MA 22000 Lesson 22
(2^{nd} half of textbook, Section 4.5)
Derivatives of Natural Logarithmic Functions

Derivative of $\ln x$**:**

$$\frac{d}{dx}(\ln x) = \frac{1}{x}$$

The derivative of ln *x* is the reciprocal of *x*.

*****There is a justification for this rule on page 237 of the textbook. We will accept it as true without proof.
There is also a rule on page 237 of the text for taking derivatives of logarithmic expressions to a base other than base *e*. You may study some of these types of problems, but none are on homework or quizzes/tests.

Let u = a function of the independent variable x. Then the derivative of u:

$$\frac{d}{dx}(\ln u) = \frac{1}{u}(u')$$

The derivative of u (a function of x) is the reciprocal of u times the derivative of u.

For the next few problems, *assume the value of the variable is positive*.

Ex 1: Find the derivative of $y = \ln(12x)$

(Chain rule must be used where $u = 12x$.)

Ex 2: Find $D_x[\ln(x^2 - 4x)]$

Ex 3: Find the derivative of *f*, where $f(x) = \ln(3x^3 - x)^{3/2}$.

Ex 4: If $y = \frac{3x+2}{2\ln x}$, find y'.

Ex 5: Find the derivative of $g(x) = (\ln 2)[\ln(12x)]$

Ex 6: $y = e^x(\ln x)$

Ex 7: $f(x) = \sqrt{e^x - \ln(3x)}$

Ex 8:

The total revenue received from the sale of x items is given by $R(x) = 20\ln(x+1)$ and the total cost to produce x items is given by $C(x) = \frac{2x}{3}$.

a) Find the marginal revenue.

b) Find the profit function.

c) Find the marginal revenue when $x = 40$.

d) Find the marginal profit when $x = 40$.

Ex 9: Suppose the demand function for x units of a certain product is

$$p = D(x) = 70 + \frac{10}{\ln x}, \quad x > 1 \text{ and } p \text{ is in dollars.}$$

a) Find the marginal revenue.

b) Approximate the revenue from one more unit when 10 units are sold.