MA 22000 Lesson 14 Notes, Section 2.1 (2nd half of text)
The Derivative and the Slope of a Graph

Tangent Line to a Graph: (You might want to read paragraphs 1 and 2 on page 82 of lesson 2.1.) We know that slope of a line is a ratio that describes the rate at which a line is rising or falling. For graphs other than lines, the rate of which the graph rises or falls changes from point to point. For example, looking at figure 2.1 on page 82 (the graph of $f(x)$, a parabola), the parabola is rising more quickly at point (x_1, x_2) than at the point (x_2, y_2). At the vertex, the graph 'levels off', then the graph begins to fall.

To determine the rate at which a graph rises or falls at a single point, we use the slope of the tangent line at the point.

Definition of the slope of a Graph: The **slope** m of the graph f at the point $(x, f(x))$ is equal to the slope of its tangent line at $(x, f(x))$, and is given by the limit definition below (provided the limit exists).

$$m = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x} \text{ or } \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Note: Some textbooks use the symbol Δx and others use the symbol h to represent a small change in x.

Examples: Find the slope of the graph of each function at the given point using the limit definition of slope above.

Ex 1: $f(x) = \frac{1}{2}x^2 - 3$ at the points $(-2, -1)$ and $(4, 5)$

Ex 2: $g(x)=x^2-3x$ at the points $(2,-2)$ and $(-3,18)$

Definition of the DERIVATIVE: The **derivative** of a function *f* at a value *x* if given by

$$f'(x)=\lim_{\Delta x\to 0}\left(\frac{f(x+\Delta x)-f(x)}{\Delta x}\right)$$

provided this limit exists. A function is **differentiable** at *x* if its derivative exists at *x*. The process of finding derivatives is called **differentiation.**

All of the following notations, in addition to $f'(x)$ can be used to denote the derivative of *y*= *f* (*x*) (the derivative of *y* with respect to *x*). The most common are

$$\frac{dy}{dx},\ \ y',\ \ \frac{d}{dx}(f(x)),\text{ and } D_x[y].$$

Ex 3: Find each derivative using the limit definition.

$a)$ $f(x)=2x^2+3x$

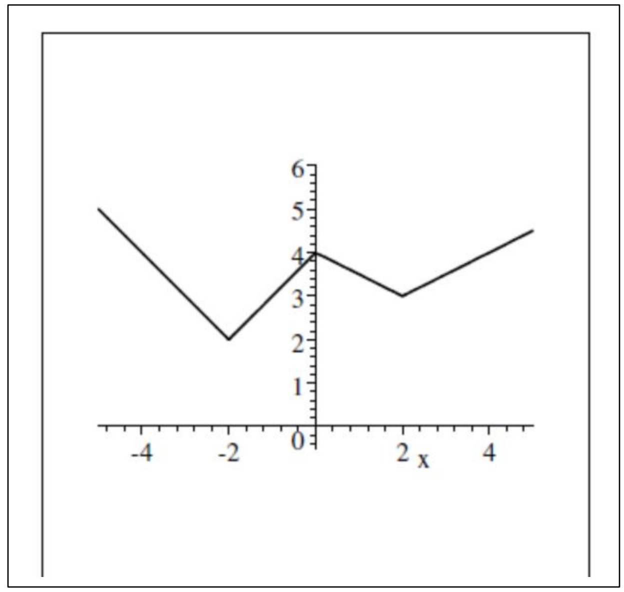
$b)$ $y=\dfrac{4}{x}$

Not every function is differentiable at every point. Here are some common examples of situations at an value *x* where a function *f* (*x*) would **not** have a derivative (not be differentiable at *x*).

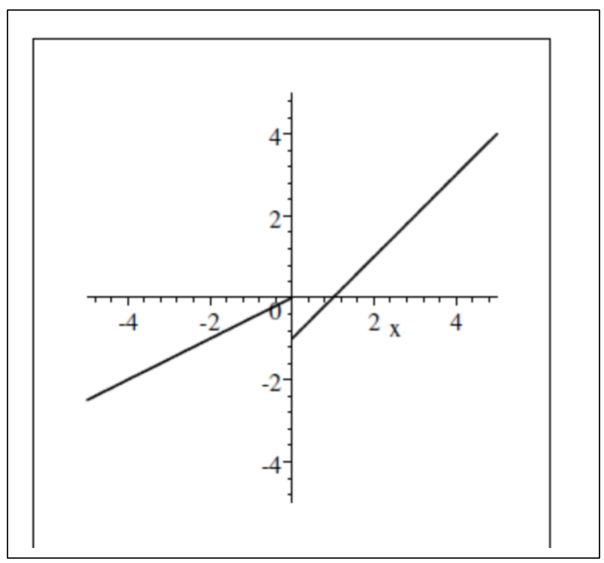
1) At that value *x*, the tangent line to the graph is a vertical line. (See picture page 89).
2) There is a discontinuity (gap) in the graph at a given value of *x*. (See page 89.) This includes an 'endpoint'.
3) At a point with a given *x* value, there is a 'cusp ' or 'node' (sharp edge, not a smooth curve). (See picture page 89.)

Describe the *x*-values at which the function if differentiable. Explain.

Ex 4:



Ex 5:



Ex 6: