MA 15200 Lesson 7 Section P.5

***There are several good study tips in this lesson on the textbook pages.**

Factoring Polynomials

1st factoring method

If two binomials are multiplied together, each polynomial is a **factor** of the product.
The first factoring method to always try is called **factoring out the greatest common factor.**
If a polynomial cannot be factored using integer coefficients, it is a **prime** polynomial.

Ex 2: Factor each polynomial by factoring out the GCF.

$a)\quad 5y^2z - 15yz^3 =$

$b)\quad 6ab^2c + 9a^2b^2c - 12abc^3 =$

$c)\quad 4(x-y) + 3c(x-y) =$

$d)\quad 11xy^2 + 3ab =$

2nd factoring method

Some 4-term polynomials can be factored using what is called the **grouping method.**

1. Group the first two terms together and factor out the GCF.
2. Group the last two terms together and factor out the GCF.
3. If there is a binomial that is a GCF of the result, that GCF is factored out and the result is a product of two binomial factors.

Ex 3: Factor each polynomial by the grouping method. Hint: You may have to reorder the terms.

$a)\quad ax - ay + 2bx - 2by$

$b)\quad 12x^3 - 20y - 16x^2y + 15x$

$c)\quad ax - 4x - 2ay + 8y$

$d)\quad 4ax + 12 - 3a + a^2$

A polynomial of the form $x^2 - y^2$ is called a **difference of squares.** A difference of squares factors as follows: $x^2 - y^2 = (x+y)(x-y)$

3rd factoring method

Ex 3: Factor each polynomial completely.

$a)\quad 4x^4 - 16 =$

$b)\quad 25 + 16a^2$

$c)\quad (x+2)^2 - 9 =$

$d)\quad 81r^4 - 16s^4 =$

Note: A sum of squares is prime, unless there is a GCF.

The following are called **perfect square trinomials** because each equals a binomial squared when factored (as shown in these formulas).

4th factoring method

$$x^2 + 2xy + y^2 = (x+y)^2$$

Perfect square

Double product of sq. roots of 1st term and last term

Perfect square

$$x^2 - 2xy + y^2 = (x-y)^2$$

Ex 4: Factor each.

$a)\quad 9a^2 - 12ab + 4b^2 =$

$b)\quad 100x^2 + 20x + 1 =$

$c)\quad 9x^2 - 15x + 4$

There are two ways commonly used to factor trinomials. Both require these first steps.

1. Write the trinomial in descending order.
2. Factor out any GCF, possibly including a (-) so that the leading term has a positive coefficient.

Some students find success with 'trial and error', simply trying different combinations until one is found such that when FOILed, the proper trinomial results.

To factor a trinomial of the form x^2+bx+c, using the 'trial-and-error' process.

Follow these steps:

1. Make your first terms have a product of x^2 (x and x)
2. Make your last terms have a product of c.
3. Find the sum of the inner and outer terms and check if it equals bx. If not, go back to steps 1 and 2 and try a different combination, until step 3 checks.

Other students like to use grouping to factor a trinomial. Sometimes I call this the product/sum method. Here are the steps for grouping or the product/sum method.

If a trinomial is of the form x^2+bx+c, find a pair of numbers r and s; such that the product (r)(s) equals c and the sum $r + s$ equals b. Then the factors are $(x+r)(x+s)$.

Ex 5: Factor each trinomial.

$a)$ $b^2+12b+20=$

Factoring trinomials is the last factoring method.

We will continue factoring trinomials in the next lesson.

$b)$ $x^2-7x+10=$

$c)$ $c^2+10c+100=$

$d)$ $3x^3-12x^2-36x=$

$e)$ $m^2-20mn+64n^2$

$f)$ x^4-x^2-6

$g)$ $x^{10}+4x^5-192$