1. Find the angle between the vectors $\mathbf{v} = 2\mathbf{i} + 2\mathbf{j} + \mathbf{k}$ and $\mathbf{w} = 2\mathbf{i} + 2\mathbf{j} - \mathbf{k}$.

   A. $\cos^{-1}\left(\frac{8}{9}\right)$ B. $\cos^{-1}\left(\frac{5}{9}\right)$ C. $\cos^{-1}\left(\frac{2}{3}\right)$ D. $\cos^{-1}\left(\frac{7}{9}\right)$ E. $\cos^{-1}\left(\frac{1}{3}\right)$

2. Find a such that $\mathbf{u} = 2\mathbf{i} - \mathbf{j} + a\mathbf{k}$ and $\mathbf{v} = \mathbf{i} + 4\mathbf{j} + 2\mathbf{k}$ are perpendicular.

   A. 3 B. 2 C. 1 D. -1 E. -2

3. If $\mathbf{w} = w_1\mathbf{i} + w_2\mathbf{j} + w_3\mathbf{k}$ is perpendicular to $\mathbf{u} = \mathbf{i} + \mathbf{j} - \mathbf{k}$ and $\mathbf{v} = 2\mathbf{i} + \mathbf{j} + \mathbf{k}$, and if $w_3 = 2$, then $w_1 =$

   A. 4 B. 2 C. -2 D. -4 E. 1

4. If $\mathbf{v} = \mathbf{i} + \mathbf{j} + \mathbf{k}$ and $\mathbf{w} = 2\mathbf{i} - \mathbf{k}$, find $|\text{proj}_{\mathbf{v}}(\mathbf{w})|$.

   A. $1/\sqrt{3}$ B. $\sqrt{3}$ C. $\sqrt{3}/5$ D. $2\sqrt{3}$ E. $\sqrt{3}/2$

5. Find the area of the triangle with vertices $P = (0, 0, 0)$, $Q = (1, 2, 1)$, and $R = (2, 1, -1)$.

   A. $\sqrt{27}$ B. $\frac{\sqrt{27}}{2}$ C. $\frac{\sqrt{11}}{2}$ D. $\sqrt{19}$ E. $\frac{\sqrt{3}}{2}$

6. The radius of the sphere $x^2 + y^2 + z^2 + 2x + 4y - 6z = 3$ is

   A. $3 + \sqrt{13}$ B. $\sqrt{13}$ C. $\sqrt{65}$ D. $3 + \sqrt{56}$ E. $\sqrt{17}$

7. The area of the region enclosed by the curves $y = x^2 + 1$ and $y = 2x + 9$ is given by

   A. $\int_{-2}^{4} (x^2 + 1 - 2x - 9)\, dx$ B. $\int_{-2}^{4} (2x + 9 - x^2 - 1)\, dx$ C. $\int_{-2}^{2} (2x + 9 - x^2 - 1)\, dx$ D. $\int_{-4}^{2} (2x + 9 - x^2 - 1)\, dx$ E. $\int_{-4}^{2} (x^2 + 1 - 2x - 9)\, dx$

8. Let R be the region between the graphs of $y = x^2$ and $y = x$. Find the volume of the solid generated by revolving R about the x–axis.

   A. $\frac{\pi}{6}$ B. $\frac{\pi}{12}$ C. $\frac{\pi}{4}$ D. $\frac{\pi}{15}$ E. $\frac{2\pi}{15}$

9. If the region in problem 8 is revolved about the y–axis, then the volume of the solid is

   A. $\frac{\pi}{6}$ B. $\frac{\pi}{12}$ C. $\frac{\pi}{24}$ D. $\frac{2\pi}{15}$ E. $\frac{\pi}{15}$

10. If R is the region bounded by the curves $x = 0$ and $x = y - y^2$, and if R is revolved around the y–axis, then the volume of the solid is

    A. $\frac{\pi}{15}$ B. $\frac{\pi}{30}$ C. $\frac{\pi}{12}$ D. $\frac{\pi}{3}$ E. $\frac{\pi}{6}$

11. A force of 4 lb. is required to stretch a spring 1/2 ft. beyond its natural length. How much work is required to stretch the spring from its natural length to 2 ft.

A. 8 ft–lbs. B. 12 ft–lbs. C. 16 ft–lbs. D. 24 ft–lbs. E. 32 ft–lbs.

12. A cylindrical tank of height 4 feet and radius 1 foot is filled with water. How much work is required to pump all but 1 foot of water out of the tank. (Density = 62.5 lbs./ft^3)

A. $9\pi(62.5)$ ft-lbs. B. $3\pi(62.5)$ ft-lbs. C. $\frac{9\pi}{2}(62.5)$ ft-lbs. D. $18\pi(62.5)$ ft-lbs. E. $6\pi(62.5)$ ft-lbs.

13. Let $f(x) = \sqrt{x}$. Find c in $[0, 9]$ such that $f(c) = f_{\text{avg}}$, where f_{avg} is the average value of $f(x) = \sqrt{x}$ on the interval $[0, 9]$.

A. $c = 4$ B. $c = 4.5$ C. $c = 5$ D. $c = 3.2$ E. $c = 6$.

14. $\displaystyle\int x(\ln x)^3 dx = \frac{x^2}{2}(\ln x)^3 - I$, where $I =$

A. $\displaystyle\frac{1}{4}\int(\ln x)^4 dx$ B. $\displaystyle\frac{1}{3}\int(\ln x)^2 dx$ C. $\displaystyle\frac{1}{3}\int(\ln x)^2 dx$ D. $\displaystyle\frac{3}{2}\int x^2(\ln x)^2 dx$ E. $\displaystyle\frac{3}{2}\int x(\ln x)^2 dx$

15. Evaluate $\displaystyle\int_0^1 xe^{3x} dx$.

A. $\displaystyle\frac{2e^3}{9}$ B. $\displaystyle\frac{1}{9}+\frac{2e^3}{9}$ C. 1 D. $\displaystyle\frac{1}{9}$ E. $\displaystyle\frac{e^3}{9} - 1$

16. $\displaystyle\int_0^{\pi/2} \sin^3 x dx =$

A. 2/3 B. 4/3 C. 0 D. 1/4 E. 1/3

17. $\displaystyle\int_0^{\pi/4} \sec^4 x \tan x \, dx =$

A. 1 B. 1/3 C. 4/3 D. 3/4 E. 2/9

18. In order to compute $\displaystyle\int \frac{dx}{(1 + x^2)^{3/2}}$ we make the substitution $x = \tan\theta$. This gives an integral in θ whose value is

A. $\frac{1}{2}\theta + \frac{1}{2}\sin\theta\cos\theta + C$ B. $\ln(\sec^2\theta) + C$ C. $\frac{1}{2}\theta + \tan^{-1}\theta + C$ D. $\frac{1}{2}\sqrt{\cos\theta} + C$ E. $\sin\theta + C$

19. $\displaystyle\int \frac{dx}{\sqrt{9-4x^2}} =$

A. $\sec^{-1}\left(\frac{3x}{2}\right) + C$ B. $\frac{1}{2}\sin^{-1}\left(\frac{2x}{3}\right) + C$ C. $\tan^{-1}\left(\frac{2x}{3}\right) + C$ D. $\frac{1}{3}\sin^{-1}\left(\frac{3x}{2}\right) + C$
E. $\sqrt{9-4x^2} + \tan^{-1}\left(\frac{2x}{3}\right) + C$

20. $\displaystyle\int \frac{x+1}{x^3-2x^2+x}\, dx =$

A. $\ln|x| + \ln|x-1| + C$ B. $\ln|x| - \ln|x-1| + C$ C. $\ln|x| - \dfrac{2}{x-1} + C$
D. $\ln|x-1| - \dfrac{2}{x-1} + C$ E. $\ln|x| - \ln|x-1| - \dfrac{2}{x-1} + C$

21. A partial fraction decomposition of $\dfrac{x+2}{x^4+2x^2}$ has the form

A. $\dfrac{A}{x} + \dfrac{B}{x^2} + \dfrac{Cx+D}{x^2+2}$ B. $\dfrac{A}{x^2} + \dfrac{Bx+C}{x^2+2}$ C. $\dfrac{A}{x} + \dfrac{B}{x^2} + \dfrac{C}{x^2+2}$ D. $\dfrac{A}{x^2} + \dfrac{B}{x^2+2}$
E. $\dfrac{A}{x} + \dfrac{B}{x^2+2}$

22. $\displaystyle\int_0^1 \frac{x+2}{x^2+1}\, dx =$

A. $\dfrac{\ln 2}{2} + \dfrac{\pi}{2}$ B. $\dfrac{\ln 2}{2}$ C. $\dfrac{\ln 2}{2} + 2\pi$ D. $2\ln 2 + \dfrac{\pi}{2}$ E. $\ln 2 + \pi$

23. Use the Trapezoidal Rule with $n = 3$ to approximate $\displaystyle\int_0^1 \frac{1-x}{1+x}\, dx$

A. $\dfrac{12}{5}$ B. $\dfrac{6}{5}$ C. $\dfrac{2}{5}$ D. $\dfrac{17}{60}$ E. $\dfrac{17}{10}$

24. Indicate convergence or divergence for each of the following improper integrals:

$$\text{(I)} \int_2^\infty \frac{1}{(x-1)^2}\, dx \qquad \text{(II)} \int_0^2 \frac{1}{(x-1)^2}\, dx \qquad \text{(III)} \int_0^1 \frac{\ln x}{x}\, dx$$

A. I converges, II and III diverge. B. II converges, I and III diverge. C. I and III converge, II diverges. D. I and II converge, III diverges. E. I, II and III diverge.

25. Find the length of the curve $y = \dfrac{2}{3}x^{3/2},\ 0 \le x \le 2.$

A. $2\sqrt{3} - 2$ B. $3\sqrt{3} - 1$ C. $\sqrt{3} - 1$ D. $\dfrac{2}{3}\left(3\sqrt{3} - 1\right)$ E. $3\sqrt{3} - 2$

26. If the curve $y = e^{2x}$, $0 \le x \le 1$, is revolved about the y–axis, then the area of the surface obtained is

A. $\displaystyle\int_0^1 2\pi\sqrt{1+4e^{4x}}\,dx$ B. $\displaystyle\int_0^1 2\pi e^{2x}\sqrt{1+e^{2x}}\,dx$ C. $\displaystyle\int_0^1 2\pi x\sqrt{1+4e^{4x}}\,dx$

D. $\displaystyle\int_0^1 2\pi e^{2x}\sqrt{1+4e^{4x}}$ E. $\displaystyle\int_0^1 2\pi e^{4x}\sqrt{1+e^{4x}}\,dx$

27. Find the centroid $(\overline{x}, \overline{y})$ of the region bounded by the x–axis and the semicircle $y = \sqrt{4-x^2}$.

A. $\left(0, \dfrac{8}{3\pi}\right)$ B. $\left(\dfrac{8}{3\pi}, 0\right)$ C. $\left(0, \dfrac{2}{3\pi}\right)$ D. $\left(\dfrac{2}{3\pi}, 0\right)$ E. $(0, 0)$

28. Evaluate $\displaystyle\lim_{n\to\infty}\left(1+\frac{(-1)^n}{n}\right)$.

A. 0 B. 1 C. -1 D. 2 E. The limit does not exist.

29. Evaluate $\displaystyle\lim_{n\to\infty}\left(n^{1/n}+\frac{1}{n!}\right)$.

A. 0 B. 1 C. e D. $1/e$ E. The limit does not exist.

30. $\displaystyle\sum_{n=0}^{\infty} 5\left(-\frac{4}{5}\right)^n =$

A. 1/9 B. 5/9 C. 25/9 D. 5 E. 25

31. If $\displaystyle L = \sum_{n=1}^{\infty}\frac{1}{2^n} + \sum_{n=0}^{\infty}\frac{(-1)^n}{2^n}$, then $L =$

A. 1/3 B. 2/3 C. 1 D. 4/3 E. 5/3

32. Find all values of p for which $\displaystyle\sum_{n=1}^{\infty}\frac{1}{(n^2+1)^p}$ converges.

A. $p > 1$ B. $p \le 1$ C. $p \ge 1$ D. $p > 1/2$ E. $p \le 1/2$

33. $\displaystyle\sum_{n=1}^{\infty}\left(1+\frac{1}{n}\right)^p$ converges for:

A. $p \le 1$ B. $p > 1$ C. $p < 0$ D. $p > 0$ E. No values of p.

34. Which of the following series converge conditionally?

(I) $\displaystyle\sum_{n=1}^{\infty}\frac{(-1)^n}{n^2}$ (II) $\displaystyle\sum_{n=2}^{\infty}\frac{(-1)^n n}{\ln n}$ (III) $\displaystyle\sum_{n=1}^{\infty}\frac{(-1)^n n}{e^n}$

A. II only. B. I and III only. C. I and II only. D. All three. E. None of them.

35. Which of the following series converge?

$$\text{(I)} \sum_{n=1}^{\infty} \frac{(-1)^n}{n^{1/4}} \qquad \text{(II)} \sum_{n=1}^{\infty} \frac{n!}{1\cdot 3\cdot 5\cdots(2n-1)} \qquad \text{(III)} \sum_{n=1}^{\infty} \frac{4}{3}\left(\frac{1}{2}\right)^n$$

A. II only. B. I and III only. C. I and II only. D. All three. E. None of them.

36. Find the interval of convergence of the power series $\displaystyle\sum_{n=1}^{\infty} \frac{3^n x^n}{n \ln n}$.

A. $-\frac{1}{3} \le x < \frac{1}{3}$ B. $-\frac{1}{3} < x \le \frac{1}{3}$ C. $0 \le x \le \frac{1}{3}$ D. $-1 \le x < 1$ E. $-3 < x < 3$

37. Find the interval of convergence of the power series $\displaystyle\sum_{n=1}^{\infty} \frac{n}{5^n}(x-2)^n$.

A. $-5 < x < 5$ B. $3 < x < 7$ C. $-2 < x < 2$ D. $-3 \le x < 7$ E. $-3 < x < 7$

38. Find the first three terms of the Maclaurin series of $f(x) = \ln(1+x)$

A. $x + \frac{x^2}{2} + \frac{x^3}{3}$ B. $x - \frac{x^2}{2} + \frac{x^3}{3}$ C. $x + \frac{x^2}{2!} + \frac{x^3}{3!}$ D. $x - \frac{x^2}{2!} + \frac{x^3}{3!}$ E. $x + \frac{2x^2}{3!} + \frac{3x^3}{4!}$

39. If $f(x) = \displaystyle\sum_{n=0}^{\infty} \frac{n^2(x-2)^n}{n+1}$, then $f^{(3)}(2) =$

A. $\frac{9}{24}$ B. $\frac{27}{2}$ C. 0 D. 27 E. $\frac{9}{4}$

40. $\displaystyle\int_0^x te^{t^3}\,dt =$

A. $\displaystyle\sum_{n=0}^{\infty} \frac{x^{2n}}{(2n)!}$ B. $\displaystyle\sum_{n=0}^{\infty} \frac{x^{3n}}{3n(n!)}$ C. $\displaystyle\sum_{n=0}^{\infty} \frac{x^{4n+1}}{(4n+1)!}$ D. $\displaystyle\sum_{n=0}^{\infty} \frac{x^{4n+1}}{(4n+1)(n!)}$ E. $\displaystyle\sum_{n=0}^{\infty} \frac{x^{3n+2}}{(3n+2)(n!)}$

41. Use the power series representation of $\sin x$ to find the first three terms of the Maclaurin series of $f(x) = x\sin(x^2)$

A. $x^3 + \frac{x^7}{3!} + \frac{x^{11}}{5!}$ B. $x + \frac{x^3}{3} + \frac{x^5}{5}$ C. $x^3 - \frac{x^7}{3!} + \frac{x^{11}}{5!}$ D. $x - \frac{x^3}{3} + \frac{x^5}{5}$ E. $x^3 - \frac{x^7}{3} + \frac{x^{11}}{5}$

42. Find the fourth term of the Maclaurin series of $f(x) = \dfrac{x^2+3}{x-1}$.

A. $-x^3$ B. $3x^3$ C. $-3x^3$ D. $-4x^3$ E. $4x^3$

43. The fourth term of the Taylor series of $f(x) = \ln x$, centered at $a = 2$, is

A. $\frac{1}{6}(x-2)^3$ B. $\frac{1}{12}(x-2)^3$ C. $\frac{1}{24}(x-2)^3$ D. $-\frac{1}{3}(x-2)^3$ E. $-(x-2)^3$

44. Using Maclaurin series and the Alternating Series Estimation Theorem, we can obtain the approximation

$$\int_0^{0.1} e^{-x^2}\,dx \approx \frac{1}{10} - \frac{1}{3000},$$

with error $\leq E$, where the value of E is

A. 10^{-5} B. 10^{-6} C. $\frac{1}{2}10^{-6}$ D. $\frac{1}{7}10^{-7}$ E. $\frac{1}{2}10^{-5}$

45. Parametric equations of a curve C are

$$x = 2\cos t, \qquad y = 3\sin t, \qquad 0 \leq t \leq \frac{\pi}{2}.$$

The curve C is:

A. A quarter of a circle. B. An ellipse. C. Half of an ellipse.
D. Half of a circle. E. A quarter of an ellipse.

46. Find the slope of the tangent line at the point $(2/3, 3)$ for the curve parameterized by $x = 2t^3/3,\ \ y = t^2 + 2t$.

A. 2/3 B. 2 C. 4/3 D. 4 E. 3

47. Find the length of the parametric curve

$$x = \frac{1}{2}t^2, \qquad y = 2 + \frac{1}{3}t^3, \qquad 0 \leq t \leq \sqrt{3}.$$

A. 21/4 B. 7/2 C. 7/3 D. 14/3 E. 8/3

48. A point P has polar coordinates $(3, \pi/4)$. Which of the following are also polar coordinates of P?

(I) $(-3, -\pi/4)$ (II) $(-3, 5\pi/4)$ (III) $(3, -7\pi/4)$ (IV) $(3, -5\pi/4)$

A. I and II only. B. I and III only. C. I and IV only.
D. II and III only. E. II and IV only.

49. The polar graph of $r = \dfrac{1}{\sin\theta + \cos\theta}$ is:

A. a parabola. B. a line. C. a cardioid. D. a rose. E. an ellipse.

50. The graph of $y^2 = 12x$ is a parabola whose focus is the point $(3, 0)$. The point $P = (12, 12)$ lies on the parabola. Find the distance from P to the directrix.

A. $\sqrt{481}$ B. $\sqrt{425}$ C. $\sqrt{306}$ D. 15 E. 12

51. The ellipse $(x-2)^2 + \dfrac{(y-1)^2}{9} = 1$ has one vertex at

A. $(1,5)$ B. $(5,1)$ C. $(2,1)$ D. $(2,4)$ E. $(2,10)$

52. Find an equation for the hyperbola with foci $(\pm 3, 0)$, and asymptotes $y = \pm \dfrac{x}{2}$.

A. $20y^2 - 5x^2 = 36$ B. $5x^2 - 20y^2 = 36$ C. $x^2 - 4y^2 = 4$
D. $4y^2 - x^2 = 4$ E. $5x^2 - 4y^2 = 1$

53. Write the complex number $\dfrac{3-4i}{1+2i}$ in the form $a + bi$.

A. $-1-2i$ B. $1+2i$ C. $2-i$ D. $3-2i$ E. $3+i$

54. Write the complex number $\sqrt{3} - i$ in polar form with argument between 0 and 2π.

A. $4\left(\cos\dfrac{\pi}{3} + i\sin\dfrac{\pi}{3}\right)$ B. $2\left(\cos\dfrac{5\pi}{6} + i\sin\dfrac{5\pi}{6}\right)$ C. $4\left(\cos\dfrac{\pi}{4} + i\sin\dfrac{\pi}{4}\right)$
D. $2\left(\cos\dfrac{11\pi}{6} + i\sin\dfrac{11\pi}{6}\right)$ E. $2\left(\cos\dfrac{\pi}{6} + i\sin\dfrac{\pi}{6}\right)$

Answers

1. D; 2. C; 3. D; 4. A; 5. B; 6. E; 7. B; 8. E; 9. A; 10. B

11. C; 12. C; 13. A; 14. E; 15. B; 16. A; 17. D; 18. E; 19. B; 20. E

21. A; 22. A; 23. C; 24. A; 25. D; 26. C; 27. A; 28. B; 29. B; 30. C

31. E; 32. D; 33. E; 34. E; 35. D; 36. A; 37. E; 38. B; 39. B; 40. E

41. C; 42. D; 43. C; 44. B; 45. E; 46. B; 47. C; 48. D; 49. B; 50. D

51. D; 52. B; 53. A; 54. D