MA 22000 Notes, Lesson 19
Textbook (calculus part) Section 2.4
Exponential Functions

In an exponential function, the variable is in the exponent and the base is a positive constant (other than the number 1).

Exponential Function: An exponential function with base *a* is defined as $f(x) = a^x$, where $a > 0$ and $a \neq 1$.

The following are **not** exponential functions. Why?
$f(x) = x^3$ $f(x) = 1^x$

*Note: If *a* (the base) in the above definition was 1, the function would be constant; a horizontal line, $y = 1$.

Exponential functions often describe what is called exponential growth or exponential decay in real life examples.

Example of an exponential function:
Many real life situations model exponential functions. One example models the <u>average amount spent(to the nearest dollar) by a person at a shopping mall after *x* hours</u> and is the function, $f(x) = 42.2(1.56)^x$. The base of this function is 1.56. Notice there is also a 'constant' (42.2) multiplied by the power. Be sure to follow the order of operations; find the exponent power first, then multiply that answer by the 42.2.

Suppose you wanted to find the amount spent in a mall after browsing for 3 hours.
Let $x = 3$.

$$f(3) = 42.2(1.56)^3$$
$$= 42.2(3.796416)$$
$$= 160.2087552$$

To the nearest dollar, a person on average would spend $160.

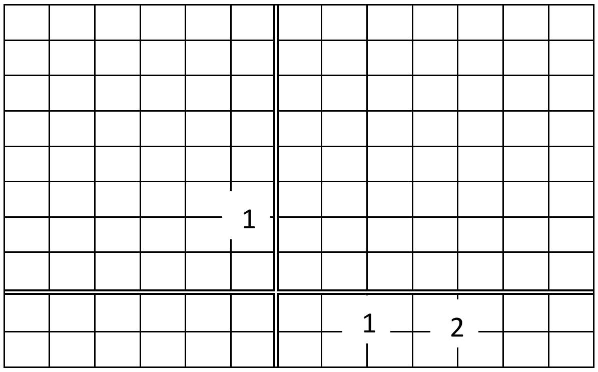
Graph of an exponential function.

Graphing Exponential Functions:
To graph an exponential function, make a table of ordered pairs as you have for other types of graphs.
Notice: If $x = 0$ for b^x, the value is 1 (zero power is 1). For a basic exponential function, the *y*-intercept is 1.

Also, notice that *y* values will always be positive, so the graph always lies above the *x*-axis.

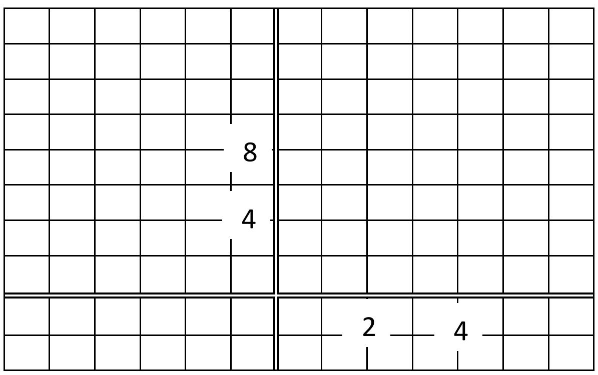
Graph each exponential function.

a) $y = \left(\frac{3}{2}\right)^x$



x	y
-2	
-1	
0	
1	
2	

b) $f(x) = \left(\frac{1}{3}\right)^x$



x	y
-1	
-2	
0	
1	
2	

What do you notice about the graphs above?

Characteristics of Exponential Functions of the form $f(x) = a^x$ (basic)

1. The domain of the function is all real numbers $(-\infty, \infty)$ and the range is all positive real numbers $(0, \infty)$ (graph always lies above the *x*-axis).
2. Such a graph will always pass through the point (0, 1) and the *y*-intercept is 1. There will be no *x*-intercept.
3. If the base *b* is greater than 1 $(b > 1)$, the graph increases left to right and is an increasing function. The greater the value of *b*, the steeper the increase (exponential growth).
4. If the base is between 0 and 1 $(0 < x < 1)$, the graph decreases left to right and is a decreasing function (exponential decay). The smaller the value of *b*, the steeper the decrease.
5. The graph represents a 1-1 function and therefore will have an inverse.
6. The graph approaches but does not touch the *x*-axis. The *x*-axis is known as an **asymptote**.

Solving exponential equations: There are a couple of ways to solve equations with the variable in an exponent. The first way is to rewrite both sides of the equation so the bases are the same.

<u>Ex 1</u>: Solve this equation: $25^{\frac{x}{2}} = 125^{x+3}$

<u>Ex 2:</u> Solve: $32^{3x-1} = 16^{5-9x}$

Compound Interest:

One of the best examples in real life where an exponential function is used is in the banking business, compound interest. You know the simple interest formula $I = Prt$. However, most banks periodically determine interest and add in the account. The amount in an account with an initial amount of *P* dollars invested at an annual interest rate of *r* (as a decimal), compounded *m* times per year for *t* years, returns a compound amount given by this formula.

$$A = P\left(1 + \frac{r}{m}\right)^{mt}$$

<u>Ex 2</u>: Find the amount and the **interest earned** on $5800 at 4.3% interest compounded semiannually for 6 years.

Definition of the number *e*: Letting *P* equal \$1 and letting *r* be 100% and *t* equal 1 year.

$A = P\left(1+\frac{r}{m}\right)^{mt}$ becomes $\left(1+\frac{1}{m}\right)^{m}$ Let the value of *m* become extremely large. Then $\left(1+\frac{1}{m}\right)^{m}$ becomes closer and closer to a number we call *e*, whose approximate value is 2.718281828. (To find the value of *e* on a TI-30XA calculator use these steps: Enter the number 1, press the 2nd key then the LN key (notice that e^x is above the LN key, so we are finding e^1. You should get the approximation given above.

As the amount of money in an account is compounded continuously, rather than periodically as with the compound interest formula earlier; the formula uses this number *e*.

Continuous Compounding Interest: $A = Pe^{rt}$

The steps to convert the regular compound interest formula to the formula above are shown on pages 82-83 in the textbook. It is a difficult process, so I will not demonstrate it in class. You can examine it on these pages, if you are interested.

<u>Ex 3</u>: Suppose \$800 is invested at 4 ½ % interest for 5 years. Find the accumulated amount in the account and the interest earned if…

(a) the money is compounded quarterly.

(b) the money is compounded continuously.

Ex 4:

Suppose that a certain type of bacteria grows rapidly in a warm spot. If 500 bacteria of these bacteria are placed in a dish in a warm location, and the number present after *x* hours is given by the model $P(x) = 500 \cdot 3^{2x}$.

Find the number of bacteria after (a) 1 hour, (b) 3 hours.