MA 15200 Lesson 22 Sections 2.3 and 2.4

I Slope of a Line

A measure of the 'steepness' of a line is called the **slope** of the line. Slope compares a vertical change (called the **rise**) to the horizontal change (called the **run**) when moving from one point to another point along a line.

Slope is a ratio of vertical change to horizontal change.

If a non-vertical line contains points (x_1, y_1) and (x_2, y_2), the slope of the line is the ratio described by

$$m = \frac{rise}{run} = \frac{\text{change in y}}{\text{change in x}} = \frac{y_2 - y_1}{x_2 - x_1}.$$

*Note: Always be consistent in the order of the coordinates.

When given two points, it does not matter which one is called point 1 and which point 2. $\frac{y_2 - y_1}{x_2 - x_1} = \frac{y_1 - y_2}{x_1 - x_2}$

There are 3 ways to find slope.

1. Using the slope formula (above)
2. Counting rise over run (when shown a graph)
3. Finding the equation in slope-intercept form (later in lesson)

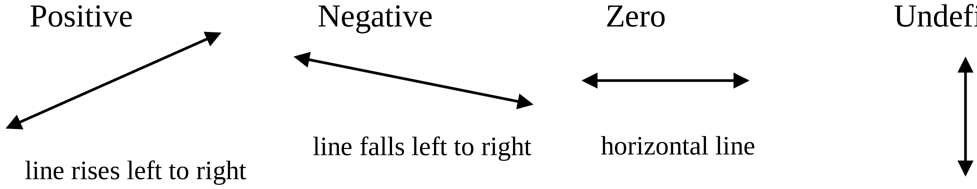
If a line is horizontal, the numerator in the slope formula will be 0 (the y coordinates of all points of a horizontal line are the same). The slope of a horizontal line is 0.

If a line is vertical, the denominator in the slope formula will be 0 (the x coordinates of all points of a vertical line are the same). A number with a zero denominator is not defined or undefined. The slope of a vertical line is undefined.

Never say 'no slope' to define the slope of a vertical line. No slope could be interpreted as 0 or undefined.

There are 4 types of slopes.

Positive Negative Zero Undefined



line rises left to right

line falls left to right

horizontal line

vertical line

Ex 1: Find the slope of a line containing each pair of points.
Describe if the line rises from left to right, falls from left to right, is horizontal, or is vertical.

$a)\quad P(2,-3),\ \ Q(-6,-12)$

$b)\quad P(-4,2),\ \ Q(5,3)$

$c)\quad P(-2,6),\ \ Q(-5,9)$

$d)\quad P(-4,10),\ \ Q(-4,-8)$

$e)\quad P(6,-2),\ \ Q(9,-2)$

II Equations of Lines, Point-Slope Form

Begin with the slope formula and drop the subscript 2's, putting them back as regular variables.

$$m=\frac{y_2-y_1}{x_2-x_1}\rightarrow\ m=\frac{y-y_1}{x-x_1}\rightarrow \text{cross multiply}\rightarrow y-y_1=m(x-x_1)$$

This is known as the point-slope form of the equation of a line.

> **Point-Slope Form**
> If a line contains the point (x_1,y_1) and has the slope m, then the equation in **point-slope form is** $y-y_1=m(x-x_1)$.

> When using point-slope form, substitute values for x_1, y_1, and m. Never substitute for *x* and *y*. These are the variables of the equation.

Ex 2: *a*) Write an equation in point-slope form for a line with a slope of $\frac{2}{3}$ and through the point (2, 12).

$b)$ Find the slope and an indicated point for a line with equation $y+2=-4(x-5)$.

Ex 3: Write an equation of a line with the given points in point-slope form, then solve for *y*.

$a)$ $m=-4,\;\; P(-3,8)$

$b)$ $m=\frac{3}{4},\; P(-2,12)$

Ex 4: Find the equation of the line through points $P(-2,-5)$ and $Q(6,-4)$ in point-slope form and then solve for *y*.

III Slope-Intercept Form

Let the point known be the *y*-intercept and call it (0, *b*).

$$y-y_1=m(x-x_1)$$
$$y-b=m(x-0)$$
$$y-b=mx$$
$$y=mx+b$$

If a line has slope *m* and a *y*-intercept at *b*, then the **slope-intercept form of the equation of the line is** $y=mx+b$ or $f(x)=mx+b$.

Ex 5: Find an equation of a line with slope $-\frac{3}{8}$ and point $(0,-6)$ in slope-intercept form.

Ex 6: Find an equation in slope-intercept form for a line with the following slope and point

$m=\frac{3}{2},\ P(-6,-1)$

If a line has a slope *m* and a *y*-intercept of *b* (point (0, *b*)), then the equation of the line can be written as $y=mx+b$. This is known as **slope-intercept form of the equation of a non-vertical line.** This can also be written as $f(x)=mx+b$ and is a **linear function.**

Ex7: Find the slope of each line given its equation.

$a)\quad y=-4x+2$

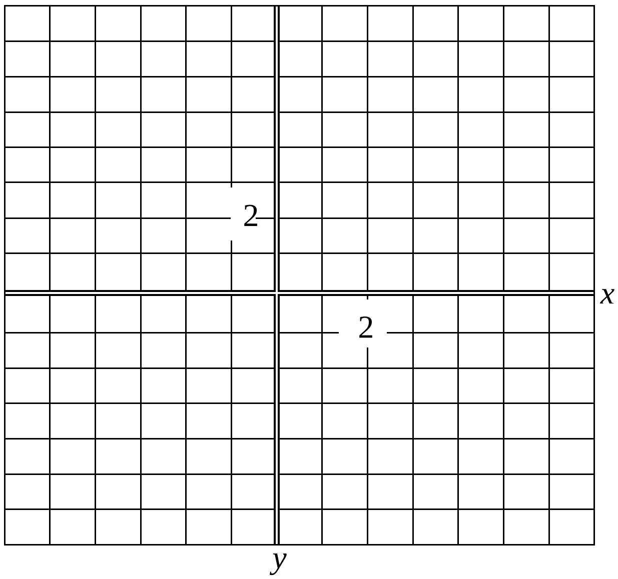
$b)\quad 7x-8y=12$

IV Graphing a Line using slope and y-intercept

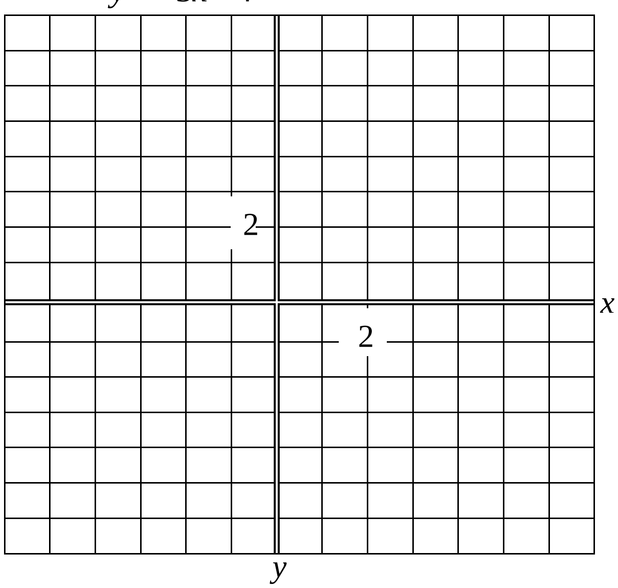
1. Plot the y-intercept on the *y*-axis (0, *b*)
2. Obtain a second point using the slope *m*. Write *m* as a fraction and use rise over run, starting at the *y*-intercept. (Note: If the slope is negative, let the rise be negative and the run positive. Move down and then right. If you let the run be negative and rise positive, move up and then left.)
3. Connect the two points to draw the line. Put arrows at each end to indicate the line continues indefinitely in both directions.

Ex 8: Graph each line.

$y = \frac{1}{2}x + 2$



$y = -3x - 4$



V Equations and Graphs of Horizontal or Vertical Lines

It a line is horizontal, the slope-intercept form is written $y = 0x + b$ or $y = b$. A vertical line cannot be written in slope-intercept form because there is no possible number for *m*. However, a vertical line would have points all with the same *x*-coordinate. So a vertical line can be written as $x = a$, where *a* is the *x*-intercept.

If *a* and *b* are real numbers, then

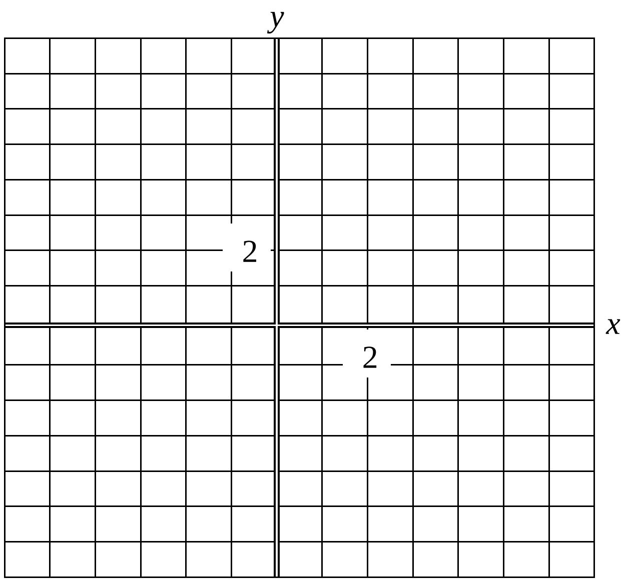
- The graph of the equation *x* = *a* is a vertical line with an *x*-intercept of *a*.
- The graph of the equation *y* = *b* is a horizontal line with a *y*-intercept of *b*.

Note: If the equation has only an *x* or only a *y*, solve for that variable. Then you will know where the intercept is and be able to graph the line.

Ex 4: Graph each line.

$a)$ $x = -3$

$b)$ $y = 4$



VI Intercepts

In lesson 19 we discussed how to identify intercepts from a graph. How are intercepts found from an equation? It is easy to determine that at least one coordinate of an intercept is zero. An *x*-intercept will have the form $(a, 0)$ and a *y*-intercept will have the form $(0, b)$. **Zero is your friend**, when it comes to intercepts.

Finding any intercepts:

1. **Find the *x*-intercept by letting *y* = 0 and solving for *x*.**
2. **Find the *y*-intercept by letting *x* = 0 and solving for *y*.**

Ex 5: Find the *x*-intercept and *y*-intercept (if they exist) for these linear equations.

$a)\quad 2x-3y=12$

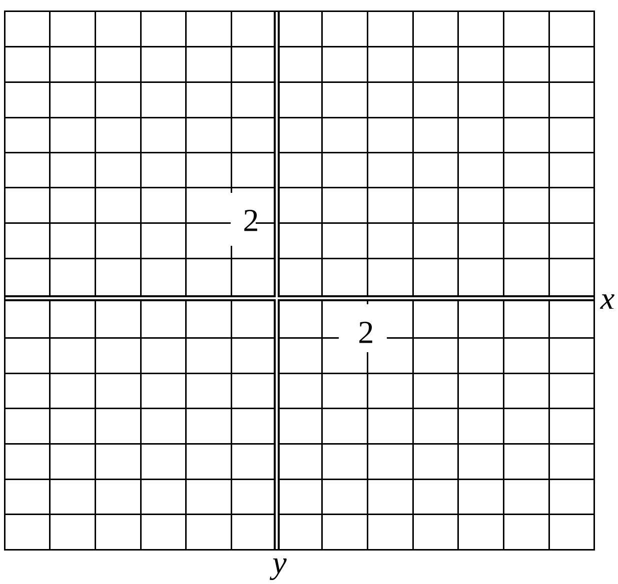
$b)\quad 5x=12y-9$

$c)\quad x=9$

Using Intercepts to Graph a Line

1. Plot the *x*-intercept.
2. Plot the *y*-intercept.
3. Draw a line through the two points that are the intercepts.

<u>Ex 6:</u> Find the intercepts and use them to graph the line. $4x-3y-12=0$



x	y