MA 22000 Lesson 23 Notes
2nd half of textbook, Section 5.1
Increasing and Decreasing Functions

A function is **increasing** if its graph goes **up** from left to right and **decreasing** if its graph goes down from left to right. When describing where a function is increasing, use interval notation of *x* values (domain values, left to right). When describing where a function is decreasing, use interval notation of *x* values (domain values, left to right).

The FIGURE 2 below (also found in your textbook on page 253) shows 3 graphs (a, b, and c) where a graph is increasing and 3 graphs (d, e, and f) where a graph is decreasing. Increasing means ‘rising’ or ‘climbing’, always left to right. Decreasing means ‘falling’ or ‘sliding down’, always left to right. In an increasing function; as the *x* values get larger, so do the *y* values or the function values. In a decreasing function; as the *x* values get larger, the *y* values (function values) get smaller. Using ‘algebra’ language, the definitions of increasing or decreasing functions are found on page 253 of the textbook.

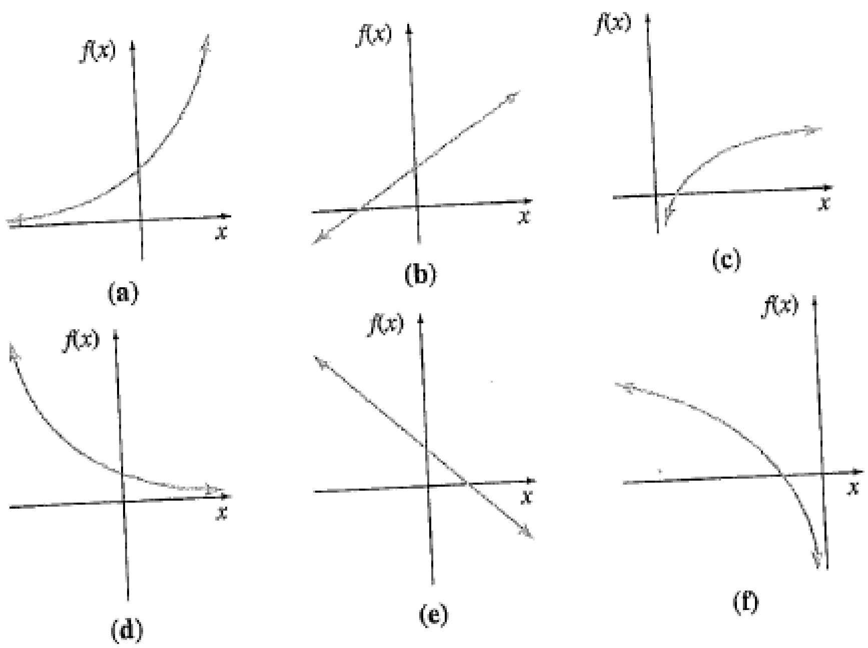
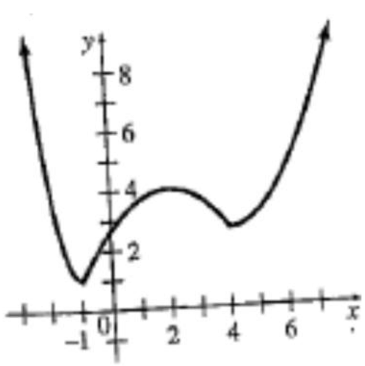



FIGURE 2

Example 1: Look at the graph below. Describe the interval(s) where the function is increasing and the interval(s) where the function is decreasing.




Increasing:______________________

Decreasing:______________________

**Using the first derivative to help determine intervals of increasing/decreasing:**

Suppose a function *f* has a derivative at each point in an open interval; then

1) if $f'(x) > 0$ for each *x* in the interval, *f* is **increasing** on that interval.
2) if $f'(x) < 0$ for each *x* in the interval, *f* is **decreasing** on that interval.
3) if $f'(x) = 0$ for each *x* in the interval, *f* is **constant** on that interval (not increasing nor decreasing).

**Critical Values and Critical Points**

To find possible intervals of increasing or decreasing, we use **critical values.** The critical values of a function are those numbers in the domain of the function for which the derivative is zero or the derivative does not exist. Critical values are the *x*-coordinates of the critical points. A **critical point** is the ordered pair whose *x*-coordinate is the critical value *c* and whose *y*-coordinate is $f(c)$; $(c, f(c))$. The critical values and a **sign chart** can be used to determine intervals of increasing, decreasing, or constant.

Example 2: Find the intervals, if any, where the following function is increasing and the intervals, if any, where the function is decreasing.

$$f(x) = \frac{2}{3}x^3 - x^2 - 4x + 2$$

1) Find the critical value(s); values of *x* where the derivative is zero (or undefined).

$$f'(x) = 2x^2 - 2x - 4$$
$$2x^2 - 2x - 4 = 0$$
$$2(x^2 - x - 2) = 0$$
$$(x-2)(x+1) = 0$$
$$x - 2 = 0 \qquad x + 1 = 0$$
$$x = 2 \qquad x = -1$$

Note: Critical points would be $(2, -\frac{14}{3})$ and $(-1, \frac{13}{3})$.

The intervals to be checked would be $(-\infty, -1)$, $(-1, 2)$, and $(2, \infty)$.

2) Make a sign chart, such as the one below.

SIGN CHART

| | $(-\infty,-1)$ | $(-1,2)$ | $(2,\infty)$ |
|---|---|---|---|
| $x-2$ | - | - | + |
| $x+1$ | - | + | + |
| Result | + | - | + |
| | INC. | DEC. | INC. |

(Example 2 continued on page 3)

3) Give the answer.

**The function is increasing on $(-\infty,-1)\cup(2,\infty)$ and decreasing on $(-1,2)$.**

Example 3: Find all open intervals where the function below is increasing, decreasing, or constant. Write answers using interval notation. $g(x)=\frac{1}{4}x^4-\frac{1}{2}x^2+2$

Example 4: Find all open intervals where the function below is increasing, decreasing, or constant. Write answers using interval notation.

$y=x^4+8x^3+18x^2-8$

Example 5: Find all open intervals where the function below is increasing, decreasing, or constant. Write answers using interval notation.

$$y = -\frac{3}{2}x + 2$$

Example 6: Find all open intervals where the function below is increasing, decreasing, or constant. Write answers using interval notation.

$$f(x) = \frac{x+3}{x-4}$$

Example 7: Find all open intervals where the function below is increasing, decreasing, or constant. Write answers using interval notation.

$$y = x^{2/3}$$

Example 8: Find all open intervals where the function below is increasing, decreasing, or constant. Write answers using interval notation.

$$g(x) = x \cdot e^{-x^2}$$

Example 9:
A manufacturer of CD players has determined that the profit *P* (*x*) (in thousands of dollars) is related to the quantity *x* of CD players produced (in hundreds) per month by the model $P(x) = -(x-4)e^{x} - 4,\ 0 < x \le 3.9$. At what production levels is the profit increasing? decreasing?

Example 10:
The percent of concentration of a drug in the bloodstream *x* hours after the drug is administered is given by $K(x) = \dfrac{4x}{3x^2 + 27}$. On what time intervals is the concentration of the drug increasing? decreasing?