf{v} = 2\mathbf{i} + 2\mathbf{j} + \mathbf{k}$  and  $\mathbf{w} = 2\mathbf{i} + 2\mathbf{j} - \mathbf{k}$ .  
A.  $\cos^{-1}\left(\frac{8}{9}\right)$     B.  $\cos^{-1}\left(\frac{5}{9}\right)$     C.  $\cos^{-1}\left(\frac{2}{3}\right)$     D.  $\cos^{-1}\left(\frac{7}{9}\right)$     E.  $\cos^{-1}\left(\frac{1}{3}\right)$
- Find  $a$  such that  $\mathbf{u} = 2\mathbf{i} - \mathbf{j} + a\mathbf{k}$  and  $\mathbf{v} = \mathbf{i} + 4\mathbf{j} + 2\mathbf{k}$  are perpendicular.  
A. 3                      B. 2                      C. 1                      D. -1                      E. -2
- If  $\mathbf{w} = w_1\mathbf{i} + w_2\mathbf{j} + w_3\mathbf{k}$  is perpendicular to  $\mathbf{u} = \mathbf{i} + \mathbf{j} - \mathbf{k}$  and  $\mathbf{v} = 2\mathbf{i} + \mathbf{j} + \mathbf{k}$ , and if  $w_3 = 2$ , then  $w_1 =$   
A. 4                      B. 2                      C. -2                      D. -4                      E. 1
- If  $\mathbf{v} = \mathbf{i} + \mathbf{j} + \mathbf{k}$  and  $\mathbf{w} = 2\mathbf{i} - \mathbf{k}$ , find  $|\text{proj}_{\mathbf{v}}(\mathbf{w})|$ .  
A.  $1/\sqrt{3}$               B.  $\sqrt{3}$                   C.  $\sqrt{3}/5$               D.  $2\sqrt{3}$                   E.  $\sqrt{3}/2$
- Find the area of the triangle with vertices  $P = (0, 0, 0)$ ,  $Q = (1, 2, 1)$ , and  $R = (2, 1, -1)$ .  
A.  $\sqrt{27}$                   B.  $\frac{\sqrt{27}}{2}$                   C.  $\frac{\sqrt{11}}{2}$                   D.  $\sqrt{19}$                   E.  $\frac{\sqrt{3}}{2}$
- The radius of the sphere  $x^2 + y^2 + z^2 + 2x + 4y - 6z = 3$  is  
A.  $3 + \sqrt{13}$               B.  $\sqrt{13}$                   C.  $\sqrt{65}$                   D.  $3 + \sqrt{56}$               E.  $\sqrt{17}$
- The area of the region enclosed by the curves  $y = x^2 + 1$  and  $y = 2x + 9$  is given by  
A.  $\int_{-2}^4 (x^2 + 1 - 2x - 9) dx$     B.  $\int_{-2}^4 (2x + 9 - x^2 - 1) dx$     C.  $\int_{-2}^2 (2x + 9 - x^2 - 1) dx$     D.  
 $\int_{-4}^2 (2x + 9 - x^2 - 1) dx$                       E.  $\int_{-4}^2 (x^2 + 1 - 2x - 9) dx$
- Let  $R$  be the region between the graphs of  $y = x^2$  and  $y = x$ . Find the volume of the solid generated by revolving  $R$  about the  $x$ -axis.  
A.  $\frac{\pi}{6}$                       B.  $\frac{\pi}{12}$                       C.  $\frac{\pi}{4}$                       D.  $\frac{\pi}{15}$                       E.  $\frac{2\pi}{15}$
- If the region in problem 8 is revolved about the  $y$ -axis, then the volume of the solid is  
A.  $\frac{\pi}{6}$                       B.  $\frac{\pi}{12}$                       C.  $\frac{\pi}{24}$                       D.  $\frac{2\pi}{15}$                       E.  $\frac{\pi}{15}$
- If  $R$  is the region bounded by the curves  $x = 0$  and  $x = y - y^2$ , and if  $R$  is revolved around the  $y$ -axis, then the volume of the solid is  
A.  $\frac{\pi}{15}$                       B.  $\frac{\pi}{30}$                       C.  $\frac{\pi}{12}$                       D.  $\frac{\pi}{3}$                       E.  $\frac{\pi}{6}$

11. A force of 4 lb. is required to stretch a spring  $1/2$  ft. beyond its natural length. How much work is required to stretch the spring from its natural length to 2 ft.
- A. 8 ft-lbs.      B. 12 ft-lbs.      C. 16 ft-lbs.      D. 24 ft-lbs.      E. 32 ft-lbs.
12. A cylindrical tank of height 4 feet and radius 1 foot is filled with water. How much work is required to pump all but 1 foot of water out of the tank. (Density =  $62.5$  lbs./ft<sup>3</sup>)
- A.  $9\pi(62.5)$  ft-lbs.      B.  $3\pi(62.5)$  ft-lbs.      C.  $\frac{9\pi}{2}(62.5)$  ft-lbs.      D.  $18\pi(62.5)$  ft-lbs.  
E.  $6\pi(62.5)$  ft-lbs.
13. Let  $f(x) = \sqrt{x}$ . Find  $c$  in  $[0, 9]$  such that  $f(c) = f_{\text{avg}}$ , where  $f_{\text{avg}}$  is the average value of  $f(x) = \sqrt{x}$  on the interval  $[0, 9]$ .
- A.  $c = 4$       B.  $c = 4.5$       C.  $c = 5$       D.  $c = 3.2$       E.  $c = 6$ .
14.  $\int x(\ln x)^3 dx = \frac{x^2}{2}(\ln x)^3 - I$ , where  $I =$
- A.  $\frac{1}{4} \int (\ln x)^4 dx$       B.  $\frac{1}{3} \int (\ln x)^2 dx$       C.  $\frac{1}{3} \int (\ln x)^2 dx$       D.  $\frac{3}{2} \int x^2(\ln x)^2 dx$       E.  $\frac{3}{2} \int x(\ln x)^2 dx$
15. Evaluate  $\int_0^1 xe^{3x} dx$ .
- A.  $\frac{2e^3}{9}$       B.  $\frac{1}{9} + \frac{2e^3}{9}$       C. 1      D.  $\frac{1}{9}$       E.  $\frac{e^3}{9} - 1$
16.  $\int_0^{\pi/2} \sin^3 x dx =$
- A.  $2/3$       B.  $4/3$       C. 0      D.  $1/4$       E.  $1/3$
17.  $\int_0^{\pi/4} \sec^4 x \tan x dx =$
- A. 1      B.  $1/3$       C.  $4/3$       D.  $3/4$       E.  $2/9$
18. In order to compute  $\int \frac{dx}{(1+x^2)^{3/2}}$  we make the substitution  $x = \tan \theta$ . This gives an integral in  $\theta$  whose value is
- A.  $\frac{1}{2}\theta + \frac{1}{2} \sin \theta \cos \theta + C$       B.  $\ln(\sec^2 \theta) + C$       C.  $\frac{1}{2}\theta + \tan^{-1} \theta + C$       D.  $\frac{1}{2}\sqrt{\cos \theta} + C$   
E.  $\sin \theta + C$

19.  $\int \frac{dx}{\sqrt{9-4x^2}} =$

- A.  $\sec^{-1}\left(\frac{3x}{2}\right) + C$     B.  $\frac{1}{2}\sin^{-1}\left(\frac{2x}{3}\right) + C$     C.  $\tan^{-1}\left(\frac{2x}{3}\right) + C$     D.  $\frac{1}{3}\sin^{-1}\left(\frac{3x}{2}\right) + C$   
 E.  $\sqrt{9-4x^2} + \tan^{-1}\left(\frac{2x}{3}\right) + C$

20.  $\int \frac{x+1}{x^3-2x^2+x} dx =$

- A.  $\ln|x| + \ln|x-1| + C$     B.  $\ln|x| - \ln|x-1| + C$     C.  $\ln|x| - \frac{2}{x-1} + C$   
 D.  $\ln|x-1| - \frac{2}{x-1} + C$     E.  $\ln|x| - \ln|x-1| - \frac{2}{x-1} + C$

21. A partial fraction decomposition of  $\frac{x+2}{x^4+2x^2}$  has the form

- A.  $\frac{A}{x} + \frac{B}{x^2} + \frac{Cx+D}{x^2+2}$     B.  $\frac{A}{x^2} + \frac{Bx+C}{x^2+2}$     C.  $\frac{A}{x} + \frac{B}{x^2} + \frac{C}{x^2+2}$     D.  $\frac{A}{x^2} + \frac{B}{x^2+2}$   
 E.  $\frac{A}{x} + \frac{B}{x^2+2}$

22.  $\int_0^1 \frac{x+2}{x^2+1} dx =$

- A.  $\frac{\ln 2}{2} + \frac{\pi}{2}$     B.  $\frac{\ln 2}{2}$     C.  $\frac{\ln 2}{2} + 2\pi$     D.  $2\ln 2 + \frac{\pi}{2}$     E.  $\ln 2 + \pi$

23. Use the Trapezoidal Rule with  $n = 3$  to approximate  $\int_0^1 \frac{1-x}{1+x} dx$

- A.  $\frac{12}{5}$     B.  $\frac{6}{5}$     C.  $\frac{2}{5}$     D.  $\frac{17}{60}$     E.  $\frac{17}{10}$

24. Indicate convergence or divergence for each of the following improper integrals:

(I)  $\int_2^\infty \frac{1}{(x-1)^2} dx$     (II)  $\int_0^2 \frac{1}{(x-1)^2} dx$     (III)  $\int_0^1 \frac{\ln x}{x} dx$

- A. I converges, II and III diverge.    B. II converges, I and III diverge.    C. I and III converge, II diverges.  
 D. I and II converge, III diverges.    E. I, II and III diverge.

25. Find the length of the curve  $y = \frac{2}{3}x^{3/2}$ ,  $0 \leq x \leq 2$ .

- A.  $2\sqrt{3} - 2$     B.  $3\sqrt{3} - 1$     C.  $\sqrt{3} - 1$     D.  $\frac{2}{3}(3\sqrt{3} - 1)$     E.  $3\sqrt{3} - 2$

26. If the curve  $y = e^{2x}$ ,  $0 \leq x \leq 1$ , is revolved about the  $y$ -axis, then the area of the surface obtained is

A.  $\int_0^1 2\pi\sqrt{1+4e^{4x}} dx$       B.  $\int_0^1 2\pi e^{2x}\sqrt{1+e^{2x}} dx$       C.  $\int_0^1 2\pi x\sqrt{1+4e^{4x}} dx$   
D.  $\int_0^1 2\pi e^{2x}\sqrt{1+4e^{4x}} dx$       E.  $\int_0^1 2\pi e^{4x}\sqrt{1+e^{4x}} dx$

27. Find the centroid  $(\bar{x}, \bar{y})$  of the region bounded by the  $x$ -axis and the semicircle  $y = \sqrt{4-x^2}$ .

A.  $(0, \frac{8}{3\pi})$       B.  $(\frac{8}{3\pi}, 0)$       C.  $(0, \frac{2}{3\pi})$       D.  $(\frac{2}{3\pi}, 0)$       E.  $(0, 0)$

28. Evaluate  $\lim_{n \rightarrow \infty} \left(1 + \frac{(-1)^n}{n}\right)$ .

A. 0      B. 1      C. -1      D. 2      E. The limit does not exist.

29. Evaluate  $\lim_{n \rightarrow \infty} \left(n^{1/n} + \frac{1}{n!}\right)$ .

A. 0      B. 1      C.  $e$       D.  $1/e$       E. The limit does not exist.

30.  $\sum_{n=0}^{\infty} 5 \left(-\frac{4}{5}\right)^n =$

A.  $1/9$       B.  $5/9$       C.  $25/9$       D. 5      E. 25

31. If  $L = \sum_{n=1}^{\infty} \frac{1}{2^n} + \sum_{n=0}^{\infty} \frac{(-1)^n}{2^n}$ , then  $L =$

A.  $1/3$       B.  $2/3$       C. 1      D.  $4/3$       E.  $5/3$

32. Find all values of  $p$  for which  $\sum_{n=1}^{\infty} \frac{1}{(n^2+1)^p}$  converges.

A.  $p > 1$       B.  $p \leq 1$       C.  $p \geq 1$       D.  $p > 1/2$       E.  $p \leq 1/2$

33.  $\sum_{n=1}^{\infty} \left(1 + \frac{1}{n}\right)^p$  converges for:

A.  $p \leq 1$       B.  $p > 1$       C.  $p < 0$       D.  $p > 0$       E. No values of  $p$ .

34. Which of the following series converge conditionally?

(I)  $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}$       (II)  $\sum_{n=2}^{\infty} \frac{(-1)^n n}{\ln n}$       (III)  $\sum_{n=1}^{\infty} \frac{(-1)^n n}{e^n}$

A. II only.      B. I and III only.      C. I and II only.      D. All three.      E. None of them.

35. Which of the following series converge?

$$(I) \sum_{n=1}^{\infty} \frac{(-1)^n}{n^{1/4}} \quad (II) \sum_{n=1}^{\infty} \frac{n!}{1 \cdot 3 \cdot 5 \cdots (2n-1)} \quad (III) \sum_{n=1}^{\infty} \frac{4}{3} \left(\frac{1}{2}\right)^n$$

A. II only.   B. I and III only.   C. I and II only.   D. All three.   E. None of them.

36. Find the interval of convergence of the power series  $\sum_{n=1}^{\infty} \frac{3^n x^n}{n \ln n}$ .

A.  $-\frac{1}{3} \leq x < \frac{1}{3}$    B.  $-\frac{1}{3} < x \leq \frac{1}{3}$    C.  $0 \leq x \leq \frac{1}{3}$    D.  $-1 \leq x < 1$    E.  $-3 < x < 3$

37. Find the interval of convergence of the power series  $\sum_{n=1}^{\infty} \frac{n}{5^n} (x-2)^n$ .

A.  $-5 < x < 5$    B.  $3 < x < 7$    C.  $-2 < x < 2$    D.  $-3 \leq x < 7$    E.  $-3 < x < 7$

38. Find the first three terms of the Maclaurin series of  $f(x) = \ln(1+x)$

A.  $x + \frac{x^2}{2} + \frac{x^3}{3}$    B.  $x - \frac{x^2}{2} + \frac{x^3}{3}$    C.  $x + \frac{x^2}{2!} + \frac{x^3}{3!}$    D.  $x - \frac{x^2}{2!} + \frac{x^3}{3!}$    E.  $x + \frac{2x^2}{3!} + \frac{3x^3}{4!}$

39. If  $f(x) = \sum_{n=0}^{\infty} \frac{n^2(x-2)^n}{n+1}$ , then  $f^{(3)}(2) =$

A.  $\frac{9}{24}$    B.  $\frac{27}{2}$    C. 0   D. 27   E.  $\frac{9}{4}$

40.  $\int_0^x te^{t^3} dt =$

A.  $\sum_{n=0}^{\infty} \frac{x^{2n}}{(2n)!}$    B.  $\sum_{n=0}^{\infty} \frac{x^{3n}}{3n(n!)}$    C.  $\sum_{n=0}^{\infty} \frac{x^{4n+1}}{(4n+1)!}$    D.  $\sum_{n=0}^{\infty} \frac{x^{4n+1}}{(4n+1)(n!)}$    E.  $\sum_{n=0}^{\infty} \frac{x^{3n+2}}{(3n+2)(n!)}$

41. Use the power series representation of  $\sin x$  to find the first three terms of the Maclaurin series of  $f(x) = x \sin(x^2)$

A.  $x^3 + \frac{x^7}{3!} + \frac{x^{11}}{5!}$    B.  $x + \frac{x^3}{3} + \frac{x^5}{5}$    C.  $x^3 - \frac{x^7}{3!} + \frac{x^{11}}{5!}$    D.  $x - \frac{x^3}{3} + \frac{x^5}{5}$    E.  $x^3 - \frac{x^7}{3} + \frac{x^{11}}{5}$

42. Find the fourth term of the Maclaurin series of  $f(x) = \frac{x^2+3}{x-1}$ .

A.  $-x^3$    B.  $3x^3$    C.  $-3x^3$    D.  $-4x^3$    E.  $4x^3$

43. The fourth term of the Taylor series of  $f(x) = \ln x$ , centered at  $a = 2$ , is

- A.  $\frac{1}{6}(x-2)^3$     B.  $\frac{1}{12}(x-2)^3$     C.  $\frac{1}{24}(x-2)^3$     D.  $-\frac{1}{3}(x-2)^3$     E.  $-(x-2)^3$

44. Using Maclaurin series and the Alternating Series Estimation Theorem, we can obtain the approximation

$$\int_0^{0.1} e^{-x^2} dx \approx \frac{1}{10} - \frac{1}{3000},$$

with error  $\leq E$ , where the value of  $E$  is

- A.  $10^{-5}$     B.  $10^{-6}$     C.  $\frac{1}{2}10^{-6}$     D.  $\frac{1}{7}10^{-7}$     E.  $\frac{1}{2}10^{-5}$

45. Parametric equations of a curve  $C$  are

$$x = 2 \cos t, \quad y = 3 \sin t, \quad 0 \leq t \leq \frac{\pi}{2}.$$

The curve  $C$  is:

- A. A quarter of a circle.    B. An ellipse.    C. Half of an ellipse.  
D. Half of a circle.    E. A quarter of an ellipse.

46. Find the slope of the tangent line at the point  $(2/3, 3)$  for the curve parameterized by  $x = 2t^3/3$ ,  $y = t^2 + 2t$ .

- A.  $2/3$     B.  $2$     C.  $4/3$     D.  $4$     E.  $3$

47. Find the length of the parametric curve

$$x = \frac{1}{2}t^2, \quad y = 2 + \frac{1}{3}t^3, \quad 0 \leq t \leq \sqrt{3}.$$

- A.  $21/4$     B.  $7/2$     C.  $7/3$     D.  $14/3$     E.  $8/3$

48. A point  $P$  has polar coordinates  $(3, \pi/4)$ . Which of the following are also polar coordinates of  $P$ ?

- (I)  $(-3, -\pi/4)$     (II)  $(-3, 5\pi/4)$     (III)  $(3, -7\pi/4)$     (IV)  $(3, -5\pi/4)$

- A. I and II only.    B. I and III only.    C. I and IV only.  
D. II and III only.    E. II and IV only.

49. The polar graph of  $r = \frac{1}{\sin \theta + \cos \theta}$  is:

- A. a parabola.    B. a line.    C. a cardioid.    D. a rose.    E. an ellipse.

50. The graph of  $y^2 = 12x$  is a parabola whose focus is the point  $(3, 0)$ . The point  $P = (12, 12)$  lies on the parabola. Find the distance from  $P$  to the directrix.

- A.  $\sqrt{481}$     B.  $\sqrt{425}$     C.  $\sqrt{306}$     D.  $15$     E.  $12$

51. The ellipse  $(x - 2)^2 + \frac{(y - 1)^2}{9} = 1$  has one vertex at  
 A. (1, 5)      B. (5, 1)      C. (2, 1)      D. (2, 4)      E. (2, 10)
52. Find an equation for the hyperbola with foci  $(\pm 3, 0)$ , and asymptotes  $y = \pm \frac{x}{2}$ .  
 A.  $20y^2 - 5x^2 = 36$       B.  $5x^2 - 20y^2 = 36$       C.  $x^2 - 4y^2 = 4$   
 D.  $4y^2 - x^2 = 4$       E.  $5x^2 - 4y^2 = 1$
53. Write the complex number  $\frac{3 - 4i}{1 + 2i}$  in the form  $a + bi$ .  
 A.  $-1 - 2i$       B.  $1 + 2i$       C.  $2 - i$       D.  $3 - 2i$       E.  $3 + i$
54. Write the complex number  $\sqrt{3} - i$  in polar form with argument between 0 and  $2\pi$ .  
 A.  $4 \left( \cos \frac{\pi}{3} + i \sin \frac{\pi}{3} \right)$       B.  $2 \left( \cos \frac{5\pi}{6} + i \sin \frac{5\pi}{6} \right)$       C.  $4 \left( \cos \frac{\pi}{4} + i \sin \frac{\pi}{4} \right)$   
 D.  $2 \left( \cos \frac{11\pi}{6} + i \sin \frac{11\pi}{6} \right)$       E.  $2 \left( \cos \frac{\pi}{6} + i \sin \frac{\pi}{6} \right)$

### Answers

1. D; 2. C; 3. D; 4. A; 5. B; 6. E; 7. B; 8. E; 9. A; 10. B  
 11. C; 12. C; 13. A; 14. E; 15. B; 16. A; 17. D; 18. E; 19. B; 20. E  
 21. A; 22. A; 23. C; 24. A; 25. D; 26. C; 27. A; 28. B; 29. B; 30. C  
 31. E; 32. D; 33. E; 34. E; 35. D; 36. A; 37. E; 38. B; 39. B; 40. E  
 41. C; 42. D; 43. C; 44. B; 45. E; 46. B; 47. C; 48. D; 49. B; 50. D  
 51. D; 52. B; 53. A; 54. D