## MA 221 F

# Final Exam Practice Problems

- 1. Calculate  $\frac{dy}{dx}$  if  $y = \cos(1-2x)$ . A.  $-\sin(1-2x)$  B.  $-2\sin(1-2x)$  C.  $2\sin(1-2x)$  D.  $\sin(1-2x)$  E.  $-2\cos(1-2x)$
- 2. Find y' if  $y = x \tan^2 x$ . A.  $2x \tan x + \tan^2 x$  B.  $2x \tan x \sec^2 x$  C.  $x \sec^2 x + \tan^2 x$  D.  $2x \tan x \sec^2 x + \tan^2 x$  E. None of these.
- 3. If  $\sin \theta = -0.5473$  and  $\pi < \theta < \frac{3\pi}{2}$ , find  $\theta$  in radians. Give your answer correct to 4 decimal places. A.  $-0.5791\,$  B.  $3.7207\,$  C.  $3.1511\,$  D.  $2.5625\,$  E.  $1.761\,$
- 4. Find the slope of the line perpendicular to the line containing the points (-2,4) and (6,-3). A. -8/7 B. -7/8 C. 1/4 D. 8/7 E. -1/2
- 5. Give the equation, in slope-intercept form, of the line which is parallel to the line 2y-6x-5=0 and passes through the point (-1,3). A.  $y=3x+10\,$  B.  $y=\frac{1}{3}x-9\,$  C.  $y=\frac{1}{3}x+2\,$  D.  $y=-3x\,$  E.  $y=3x+6\,$
- 6. If  $f(x) = \frac{x}{x^2 + 1}$ , find  $\frac{1}{f(3)}$ . A. 3/10 B. 3/16 C. 16/3 D. 10/3 E. 1/3
- 7. If  $f(x) = \frac{2}{x}$  then  $\frac{f(x + \Delta x) f(x)}{\Delta x} = A$ .  $\frac{-2}{x^2}$  B.  $\frac{2}{x + \Delta x} \frac{2}{x}$  C.  $\frac{-2}{x(x + \Delta x)}$  D.  $\frac{2}{x(x + \Delta x)}$  E.  $\frac{-2}{x^2 + \Delta x}$
- 8.  $\lim_{x \to 1} \frac{x^2 + 4x 5}{x^2 1} =$ A. 3 B. 1 C.  $\infty$  D. 0 E. 5
- 9. If the tangent line to the graph of y = f(x) at (2,3) has equation x y + 1 = 0, then f'(2) = A. 1 B. 3/2 C. 2/3 D. -2/3 E. -3/2
- 10. The derivative of  $\frac{x^2+1}{x+5}$  is

  A.  $\frac{(x+5)(2x)-(x^2+1)}{(x+5)^2}$  B. 2x C.  $\frac{(x+5)(2x)}{(x^2+1)^2}$  D.  $\frac{(x^2+1)+(x+5)(2x)}{(x+5)^2}$  E.  $\frac{(x^2+1)-(x+5)(2x)}{(x+5)^2}$
- 11. If  $y = (3 x^2)^3$  then y'' = A.  $-6x(3 x^2)^2$  B.  $24x^2(3 x^2) 6(3 x^2)^2$  C.  $6(3 x^2)$  D.  $24x^2(3 x^2)$  E. None of these.
- 12. The line tangent to the graph of  $f(x) = x \frac{1}{x}$  at (2, 3/2) has slope A. 3/4 B. 3/2 C. 0 D. 1/4 E. 5/4
- 13. A point moves along the x-axis in such a way that its distance, s, from the origin at time  $t \ge 1$  is given by  $s = (t^3 t)^{3/2}$ . Find the velocity of the point when t = 2. Give your answer correct to two decimal places.
  - A. 3.67 B. 3.83 C. 40.42 D. 36.74 E. 21.08
- 14. Differentiate  $y = \sin(x^2)$ .  $\frac{dy}{dx} =$ A.  $\cos(x^2)$  B.  $2x \cos(x^2)$  C.  $-\cos(x^2)$  D.  $\cos(2x)$  E.  $-2x \cos(x^2)$

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15. Find y' if  $y = x \tan x$ .

A. 
$$\sec^2 x$$
 B.  $x \sec^2 x$  C.  $x \tan 1 + \tan x$  D.  $1 + \sec^2 x$  E.  $x \sec^2 x + \tan x$ 

16. Find f'(x) if  $f(x) = \frac{x}{\cos(4x)}$ . f'(x) =

Find 
$$f'(x)$$
 if  $f(x) = \frac{x}{\cos(4x)}$ .  $f'(x) =$ 

A.  $\frac{\cos(4x) + 4x\sin(4x)}{\cos^2(4x)}$  B.  $\frac{-1}{4\sin(4x)}$  C.  $\frac{\cos(4x) - x\sin(4x)}{\cos^2(4x)}$  D.  $\frac{\cos(4x) + x\sin(4x)}{\cos^2(4x)}$  E.  $\frac{-4x\sin(4x)}{\cos^2(4x)}$ 

- 17. The maximum value of  $f(x) = x^3(40-x)^2$  on the closed interval,  $0 \le x \le 40$ , occurs at x =A. 20 B. 15 C. 35 D. 18 E. 24
- 18. Which of the following best describes the graph of  $y = 4x^3 3x^4$ ? The graph has a
  - A. relative maximum point and two points of inflection.
  - B. relative maximum point, a relative minimum point and one point of inflection.
  - C. a relative minimum point and two points of inflection.
  - D. a relative minimum point, two relative maximum points and two points of inflection.
  - E. None of these.
- 19. Which of the following best describes the graph of  $f(x) = \frac{x^2 + 1}{x^2 1}$ ?
  - A. Vertical asymptotes x = 1, x = -1, and symmetric to the x-axis.
  - B. Vertical asymptotes x = 1, x = -1 but not symmetric to either the x or the y-axis.
  - C. Vertical asymptotes x = 1, x = -1 and symmetric to the y-axis.
  - D. No vertical asymptotes and symmetric to the y-axis.
  - E. Vertical asymptote x = 1 and symmetric to the x-axis.
- 20. Find the area of the region bounded by the curves  $x^2 + 4y = 0$  and  $x^2 4y 8 = 0$ . A. 2/3 B. 16/3 C. 6 D. 4/3 E. 10/3
- 21. What is the area of the largest rectangle with sides parallel to the axes which can be inscribed in the first quadrant under the parabola  $y = 4 - x^2$ ? (Give your answer correct to 2 decimal places.)

22. A box with square base and no top is to have a volume of 108 in.3. What is the smallest possible surface area of such a box.

23. Find the value of dy and  $\Delta y$  for  $y=2x^3-4x, x=2$  and  $dx=\Delta x=0.1$ . Give your answer correct to two decimal places.

A. 
$$dy = 2, \Delta y = 2.12$$
 B.  $dy = 2, \Delta y = 2.46$  C.  $dy = 2.12, \Delta y = 2$  D.  $dy = 2.46, \Delta y = 2$  E. None of these.

- 24. Calculate  $\lim_{x\to\infty} \frac{2+3x-2x^3}{3-4x+x^3}$ . A. 2 B.  $\infty$  C. 2/3 D. -3/4 E. -2
- 25. Evaluate  $\int \sqrt{2x+1} dx$

A. 
$$\frac{2}{3}(2x+1)^{3/2} + C$$
 B.  $\frac{1}{3}(2x+1)^{3/2} + C$  C.  $(2x+1)^{-1/2} + C$  D.  $2(2x+1)^{1/2} + C$  E. None of these.

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- 26. Evaluate  $\int_{1}^{2} (6\sqrt{x} \frac{1}{2\sqrt{x}}) dx$ . Give your answer correct to 2 decimal places. A. 9.90 B. 6.90 C. 5.66 D. 7.35 E. None of these
- 27. An object is thrown vertically downward from the top of a building 200 ft high with an initial velocity of 40 ft/sec. Find its velocity when it hits the ground.  $(s = -16t^2 - 40t + 200.)$ A. -40 ft/sec B. -200 ft/sec C. -120 ft/sec D. -80 ft/sec E. None of these.
- 28. Calculate the area bounded by the parabola  $y = x^2$  and the line y = x + 2. A. 9/2 B. 10/3 C. 7/6 D. 15/2 E. None of these.
- 29. Calculate the volume generated by revolving the area bounded by  $y = \sqrt{x}$ , the x-axis and x = 4 about the y-axis. (Express your answer as a definite integral.)

A. 
$$\pi \int_0^4 x dx$$
 B.  $\pi \int_0^4 \sqrt{x} dx$  C.  $2\pi \int_0^4 (4-x)\sqrt{x} dx$  D.  $2\pi \int_0^4 x^2 dx$  E.  $2\pi \int_0^4 x^{3/2} dx$ 

30. Find the function, y, satisfying the following conditions:  $\frac{dy}{dx} = 3x^2 - 1$ , and the graph of y passes through the point (1,3)

A. 
$$y = x^3 - x + 3$$
 B.  $y = x^3 - x$  C.  $y = 6x - 3$  D.  $y = 6x$  E.  $y = 3x^3 - x + 1$ 

31. Calculate the volume generated by revolving the area bounded by  $y = \sqrt{x}$ , the y-axis, and the line y = 2, about the x-axis. (Express your answer as a definite integral.)

A. 
$$\pi \int_0^4 (2 - \sqrt{x})^2 dx$$
 B.  $\pi \int_0^4 (4 - x) dx$  C.  $2\pi \int_0^4 (2 - \sqrt{x}) dx$  D.  $2\pi \int_0^4 (4\sqrt{x} - x) dx$  E.  $\pi \int_0^4 x dx$ 

- 32. If f'(x) = 4x 3 and f(0) = 7 calculate f(2). A. 5 B. 2 C. 7 D. 9 E. 3
- 33. Calculate the centroid of a quarter circle of radius r.

Calculate the centroid of a quarter circle of radius 
$$r$$
.

A.  $\overline{x} = \frac{r}{3\pi}, \overline{y} = \frac{r}{3\pi}$  B.  $\overline{x} = \frac{4r}{3}, \overline{y} = \frac{4r}{3}$  C.  $\overline{x} = \frac{4r}{\pi}, \overline{y} = 0$  D.  $\overline{x} = \frac{4r}{\pi}, \overline{y} = \frac{4r}{\pi}$  E.  $\overline{x} = \frac{4r}{3\pi}, \overline{y} = \frac{4r}{3\pi}$ 

34. Calculate the x-coordinate of the centroid,  $\bar{x}$ , of the area given in problem 26, if the area is 16/3 square units.

A. 
$$\overline{x} = 2$$
 B.  $\overline{x} = 3/2$  C.  $\overline{x} = 9/5$  D.  $\overline{x} = 12/5$  E.  $\overline{x} = 1/5$ 

- 35. Find the work done in pumping the water out of the top of a cylindrical tank 5 ft in radius and 10 ft high, if the tank is initially half full of water, which weighs 62.4 lb/ft<sup>3</sup>.
  - A.  $93,750\pi$  ft-lb B.  $58,500\pi$  ft-lb C.  $7,800\pi$  ft-lb D.  $15,600\pi$  ft-lb E. None of these.
- 36. A spring of natural length 12 ft. requires a force of 6 lb. to stretch it 2 ft. Find the work done in stretching it 6 ft. (F = kx)

37. A vertical rectangular floodgate on a dam is 5 ft long and 4 ft deep. Find the force on the floodgate if its upper edge is 3 ft below the surface. (The weight of water is 62.4 lb/ft<sup>3</sup>.) Give your answer correct to the nearest integer.

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38. A horizontal tank with vertical circular ends is filled with oil. If the radius of each end is 2 m, find the force on one end of the tank. (Assume w is the weight of the oil.) Express your answer as a definite integral. (Hint: Assume that the origin is at the center of one of the circular ends.)

A.  $2w \int_0^2 y \sqrt{4-y^2} dy$  B.  $w \int_{-2}^2 \sqrt{4-y^2} dy$  C.  $2w \int_{-2}^2 (2-y) \sqrt{4-y^2} dy$  D.  $2w \int_{-2}^2 (2-y) dy$  E. None of these.

39. Grant and Stadium Streets are straight and perpendicular to each other. A black 1997 Porsche 911 is going on Grant Street toward the intersection of the two streets at 60 mph (miles per hour), and a red 1993 Volkswagen Golf is going on Stadium Street toward the same intersection at a rate of 40 mph. At what rate is the distance between the two cars decreasing when the Porsche is 1/2 mile from the intersection and the Golf is 3/8 mile from it?

A. 40 mph B. 56 mph C. 60 mph D. 72 mph E. 32 mph

- 40. The line perpendicular to the graph of  $f(x) = x \frac{1}{x}$  at  $(2, \frac{3}{2})$  has slope A. 3/4 B. 3/2 C. 0 D. 1/4 E. -4/5
- 41. Find the center C and radius r of the circle whose equation is

$$x^2 + y^2 - 10x + 6y + 30 = 0$$

A. 
$$C = (-5,3); r = 2$$
 B.  $C = (-5,3); r = 4$  C.  $C = (5,-3); r = 8$  D.  $C = (5,-3); r = 4$  E.  $C = (5,-3); r = 2$ 

- 42. If  $y^3 + x^2 = 9$  and  $\frac{dx}{dt} = 5$ , find  $\frac{dy}{dt}$  when x = 1. A. -5/6 B. 2/3 C. -10 D. 1/3 E. 10/3
- 43. Water is flowing into a tank which is in the shape of a right circular cylinder standing on its circular base. If the water is flowing in at a rate of 80 cu. ft. per min. and the radius of the base of the tank is 4 ft., how fast is the water rising when the water is 10 ft. deep?

A. 
$$\frac{\pi}{5}$$
 ft/min B.  $5\pi$  ft/min C.  $\frac{50}{\pi}$  ft/min D.  $\frac{5}{\pi}$  ft/min E.  $50\pi$  ft/min

44. Find the mean value of  $f(x) = x^{1/3}$  on the interval [0,2].

A. 
$$\frac{3}{2^{2/3}}$$
 B.  $\frac{3}{4}$  C.  $\frac{1}{2^{4/3}}$  D.  $\frac{3}{2^{1/3}}$  E.  $\frac{3}{2^{5/3}}$ 

45. Find the root mean square of  $f(x) = \sqrt{x}(1-x)$  on the interval [0, 2]. A.  $1/\sqrt{3}$  B. 2/3 C.  $2/\sqrt{3}$  D.  $\sqrt{3}/2$  E.  $1/\sqrt{6}$ 

#### Answers

1. C; 2. D; 3. B; 4. D; 5. E; 6. D; 7. C; 8. A; 9. A; 10. A; 11. B; 12. E; 13. C; 14. B; 15. E; 16. A; 17. E; 18. A; 19. C; 20. B; 21. C; 22. D; 23. A; 24. E; 25. B; 26. B; 27. C; 28. A; 29. E; 30. A; 31. B; 32. D; 33. E; 34. D; 35. B; 36. A; 37. D; 38. C; 39. D; 40. E; 41. E; 42. A; 43. D; 44. E; 45. A.