MA 15200 Lesson 4 Section P.2 (part 2) and Section P.3 (part 1)

I Writing Numbers in Scientific Notation

A number is written in **scientific notation** when it is in the form $a \times 10^n$ where $1 \le |a| < 10$ and n is an integer. If the original number is 10 or greater, the exponent of the base 10 will be positive. If the original number is less than 1, the exponent of the base 10 will be negative. If the original number is at least 1, but less than 10, the exponent of the base 10 will be zero.
A positive exponent of 10 corresponds to a 10 or larger number. A negative exponent of 10 corresponds to a decimal number (less than 1).

Ex 1: Write each standard number in scientific notation.

a) It has been estimated that in the year 2020, the world population will be approximately 7,516,000,000 (Source: U.S. Bureau of the Census, International Data Base)

b) A grain of sand has a mass of 0.0648 grams.

Ex 2: Write each scientific notation number as a standard number.

a) The lightest known particle in the universe, a neutrino has a maximum mass of 1.8×10^{-36} kg. (Source: Guinness Book of World Records, 2004)

b) A ton of 5-dollar bills is worth $\$4.54\times10^{6}$.

II Computations with Scientific Notation

- Multiply or Divide the Numeral parts.
- Use the Product or Quotient Rules of Exponents with the base 10.
- ‘Re-adjust’ if necessary to write number in scientific notation.
- If directed, convert scientific notation to a standard decimal number.

Ex 3: Use scientific notation to simplify these computations. Write answers in scientific notation and in decimal notation.

a) $(3.8\times10^{-4})(4.1\times10^{8})$

b) $\frac{48,000}{0.00032}$

c) $\frac{(45,000,000,000)(212,000)}{0.00018}$

d) $\frac{(65,000)(45,000)}{(250,000)(0.00001)}$

<u>Ex 4</u>: Use scientific notation to solve these application problems.

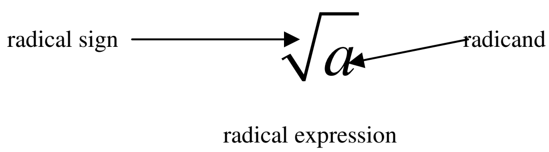
a) The mass on one proton is 1.67248×10^{-24} gram. Find the mass of one billion of these protons.

b) A sheet of plastic shrink wrap has a thickness of 0.00015 mm. The sheet is 1200 mm by 79 mm. Find the volume of the sheet in cubic mm.

c) In a certain country, taxes for 2008 of $\$1.8\times10^{12}$ were collected. If there were 244 million people, what was the average amount of taxes per person? Round to the nearest thousandth in scientific notation and convert that number to a standard decimal number.

<u>I Square Roots</u>

If $b^2 = a$, then b is a square root of a.
If a is a nonnegative real number, the nonnegative number b such that $b^2 = a$, denoted by $b = \sqrt{a}$, is the **principal** square root of a.



Ex 1: Evaluate each. If not real, write 'not real'.

$a)\ -\sqrt{81}$

$b)\ \sqrt{\dfrac{25}{36}}$

$c)\ \sqrt{36+64}$

$d)\ \sqrt{36}+\sqrt{64}$

$e)\ \sqrt{-49}$

Many times students believe that $\sqrt{a^2} = a$. However, the principal square root is always positive. Examine the following.

$\sqrt{8^2} = \sqrt{64} = 8$

$\sqrt{(-8)^2} = \sqrt{64} = 8,$ not -8

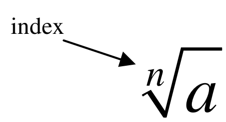
$-\sqrt{8^2} = -\sqrt{64} = -8$

$\sqrt{-8^2} = \sqrt{-64}$, which is not real

> In general:
> $$\sqrt{a^2} = |a|$$
> Therefore, we will always assume that variables represent positive numbers in order to avoid using absolute value signs.

<u>II Other Types of Roots</u>

$\sqrt[n]{a} = b$ means that $b^n = a$ If n is even, then a and b must be positive. If n is odd, a and b can be any real numbers.



> If no index is written, the root is assumed to be a square root.

Ex 2: Evaluate each. If not real, write 'not real'.

$a)\ \sqrt[3]{-125}$

$b)\ \sqrt[4]{-81}$

$c)\ \sqrt[6]{64}$

$d)\ \sqrt[3]{\frac{27}{8}}$

$e)\ \sqrt{0.04}$

$f)\ \sqrt[5]{-32}$

III <u>The Product and Quotient Rules of Radicals</u>

If all expressions represent real numbers,

$$\sqrt[n]{a}\cdot\sqrt[n]{b}=\sqrt[n]{ab} \text{ and } \sqrt[n]{ab}=\sqrt[n]{a}\cdot\sqrt[n]{b}$$

$$\frac{\sqrt[n]{a}}{\sqrt[n]{b}}=\sqrt[n]{\frac{a}{b}} \text{ and } \sqrt[n]{\frac{a}{b}}=\frac{\sqrt[n]{a}}{\sqrt[n]{b}} \qquad (b\neq 0)$$

Note: These properties are for multiplication and division. Similar statements are not true for addition or subtraction. ($\sqrt[n]{a+b}\neq\sqrt[n]{a}+\sqrt[n]{b}$, for example)

Ex 3: Use the product or quotient rules of radicals (if you can) to write as one radical. Simplify, if possible.

$a)\ \sqrt{3}\cdot\sqrt{10}=$

$b)\ \frac{\sqrt[3]{54}}{\sqrt[3]{2}}=$

$c)\ \sqrt{5}\cdot\sqrt[3]{2}=$