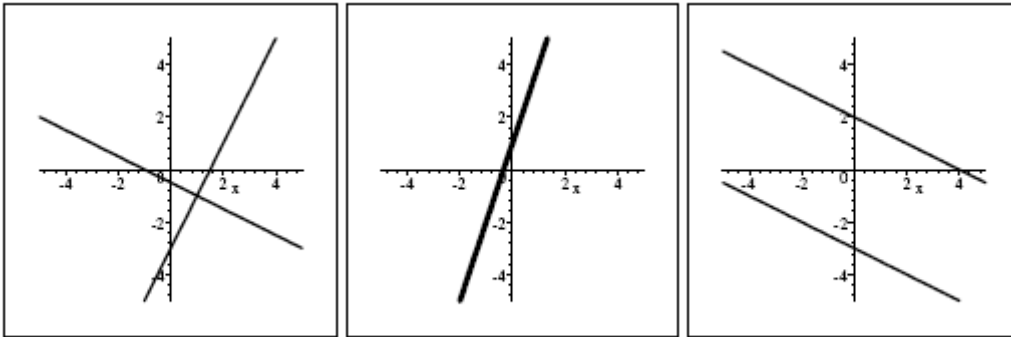


MA 15200 Lesson 32, Section 5.1

A linear equation with two variables has infinite solutions (many ordered pairs, example $2x - 3y = 6$) However, if a second equation is also considered, and we want to know what ordered pairs they may have in common, how many solutions may there be? When two linear equations are considered together, it is known as a **system of linear equations or a linear system**. The **solution(s)** of the system is/are any **ordered pair(s)** they have in common.

Consider graphing two lines. How many points may they have in common?



$$2x - y = 3, x + 2y = -1$$

$$y = 3x + 1, 6x - 2y = -2$$

$$y = -\frac{1}{2}x - 3, x + 2y = 4$$

As demonstrated in the graphs above, there are 3 situations.

1. The graphs may intersect at one point. If the lines of the equations intersect, the solution is one ordered pair.
2. The graphs may intersect at an infinite number of points. If the lines of the equations form the same line, the solution is an infinite number of ordered pairs.
3. The graphs may not intersect. If the lines of the equations do not intersect, there is no solution.

Ex 1: Given the ordered pair, determine if it is a solution of the shown system.

$$a) \begin{cases} 2x + 5y = 16 \\ 3x + y = -2 \end{cases} \quad (-2, 4)$$

$$b) \begin{cases} 5x + 2y = 1 \\ 10x + 5y = 1 \end{cases} \quad \left(\frac{3}{5}, 1\right)$$

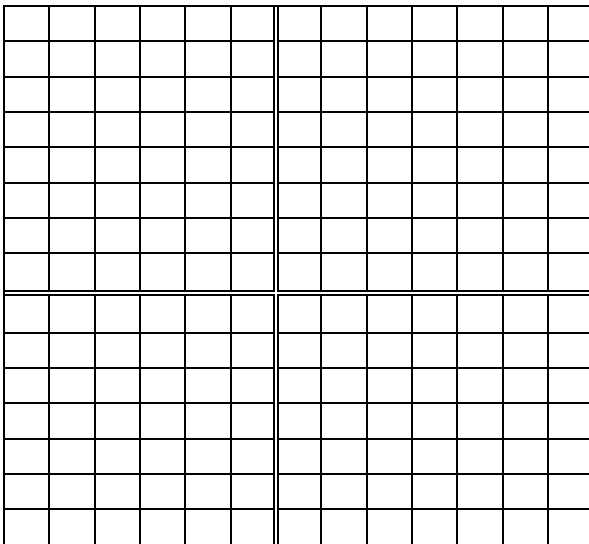
If an ordered pair is a solution of the system of equations, it is a solution of both equations. In other words, substituted in both equations yields a true statement.

There are three methods that may be used to find the solution of a system of linear equations. Graphing the lines could lead to an ordered pair where the lines intersect. The ordered pair is the solution. Two more algebraic methods are covered in this lesson.

1. **The Graphing Method**-Graph each line and identify any common point (ordered pair), or determine if the lines are the same or if the lines are parallel.
2. **The Substitution Method**
3. **The Addition Method**

For the next example we will use the graphing method. The graphing method is not really the best method. Graphing is not very accurate. And, if the solution has coordinates that are fractions or decimals, the coordinates of the point of intersection (if there is one) would be difficult to determine. There is a good study tip on using the graphing method on page 483 of the textbook.

Ex 1: Find the solution to the system $\begin{cases} 3x + 2y = 2 \\ 2x - 3y = -16 \end{cases}$ by graphing.



Substitution Method:

1. Solve either of the equations for one variable in terms of the other.
2. Substitute the expression found in step 1 for that variable in the *other equation*. This will result in an equation with only one variable.
3. Solve the equation for that variable.
4. Back-substitute the value found in step 3 into one of the original equations and solve for the remaining variable.
5. Check and write the solution as an ordered pair or identify x and y .

Ex 2: Solve each system of linear equations using the substitution method.

$$a) \begin{cases} 2x + 3y = 0 \\ y = 3x - 11 \end{cases}$$

$$b) \begin{cases} 4x + 5y = 4 \\ 8x - 15y = 3 \end{cases}$$

Note: It is easier to use the substitution method when a variable of one of the equations has a coefficient of 1 or -1. This avoids the use of fractions.

Examine these examples.

$$A \quad \begin{cases} x + 3y = 4 \\ y = -\frac{1}{3}x + \frac{4}{3} \end{cases}$$

$$x + 3\left(-\frac{1}{3}x + \frac{4}{3}\right) = 4$$

$$2x + 6\left(-\frac{1}{3}x + \frac{4}{3}\right) = 8$$

$$2x - 2x + 8 = 8$$

$$8 = 8$$

$$B \quad \begin{cases} y = -x + 3 \\ x + y = 7 \end{cases}$$

$$x + (-x + 3) = 7$$

$$x - x + 3 = 7$$

$$3 = 7$$

You will notice that both variables '**dropped out**' in both example A and example B.

A In example A, a true statement was the result ($8 = 8$). This indicates that there are several solutions, because the statement is true. Graphically, these are the same line. There are an **infinite** number of solutions, every point on the line. The

general solution may be written as a set this way: $\left\{ (x, y) \mid y = -\frac{1}{3}x + \frac{4}{3} \right\}$.

Although there are an infinite number of solutions in such a system, this does not mean that any ordered pair is a solution. It must be an ordered pair that satisfies the equation of the line.

B In example B, the result was a false statement ($3 = 7$). Because the result was false, there is no ordered pair that is common to both equations. The system has **no solution**. Graphically, these lines would be parallel.

This system is called an **inconsistent system**.

Ex 3: Solve the system $\begin{cases} 3x - y = 12 \\ y = 3x - 12 \end{cases}$.

Addition Method: As with the substitution method, the goal of the addition method is to eliminate one of the variables.

1. Write both equations in standard form ($Ax + By = C$).
2. If necessary, multiply one or both of the equations by appropriate nonzero number(s) so that the sum of the x -coefficients or the sum of the y -coefficients is 0.
3. Add the equations in step 2. The sum will be an equation in one variable.
4. Solve the equation.
5. Back-substitute the value you found in step 4 into either of the given equations and solve for the remaining variable.
6. Check. Write the solution as an ordered pair or give values of x and y .

Ex 4: Solve each system.

$$a) \begin{cases} 2x + 3y = 8 \\ 5x - y = 3 \end{cases}$$

$$b) \begin{cases} 3x - 3y = y - 9 \\ 5(x + y) = -15 \end{cases}$$

$$c) \begin{cases} 3x = 4(2 - y) \\ 3x - 6 + 4y = 0 \end{cases}$$

Ex 5: Three times a first number decreased by 5 times a second number is -9 . The first number is one less than the second number. Find the numbers.

Ex 6: For a linear function $f(x) = mx + b$, $f(2) = -1$ and $f(5) = 8$. Find the values of m and b .