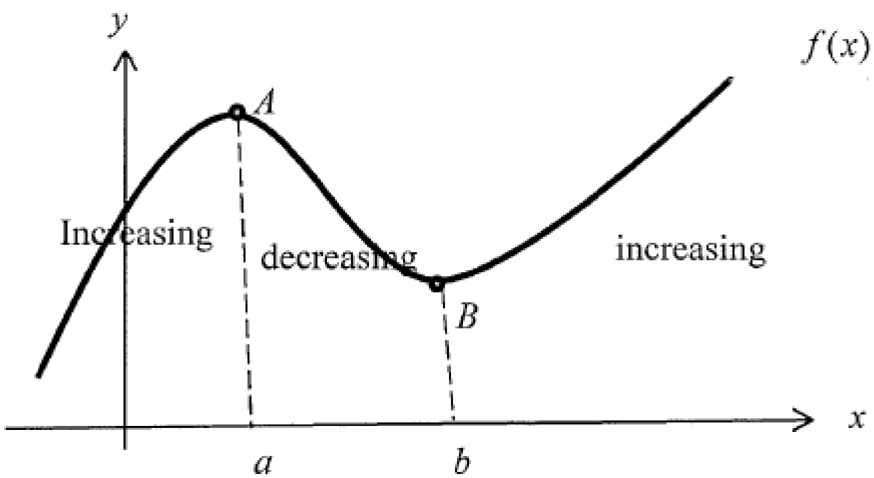
MA 22000 Lesson 24 Notes
(2^{nd} half of text, section 5.2)
Relative Extrema

In the last lesson, we found intervals where a function was increasing or intervals where that function was decreasing. At a point, where a function changes from increasing to decreasing, there will be a **relative maximum** (a function value larger than all other close function values). At a point, where a function changes from decreasing to increasing, there will be a **relative minimum** (a function value smaller than all other close function values). Examine the picture below of function *f*.



****Important****

At $x = a$ there is relative maximum. (Notice this point is 'higher' than other close points.) If point *A* has coordinates (2, 7), we would say there is a relative maximum of 7 and it occurs when $x = 2$; or we could say the relative maximum is $f(2) = 7$.

At $x = b$ there is a relative minimum. (Notice this point is 'lower' than all other close points.) If point *B* has coordinates (5, 3), we would say there is a relative minimum of 3 and it occurs at $x = 5$; or we could say the relative minimum is $f(5) = 3$.

*****Relative extrema (relative minimums or relative maximums) will only occur at critical values for the function. That is to say, they only occur when the derivative of the function equals zero or is undefined.*****

Textbook definition of a **Relative Maximum or Minimum:**

Let c be a number in the domain of a function f. Then $f(c)$ is a relative (or local) maximum for f if there exist an open interval (a, b) containing c such that $f(x) \leq f(c)$ for all x in (a, b), in other words for all close x values.
Likewise, $f(c)$ is a relative (or local) minimum for f if there exists an open interval (a, b) containing c such that $f(x) \geq f(x)$ for all x in (a, b), in other words for all close x values.

A function has a **relative (or local) extremum** (plural extrema) at c if it has either a relative maximum or a relative minimum there. Our textbook does consider it is possible for an endpoint of a closed interval to be a relative extremum. (The open intervals above could be closed intervals.)

Look at example 2 (figure 15) in the calculus part of textbook on page 264. There are relative extrema at x_1, x_2, x_3, and x_4. Figure 16 on the same page (and below) has two relative maximums and two relative minimums indicated. Notice that the tangent lines at these points have a slope of 0 or no tangent line exists because the point has a 'corner' or is an endpoint.

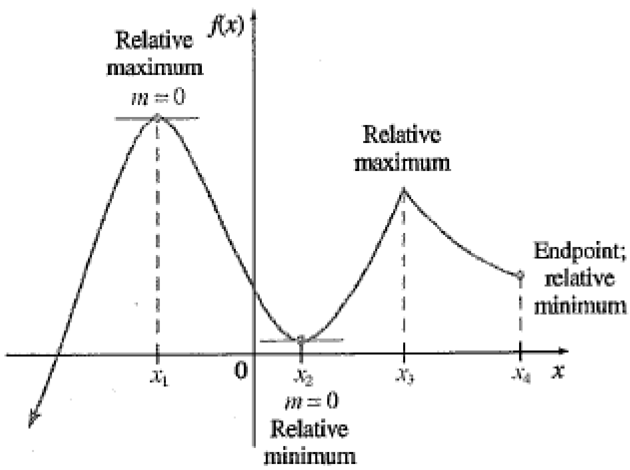


FIGURE 16

If a function f has a relative extremum at c, then c is a critical value or c is at an endpoint of the domain. In other words, a relative extremum occurs at critical values (derivative equals 0 or is undefined) of the domain or endpoints of the domain.

CAUTION: Not every critical value indicates a relative extremum. (See figure 17 on page 265 of the text. There is a critical value when $x = 0$, but there is no relative maximum or relative minimum at $x = 0$.)

FINDING RELATIVE EXTREMA (1st Derivative Test):
Let c be a critical value for a function f that is continuous on an open interval (a, b) and each value in this open interval, except possibly c, and c is the only critical value for f in (a, b).

1. $f(c)$ is a relative maximum of f if the derivative $f'(x)$ is positive in the interval (a, c) and negative in the interval (c, b). In other words, the function is increasing to the left of $x = c$ and decreasing to the right of c.
2. $f(c)$ is a relative minimum of f if the derivative $f'(x)$ is negative in the interval (a, c) and positive in the interval (c, b). In other words, the function is decreasing to the left of $x = c$ and increasing to the right of c.

***There is a good picture at the top of page 266 (calculus part of the text) that shows the signs in the open intervals and possible 'looks' for the graph of the function.

For the following problems; find intervals of increasing or decreasing and the ordered pairs (points) where there are relative extrema and identify.

Example 1: $f(x) = -x^3 - 2x^2 + 15x + 10$

Example 2: $y = \frac{1}{5}x^5 - x$

Example 3: $g(x) = x^4 - 2x^3$

Example 4: $h(x) = x + \frac{1}{x}$

Example 5: $f(t) = -t^3 - 12t^2 - 48t - 64$

Example 6: $y = x^{2/3} - x^{5/3}$

Example 7: $h(x) = x^2 e^x$

(This next problem is actually example 4 from page 269 of the textbook.)
Example 8:
A small company manufactures and sells bicycles. The weekly cost function has been determined to be $C(q) = 10 + 5q + \frac{1}{60}q^3$ and the weekly demand function (price) is $p = D(q) = 90 - q$ (in dollars).

(a) Find the maximum weekly revenue that is possible.

(b) Find the maximum weekly profit that is possible.