MA 15200 Lesson 3 Section P.2 (part 1)

I Review of Exponential Expressions

Exponential Expression: $x^n = \underline{x \cdot x \cdot x \ldots x}$
n factors of x

x is called the **base**, n is called the **exponent**, x^n is called a **power**

Ex 1: Write the following without exponents.

$a)\quad 8^3 =$

$*b)\quad (-3)^4 =$

$*c)\quad -3^4 =$

$d)\quad -12x^3 =$

$e)\quad (3m)^3 =$

Ex 2: Write with exponents.

$*a)\quad 2a \cdot a \cdot a \cdot a \cdot a =$

$*b)\quad (2a)(2a)(2a)(2a)(2a) =$

$c)\quad -(-n)(-n)(-n)$

*Note: There is a difference between ax^n and $(ax)^n$

If you have a power, such as 12.6^7, a scientific calculator with a **power key** can be used to evaluate the power. The power key may look like $\boxed{y^x}$ or $\boxed{\wedge}$. You are responsible to know how to use this key on *your calculator*.

Example: $12.6^7 \approx 50{,}418.952.18$

II Rules of Exponents

A) Product Rule for Exponents: $b^m b^n = b^{m+n}$

Note: There must be the same base and the base is kept.

Examples: $(a)\quad m^4 m^8 m^{-3} = m^9 \qquad (b)\quad 9^6 \cdot 9^7 \cdot 9 = 9^{14}$

B) *Power Rules for Exponents: $(b^m)^n = b^{mn} \qquad (ab)^n = a^n b^n \qquad \left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$

***Includes what the textbook calls Powers to Powers, Product-to-Powers, and Quotient-to-Powers Rules

Note: Power Rules demonstrate how parentheses are removed from an exponential expression.

Examples: $(a)\ (4^3)^4 = 4^{12}$ $(b)\ (3xy^2)^3 = 27x^3y^6$ $(c)\ \left(\frac{-4}{xy^3}\right)^4 = \frac{256}{x^4y^{12}}$

C) Zero Exponent Rule: $b^0 = 1$

Proof: $x^0x^3 = x^3$ and $1\cdot x^3 = x^3 \rightarrow x^0 = 1$

Examples: $(a)\ 2x^0y^2 = 2y^2$ $(b)\ (3xy)^0y = y$

D) Negative Exponent Rules: $b^{-n} = \frac{1}{b^n}$, $\frac{1}{b^{-n}} = b^n$, $\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$

Proof: $x^{-n}x^n = x^0 = 1$ and $\left(\frac{1}{x^n}\right)x^n = \frac{x^n}{x^n} = 1 \rightarrow x^{-n} = \frac{1}{x^n}$

Examples: $(a)\ (a^2)^{-4} = a^{-8} = \frac{1}{a^8}$ $(b)\ \frac{mn^{-2}}{a^{-6}b^2} = \frac{a^6m}{b^2n^2}$

Note: Students often think a negative exponent indicates a negative number. This is not true, necessarily! Think reciprocal when you see a negative exponent.

E) Quotient Rule for Exponents: $\frac{x^m}{x^n} = x^{m-n}$

Proof: $\frac{x^m}{x^n} = x^m\left(\frac{1}{x^n}\right) = x^mx^{-n} = x^{m-n}$

Examples: $(a)\ \frac{3r^4}{8r^8s} = \frac{3r^{-4}}{8s} = \frac{3}{8r^4s}$ $(b)\ \frac{9x^2yz^{-2}}{12x^4y^{-5}z^4} = \frac{3x^{-2}y^6z^{-6}}{4} = \frac{3y^6}{4x^2z^6}$

Note: When using the quotient rule, the powers **always are in the numerator** initially.

III Simplifying Using the Rules of Exponents

The properties or rules of exponents are used to simplify exponential expressions. An exponential expression is simplified when

- No parentheses appear. Use $(ab)^n = a^nb^n$ or $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$
- No powers are raised to powers. Use $\left(b^m\right)^n = b^{mn}$
- Each base occurs only once. Use $b^mb^n = b^{m+n}$ or $\frac{b^m}{b^n} = b^{m-n}$
- No negative or zero exponents appear. Use
  $b^{-n} = \frac{1}{b^n}$ or $\frac{1}{b^{-n}} = b^n$ or $\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$ or $b^0 = 1$

Use the steps above to simplify.

Ex 3: $(x^4y^{-2})^3(4xy^5)^{-1} =$

Ex 4: $\left(\dfrac{-2a^4b^{-1}a^4}{3b^{-8}ba^{-4}}\right)^{-2} =$

Ex 5: $\dfrac{(3x^2yz^5)^{-2}x^{-2}z^3}{x^4(y^2z^{-3})^2} =$

Ex 6: $\left(\dfrac{2x^{-3}}{y^2}\right)^3\left(3^{-1}x^{-4}y^2\right) =$

Ex 7: True or False?

$a)\quad 2^{-4} = -16$

$b)\quad x^4 = 4x$

$c)\quad (-3)^4 = -81$

$d)\quad 3^{-4} = \frac{1}{81}$

$e)\quad (-5)^{-2}$ is negative.

$f)\quad (-4a)^3 = -4a^3$

$g)\quad -8x^2 = 64x^2$

Ex 8: Evaluate.

$$4^0 + 3^{-1} + \left(\frac{1}{2}\right)^2$$