MA 15200 Lesson 31, Section 4.4

An **exponential equation** is an equation with a variable in one of its exponents.

A **logarithmic equation** is an equation with one or more logarithmic expressions that contain a variable.

Solving an exponential equations: (There are 2 types of exponential equations.)

A If you can write the left and right sides using the same base, then the exponents are the same. For example:

$$8^{3x} = 16^{5x-1}$$
$$\left(2^3\right)^{3x} = \left(2^4\right)^{5x-1}$$
$$2^{9x} = 2^{4(5x-1)}$$
$$9x = 20x - 4$$
$$4 = 11x$$
$$\frac{4}{11} = x$$

If $b^M = b^N$, then $M = N$

If possible, try to write both exponential expression with the same base.

1) Write expression as $b^M = b^N$
2) Set $M = N$
3) Solve for the variable.

This cannot always be done, however.

If two quantities are equal, the logs to the same base of those quantities are equal.

Therefore: $\log_b M = \log_b N \Leftrightarrow M = N$

B Therefore, to solve an exponential equation:

1. Write with exponential expression on one side.
2. Take either the common logs or the natural logs of both sides.
3. Use the properties of logs and solve.
4. Use a scientific calculator if asked to approximate.

Ex 1: Solve each equation. Approximate to 4 decimal places, if necessary.

$a)$ $8^{x-2} = 2^{2x}$ $b)$ $3^{x^2-2x} = \frac{1}{3}$

$c)$ $9^x = 13$

$d)\quad 10^{x-1} = 25$ $\qquad$ $e)\quad e^{3x-2} - 5 = 256$

Solving logarithmic equations:

There are two types of logarithmic equations.

1. Type 1
   - Express the equation in the form $\log_b M = \log_b N$, a single logarithm on each side. You may have to use the properties of logarithms.
   - Use the 1-1 property (If $\log_b M = \log_b N$, then $M = N$). In other words, set the arguments equal.
   - Solve for the variable.
   - **Check** the proposed solution(s) in the original equation. Arguments should all be positive.
2. Type 2
   - Express the equation in the form $\log_b M = c$.
   - Rewrite the equation in exponential form $b^c = M$
   - Solve for the variable.
   - **Check** all proposed solution(s) in the original equation. Arguments should all be positive.

The coefficient of each logarithm should be one.

When solving logarithmic equations, always remember that **any value for the variable must make any arguments be positive**. A logarithm of a negative number does not exist. Any possible solution that makes a 0 or negative argument must be disregarded.

Ex 2: Solve each of these equations.

$a)\quad \log_2(x-4) - \log_2(3x-10) = -\log_2 x$

$b)\quad \ln(3x+2)=\ln(4x+10)$

$c)\quad \ln(x-3)=\ln(7x-23)-\ln(x+1)$

<u>Ex 3:</u> Solve each equation.

$a)\quad 2\log_3(7+x)=4$

$b)\quad \log x+\log(x+9)=1$

$c)$ $2\log_3 x - \log_3(x-4) - \log_3 2 = 2$

$d)$ $\log x = 1 - \log(x-9)$

e) $\log \dfrac{5x+2}{2(x+7)} = 0$